THE EFFECTS OF COOPERATIVE LEARNING ON MOTIVATION AND PERFORMANCE OF GRADE 11 HIGHER LEVEL MATHEMATICS LEARNERS IN THE OSHANA EDUCATION REGION

A THESIS SUBMITTED IN PARTIAL FULFILMENT

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By:

Frans N. Haimbodi

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Main Supervisor: Prof. C.D. Kasanda (UNAM)

Co-Supervisor: Dr. H.M. Kapenda (UNAM)

APPROVAL PAGE

This research has been examined and is approved as meeting the required standards for partial fulfilment of the requirements of the degree of Master of Education.

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Internal Examiner	Date
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Internal Examiner	Date
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Dean: Faculty of Education	Date
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External Examiner	Date

DECLARATION

I, Frans Ndemupondaka Haimbodi, declare hereby that **"The effects of cooperative learning on performance and motivation of grade 11 higher level Mathematics learners in the Oshana Education Region"** is a true reflection of my own research, and that this work, or part thereof has not been submitted for a degree in any other institution of higher education.

Date: ……..

Frans Ndemupondaka Haimbodi

DEDICATION

To my son, Tonata Justice Haimbodi who was born during the first year of this course.

To my mother, Victoria Ndamana Petrus for her unfailing love, good upbringing and unconditional support from birth to today.

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Lastly but most crucially, I want to glorify the Almighty for not only giving me sanity, but also the strength to undertake this project.

ABSTRACT

The national Grade 12 Mathematics results have been poor over the years since independence. Recent studies on factors contributing to poor results in Mathematics pointed to ineffective teaching methods, insufficient resources and learners' low motivation to study.

This study sought to determine the effects of cooperative learning on the performance and motivation of the Grade 11 learners doing Mathematics on higher level in the Oshana education region. The study used a quasi-experimental design. Two Grade 11 classes (each comprising 31 learners) doing Mathematics on a higher level from one school in the Oshana region were purposefully selected; one as a control group and the other as an experimental group.

Motivation was measured using a modified Fennema-Sherman Attitude Scale, and concentrated on three subscales; Usefulness of Mathematics, Confidence in Mathematics and Effectance Motivation in Mathematics.

The instruments used to measure the performance in Mathematics were a pre-test and post test. Prior to collection of the data, a pilot study was carried out in a different school to gather information on the appropriateness of the instruments and other administrative logistics, in order to improve the quality and efficiency of the study instruments (Lancaster, Dodd, & Williamson, 2004). Discrepancies where found in the instruments

and corrected before the main study. During the main study, the experimental and control groups were separately taught a topic from the higher level Mathematics syllabus, *Differentiation*.

The t-test was used to find out whether significant differences existed in the motivation and performance of the control and experimental groups. The results showed that significant differences in performance and in the motivation level of the experimental and control group existed at the 0.01 significant level. The findings suggested that cooperative learning improved learners' performance in Mathematics and also increased their motivation of learning Mathematics.

The study recommended that Mathematics teachers should place emphasis on learners' understanding of particular concepts, guiding learners in active learning, providing opportunities for discussion and elaboration and encouraging them to work with peers to enhance learners' motivation and academic performance.

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LIST OF ACRONYMS

BETD	Basic Education Teacher Diploma
BES III	Basic Education Support III
DNEA	Directorate of National Examinations and Assessment
F-SMAS	Fennema-Sherman Mathematics Attitude Scales
IGCSE	International General Certificate of Secondary Education
INSTANT	In-Service Training Assistance to Namibian Teachers
MASTEP	Mathematics and Science Teachers Extension Programme
MEC	Ministry of Education and Culture
NIED	National Institute for Educational Development
NSSC	Namibian Secondary School Certificate
PoN	Polytechnic of Namibia
UNAM	University of Namibia

CHAPTER ONE: BACKGROUND

Introduction

The Namibian Government attaches great significance to the teaching of Mathematics in Namibian schools. "Mathematics is indispensable for the development of science, technology and commerce" (National Institute for Education Development, [NIED], 2010a, p.18). Mathematical skills, knowledge, concepts and processes, enable the individuals to investigate, model, and interpret numerical and spatial relationships and patterns that exist in the world (NIED, 2009; Iyambo, 2010). Mathematics is an entry requirement at tertiary institutions for courses such as medicine, geology, engineering and information technology and Namibia needs experts in these fields in order to accelerate development and economic growth (Iyambo, 2010). The value attached to Mathematics led to the reform policy that Mathematics was to become a compulsory subject for every child in Namibian schools (NIED, 2010a), at the beginning of 2012.

The Ministry of Education and Culture [MEC] (1993) points out that education in Namibia should utilise a learner centred approach – a methodology that promotes learning through active learner participation.

> Our children need to learn to think independently and critically. They must master strategies for identifying, analysing, and solving problems. Most important, they must develop self confidence. Our teaching must be learner-centred: a methodology that promotes learning through understanding and

> practice directed towards autonomous mastery of living conditions (MEC, 1993:120).

In learner centred education, teachers need to view learners as active human beings, and should thus structure learning activities that incite curiosity in learners to explore and gain knowledge and skills to master their surrounding world. The proponents of learner centred education maintain that learners go to school with a wealth of knowledge and social experiences gained from interactions with the environment and co-inhabitants in their communities (NIED, 2003). NIED (2010a) therefore urges teachers to ensure that the learning process allows learners to communicate and interact with their fellows so that they can learn from each other.

The learner centred education presupposes that teachers have a holistic view of the learner, valuing the learner's experiences as the starting point of their lessons (NIED, 2003). NIED (2010a) thus urges teachers to select the content and teaching methods based on the analysis of learners' needs, use local and natural resources as alternatives or supplement to ready-made study materials and thus develop their own and the learners' creativity. Amutenya (2002) noted that active learner participation in classrooms directs learning by enabling learners to understand, share information and learn through a productive process. She further adds that teachers must have a sense of commitment, confidence, a reflective attitude, critical curiosity, problem-solving skills and a sense of empowerment to enable them to employ active learning strategies in their classrooms.

Mathematics Education in Namibia

The teaching of Mathematics in Namibia has been a challenge since independence in 1990 (NIED, 2009). The learners' performance in Mathematics at the Grade 12 level has been unimpressive. Several studies have been conducted in the country to investigate the expertise of teachers and seek for possible solutions (Haufiku, 2008). According to Haufiku, these studies identified teachers' inadequate Mathematics content knowledge in Mathematics teachers as a predicament to Mathematics education, and as a result interventions to upgrade the skills of the Mathematics teachers were made. These interventions included the In-service Training and Assistance for Namibian Teachers (INSTANT), the Mathematics and Science Teachers Extension Programme [MASTEP] and the Basic Education Support III [BES III] (Haufiku, 2008). These interventions also aimed to train Mathematics teachers to use the learner centred-approach in their classrooms. According to Peters (2006), these interventions did not improve the teaching of Mathematics in the country and learner-centred education became a threat to teachers as they had no clear notion of what direction the education system was taking. The Mathematics teachers thus switched to group work instructions confusing it with learner-centred education (Peters, 2006).

The learner-centred approach is still being enforced in Namibian schools; "preparation for a knowledge-based society requires a learner-centred approach to teaching and learning" (NIED, 2008, p. 4). Nonetheless, Namibia has been experiencing poor performance in Mathematics (Iiyambo, 2010; NIED, 2010a). A total of 15809 pupils set

for Grade 12 Mathematics examinations in 2011, 96.48% of these pupils wrote Mathematics on ordinary level (Directorate of National Examinations and Assessments [DNEA], 2011). Despites that a higher number of these learners wrote ordinary level, only 20% of them obtained C - symbol and above. The University of Namibia (UNAM) requires a minimum C – symbol for entry to science related fields.

Namibian Mathematics teachers are being encouraged to employ pedagogical methods that promote the active involvement of students in their own learning with the hope of improving the national results in Mathematics (Iyambo, 2010). Iyambo maintains that it is necessary that teachers employ instructional approaches that allow learners to participate actively in their own learning and create a social setting in which learners learn problem solving skills through interactions with their fellows.

According to Peters (2006), group work is almost the only method that teachers related with learner-centred education. Learners were allocated to groups in most of the Mathematics lessons, however, and in many such situations learners were not guided to collaborate effectively as they seek to acquire knowledge, skills and values through classroom activities (NIED, 2003).

Cooperative learning is one of the learner centred teaching methods that could be used to teach Mathematics. It is an instructional use of a group of three to five so that learners work together to maximize their own and each others' learning (Dunne & Bennett, 1990). Phillip (1999) defines cooperative learning as an instructional strategy in which

groups of three to five learners work together on activities that are carefully designed to promote comprehension as well as individual responsibility.

Phillip (1999) justifies that when students work in a small group (3 to 4 learners) with their peers it provokes discussions about plausible choices on different strategies to solve a problem, and necessitates a discussion on the merits of those strategies if one approach is to be settled on. When a student works on a problem alone, the first plausible option is most likely to be chosen and the discussion on the merits of different approaches which should take place internally, may not take place (Phillip, 1999).

Iyambo (2010) urges teachers to motivate their learners to study Mathematics and be able to further their education in science related fields such as geology, engineering and Information Technology. According to Nwihim (2007), motivation is an internal process that activates, guides and maintains the behaviours of an individual. A learners is motivated to learn if that learner has an interest to explore the meanings in academic activities and hence tries to find and study more related activities until he/she masters the content (Good & Brophy, 1997). Learning motivation have been linked to increased learning engagement and higher levels of student success (Broussard, 2002). Broussard argues that motivated learners tend to have autonomy over their learning and higher self-efficacy which makes them feel worth achieving higher academic scores.

When a person is motivated to learn, they become engaged and positively committed in studying course materials and working on activities (Good & Brophy, 1997). Learners are motivated by the methods that take into consideration all of the learners' concerns

and the factors that have an effect on the learners' lives, and not just educational needs (Stamler, 2007). Motivation is thus an important factor in learning Mathematics as without a positive commitment to learning, students are likely to do poorly in their school subjects.

According to Nwihim (2007), the decline of performance in school subjects might be due to the lack of motivation in learners. Cobb (2005) argues that when learners work cooperatively, they share their ideas and listen to other learners' perspectives, seek new ways of clarifying differences, resolving problems, and constructing new understandings and knowledge. The result is that students attain higher academic outcomes and are more motivated to achieve than they would be if they worked alone.

Theoretical framework

This study draws upon the theory of social constructivism. Vygotsky (1986) as cited in Fosnot (2005) argues that a key factor in social constructivism is that the children's development is enhanced by participating in activities that are slightly above their level of competence with mastery occurring as a result of help from others; which is enhanced as students are given opportunities to teach each other and practise in a social context. Vygotsky (1978) maintains that learning is a social process in which students actively participate and contribute with ideas and arguments. Learners who solve problems in groups, if structured effectively, gain better understanding and achieve better results than learners who work individually. This technique also allows learners to take responsibility for their own learning. It is claimed that during group work learners

achieve far more than they would when working individually (Cooper, 2010). Interactions among students are crucial to cooperative learning and it is the interactions that occur in the groups that help to inspire the learning motivation (Cobb, 2005).

Vygotsky (1978) believes that knowledge is constructed using prior-knowledge, through language as well as experiences, beliefs and culture, in this way meaningful learning has to take place. Vygotsky also explains that the learner is capable of constructing new knowledge with the help of others who are more knowledgeable. This means that learners learn best through interacting with their peers, teachers and others. Therefore, constructivism is an approach to teaching and learning which emphasizes that learning is both an individual and social process.

Cooper (2010) maintained that Vygotsky‘s theory is possibly the most useful theoretical framework if one wants to study learning in small groups and concluded that Mathematics educators should encourage small group work in their teaching.

The study also draws upon the motivational theories especially Harter’s effectance (or Mastery) motivation theory. Effectance motivation is defined as a general tendency to interact with and to express influence over the environment, enhancing perceptions of competence and perceived internal control over outcomes, giving the individual pleasure, and ultimately increasing motivation (Harter, 1983). Harter proposed a model of effectance motivation, describing the effects of both success and failure experiences on effectance motivation. The goals of effectance motivation are acquiring competence and influencing one’s environment (Broussard, 2002).

Johnson & Johnson (1989) found that as learners become more engaged in their learning, their motivation increases. To be motivated to learn, learners need ample opportunity to interact with each other as well as steady encouragement and support of their learning efforts. By placing learners in groups and giving them tasks that require interdependence, each member of the group becomes accountable for achieving a shared goal. Learners are then motivated by the team effort as well as by seeing their own contributions accepted by the group. The active exchange of ideas within small groups of students does not only increase interest among the students, but also promotes critical thinking and the discovery of knowledge (Johnson & Johnson 1989).

Harter's effectance motivation theory is important for this study because it includes the effects of learner interaction with their environment on motivation to study. Learners with effectance motivation want to gain competence in their school subjects (Broussard, 2002). Their goal is to actually learn the content, and classrooms oriented to this goal will encourage learners to master tasks and develop intellectually. Cooperative learning therefore gives room for the development and nursing of effectance motivation.

Self-Efficacy Theory

The self-efficacy construct is a major part of Bandura's (1997) broader social cognitive model of learning and development. Bandura defines self-efficacy as the confidence learners have in their ability to organize and accomplish a given task or to solve a Mathematics problem. He emphasizes self-efficacy perceptions as a major influence on individuals' achievement strivings, including performance, choice, and persistence.

Bandura's (1997) distinguished two kinds of expectancy beliefs as the outcome expectations, or beliefs that certain behaviours, like practice, will lead to certain outcomes, like improved performance, and efficacy expectations, or beliefs about whether one can perform the behaviours necessary to produce the outcome. Learners may believe that certain activities will produce a good result, but may not believe they can do these activities. Bandura therefore proposed that individuals' efficacy expectations rather than outcome expectancies are the major determinant of goal setting, activity choice, willingness to expend effort, and persistence (Bandura, 1997).

The self efficacy theory connects with the present study as Bandura (1997) proposed that verbal encouragement by others can increase self-efficacy. When teachers employ cooperative learning strategies in their classrooms, learners are given chances to collaborate and may encourage each other since the success of each group depends on the success of individual members.

Expetancy-Value theory

The study also drew upon the Expectancy-Value Theory of Motivation (Hodges, 2004).This is a general notion that learners expect certain outcomes from behaviours and the more learners value a behaviour, the more they are likely to perform well in it (Hodges, 2004). Learners want to score good grades and when motivated tend to study comprehensively and perform well. Hodges (2004) further argued that Expectancy-

Value theory depends on the self-esteem of learners and is assured through valuing the expected results of the activities.

Learners are likely to do well in Mathematics when they value the subject. This can range from seeing the subject as important in their lives after school and the subjects' applicability to situations in real live. The aspect of cooperative learning entails tasking learners to work with materials and discover meaning through collaborations. This may allow learners to relate to real life contexts and subsequently value the outcome of Mathematics activities.

In the experimental group of this study, the social constructivism theory was used as it relates to "individuals cooperatively building or constructing their own notions of reality out of their experiences, and that these constructions result in knowledge" (Malin, 2007). Malin noted that cooperative learning provided the environment and opportunity for learners, with their teachers, to engage, explore and integrate concepts with previous knowledge and help each other to construct new meaning and understanding.

Statement of the problem

The Grade 12 national Mathematics results has been very poor over the past few years. In 2011, 80% of the pupils who wrote Mathematics obtained symbols below C, which is the minimum entry requirement to science related fields at UNAM and the Polytechnic of Namibia (PoN).

A higher number of Grade 12 pupils have been taking Mathematics on ordinary level. In 2011, a total of 15809 pupils wrote Mathematics examinations but only 560 pupils, equivalent to 3.54%, wrote Mathematics on higher level (DNEA, 2011). It appears that many schools in Namibia do not offer Mathematics on higher level. Of the 560 pupils that wrote Mathematics on higher level, only 31 pupils were from the Oshana education region, which was 5.53% of the total, all the 31 learners were graded (DNEA, 2011). In 2011, only two schools in the Oshana education region offered Mathematics on higher level. According to Iyambo (2010), other than the poor performance of learners at ordinary level Mathematics, there is a dilemma that too few learners take up Mathematics at higher level countrywide. It appears that many learners did Mathematics on ordinary level and obtained poor symbols.

Recent studies (NIED, 2010a; Nambira, Kapenda, Tjipueja, & Sichombe, 2009) investigated reasons for poor performance in Mathematics and among their findings listed lack of proper teaching methods, insufficient resources and low motivation to study. It might be that the poor performance of learners in Mathematics is due to inappropriate teaching methods (NIED, 2010a) and the lack of motivation to study Mathematics might be the factor deterring learners from writing Mathematics on a higher level. The purpose of this study was to determine the effects of cooperative learning on learners' motivation and performance in higher level Mathematics in the Oshana Education Region.

Research questions

This study sought to determine the effects that cooperative learning has on the learners' performance and motivation to study higher level Mathematics in the Oshana education region. The study was guided by the following two questions:

1. What are the effects of cooperative learning on the performance of Grade 11 learners in higher level Mathematics in Oshana education region?
2. What are the effects of cooperative learning on the motivation of Grade 11 learners to study higher level Mathematics in Oshana education region?

Hypotheses

In this study, two null hypotheses were tested at $\alpha = 0.01$ significance level.

1. H_o: There is no significant difference between the performance of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

   H_1: There is a significant difference between the performance of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

2. H_o: There is no significant difference between the level of motivation of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

H_1: There is a significant difference between the level of motivation of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

Significance of the study

The results of this study provided information on the effects that cooperative learning has on the performance of learners in higher level Mathematics and on the learners' motivation towards studying Mathematics. These results may benefit the Mathematics teachers in the Oshana education region by providing them with a cooperative learning approach which has a positive influence on the academic performance of learners. The results might also help the Mathematics teachers in the Oshana Education Region in enhancing the motivation of their learners to do Mathematics on higher level.

Limitations of the study

The following were the limitations of this study:

- This study involved a case of one school in the Oshana education region. Therefore, the results may not be generalised across Namibia.
- The responses to the instrument which sought to determine the motivation level of learners were to be ticked on a Lickert scale. Some learners might have ticked without necessarily reading the items of the scale. Such responses could have misinformed the researcher on the effects of cooperative learning on the motivation of the higher level Mathematics learners in the Oshana Education

Region. However, it was assumed by the researcher that all responses given by participants represented the truth since participants were encouraged to be honest, even though it may not be the case.

Delimitations of the study

The study was restricted to the Grade 11 higher level Mathematics learners in one school in the Oshana Education region in 2012.

Definition of terms

Effects – 'refers to the changes the activities bring about, what is caused by the intervention on the target area and group, e.g. improved learning in schools' (Balanskat, Blamire, & Kefala, 2006, p.24). In this study the researcher used the word "effects" to refer to the changes cooperative learning had on Grade 11 learners' performance in higher level Mathematics and to refer to the influence that cooperative learning had on the learners' motivation towards learning of higher level Mathematics.

Performance – refers to the academic accomplishment of a given task measured against preset known standards of accuracy, completeness and speed (Cobb, 2005). In this study the word performance refers to the scores/marks of the Grade 11 learners in higher level Mathematics assessments in the Oshana Education Region. Academic achievement and academic performance are used interchangeably, as there is no real difference or distinction between the two concepts in the literature.

Motivation: Is an internal process that activates, guides and maintains behaviour over time (Baron, 1988 & Schunk, 1990:121 *cited* in Nwihim, 2007, p.7). In this study, motivation refers to the desire of the Grade 11 learners in the Oshana Education Region to achieve a goal and the enthusiasm to work towards achieving that goal, in this case good performance in higher level Mathematics.

Small groups: In this study, small groups referred to a group of 3-5 learners working together to enhance each other's learning (Johnson et al., 1989).

CHAPTER TWO: LITERATURE REVIEW

This chapter reviews the literatures related to this study. This study sought to determine the effects of cooperative learning on the performance and motivation of higher level Mathematics learners in the Oshana Education Region in Namibia. This chapter begins by giving an overview of Mathematics education in Namibia, reviewed researches on Mathematics education in Namibia, then discussed cooperative learning, academic performance and motivation.

Overview of Mathematics education in Namibia

Upon independence in 1990, the Namibian government embarked on reforming the education system to train its citizens into economic productive individuals. The Ministry of Education and Culture adopted the educational reform policy "T*owards Education for All"* (MEC, 1993) and introduced a new teacher education programme, the Basic Education Teacher Diploma (BETD) in order to develop professional experts who could promote the needed change in educational reform (Ilukena, 2008).

The reform policy required teachers to holistically develop learners into individuals who could identify, analyse and solve problems independently (MEC, 1993). The policy expected teachers to develop new visions, new instructional approaches and creativity in selecting teaching aids to allow learners to construct their own knowledge through the manipulation of available resources (MEC, 1993).

Other than the BETD programme, the INSTANT, BES III and MASTEP were designed to address the content knowledge and instructional skills of Mathematics teachers in the country (Haufiku, 2008).

The INSTANT project

The In-service training and assistance for Namibia teachers (INSTANT) project was established after the independence of Namibia with the aim to guide the educational reforms in science and Mathematics in secondary education (Clegg, 2005). The project operated from 1991 to 1997 and placed emphasis on strengthening the content knowledge of the Mathematics and science teachers in the country by improving the effectiveness of the teaching and learning in Mathematics and science (Clegg, 2005).

According to Clegg (2005), the INSTANT project lacked a proper defined guideline of how it would reach its objectives and resorted to solving problems of the Ministry of Basic Education and Culture which were outside the mandate of the INSTANT.

BES III

The Namibia Basic Education Support phase III (BES III) project began in 2005 and targeted to provide a full range of services to the Namibian education community which among other objectives included providing in-service support for language, Mathematics, and science teachers and providing seminars on language, math, and science to pre-service teachers at teacher training institutions (Namibia Basic Education Support phase III [BES III], 2005).

The BES III builds on the foundation established by BES II, through which the Ministry of Education developed innovative school management systems, long-term professional development programs, and effective information and assessment practices (BES III, 2005). BES III worked in the six remote northern regions (Oshana, Ohangwena, Oshikoto, Omusati, Kavango and Caprivi) of the country where almost 70 percent of all Namibian school children lived and where poverty levels were high. Major project activities included designing materials and teaching strategies to improve reading, writing and numeracy skills; implementing continuous assessment tools to measure learner performance more effectively and establishing a sustainable and ongoing system of professional development (BES III, 2005).

MASTEP

The Mathematics and Science Teachers Extension Programme (MASTEP) was a two year upgrading part-time Diploma in Education programme offered by the UNAM. The programme consisted of content and methodological upgrading aimed for the teachers of Mathematics and science (Ministry of Basic Education, Sports and Culture [MBESC], 2002). The importance of the MASTEP was seen in the light of insufficient qualified teachers in Mathematics and science subjects at secondary education level.

Performance in Grade 12 Mathematics examinations in Namibia.

A study carried out by MBESC (2002) reports that learners have been under achieving in Mathematics. A few years later, Peters (2006) noted that only 34.9 % of the learners who wrote International General Certificate of Secondary Education [IGCSE]

Mathematics scored a D symbol or above. According to Iyambo (2010), the passing rate in Mathematics from 2005 to 2009 was unimpressive and below 40%. Iyambo added that another predicament Namibia faced was that only a small number of learners registered for Mathematics on higher level. The Mathematics national results for 2011 showed 80% of the learners obtained lower than the C – symbol (DNEA, 2012). Despites the INSTANT, BES III and MASTEP interventions, Mathematics education in Namibia has been a challenge and learners have performed poorly in Mathematics over the years.

Research on Namibian Mathematics Education

Peters (2006) investigated the teaching strategies of Mathematics teachers in Windhoek schools. Peters suspected that the teaching approaches used by the Mathematics teachers could be the contributing factors to poor performance in Mathematics. Her findings revealed that the teaching strategies of teachers had an effect on the learners' performance and on the motivation of learners to study the Mathematics. Peters (2006) thus recommends that Mathematics teachers design instructions that involve active learners' participation and the ones were learners can view Mathematics as a subject that gives them power to solve problems in real life. She suggests that learning activities be contextualised to enhance understanding.

Ilukena (2009) sought to determine whether there is a need to implement a complementary course in Mathematics education for teachers in Namibia. He found that many schools had Mathematics teachers who were not qualified to teach Mathematics at

a secondary level. Ilukena (2009) also found that some teachers had low content knowledge and recommends for a complementary course to be implemented in order for Mathematics teachers in Namibia to upgrade their content and teaching skills.

Nambira et al. (2009) did a study that sought to determine reasons for poor performance in Mathematics, and found that the low performance in Mathematics lies in the teaching approaches, lack of learning resources and the implementation of the syllabus. Similar results were earlier found by DNEA (2004) cited in NIED (2010a) in a study to determine reasons for poor performance in Mathematics, and the study results includes shortage in learners' motivation to learn, availability of teaching materials and methods of presentation.

In a study conducted by NIED (2010a), learners were asked for suggestions to improve their performance and among others mentioned that teachers should adjust their teaching approaches and take views of learners into consideration. Nambira et al. (2009) and NIED (2009) all seem to highlight a need for a better teaching mechanism that enhances learning. "The main challenges facing the attainment of high performance in Mathematics lie on the teaching and learning of Mathematics, the implementation of the syllabus" (NIED, 2009). These studies, therefore, gives support to a study which sought to determine the effects of a teaching method on performance.

Cooperative learning

Cooperative learning is defined in a variety of ways. According to Johnson, Johnson, and Holubec (2008), cooperative learning is a group of three to five learners who work together as a team to solve a problem, complete a task, or achieve a goal. The main description being working together to accomplish a goal, cooperative learning procedures are designed to engage learners actively in the learning process through inquiry and discussion with their peers in small groups (Johnson et al., 2008).

Vygotsky (1978) claimed that socialisation is the foundation of cognitive development. According to Vygotsky, socialisation facilitates learning because the process of working with others offers a learner an opportunity to operate within his or her "zone of proximal development". The zone of proximal development has been defined as the distance between the current level of development as indicated by what a learner can do without assistance and the level of potential development as indicated by what a learner can accomplish with assistance from peers (Liao, 2005). Liao adds that the rationale that social interaction with peers enhances learning lies on the fact that cooperation with peers allows learners to work closely within one another's levels of proximal development. When learners work closely within one another's levels of proximal development, they can receive explanations that are presented to them in a simpler and more comprehensible fashion than if they were provided by a person of a different mental age. The process of cooperation thus benefits learners academically.

According to Johnson, Johnson and Holubec (1994) cited in Regnier (2009), cooperative learning is incorporated mainly by splitting learners into groups of three to five to work on assignments until all group members understand it. In these groups, learners are expected to discuss ideas, help each other to reveal links and clarify concepts and then complete the tasks. The group work is carefully organised and structured so as to promote the participation and learning of all group members in a cooperative context (Regnier, 2009).

Johnson et al (1994) as cited by Regnier (2009) listed five components of cooperative learning that needs to be considered for cooperative learning to be effective. The five components are positive interdependence, face-to-face interaction, individual accountability, interpersonal skills and group processing.

Positive interdependence

Johnson et al. (1992) and Kagan (1994) both cited in Regnier (2009) stressed that the positive interdependence is the most important element of cooperative learning. Positive interdependence is the need for learners to perceive that they are linked with their group mates in such a way that they will not succeed unless everyone else succeeds and that they must work together to achieve the goal. The success of the whole group depends on the success of each member and vice versa.

Face-to-face interaction

Cooperative learning greatly emphasises learner interactions (Liao, 2005). It promotes a context where learners argue, elaborate and explain by linking current materials to what is learned before. Learners have to sit in circles and interact face-to-face as this gives them an opportunity to negotiate and discuss their learning together (Zourez, 2010).

Individual accountability

According to Chen (2005), individual accountability occurs when every team member feels in charge of his/her own learning and those of other group members and hence makes active contributions to the group. Individual accountability is stressed when the performance of each member can be seen by the rest of the group members so that the group knows who needs more help in completing the task. The group then, in turn, helps that member at the benefit of everyone. Randomly selecting one learner' scores to represent the entire group or averaging the scores of the group members are common ways of promoting individual accountability.

Interpersonal skills

This refers to the way learners interact with teammates when mediating disagreements, encouraging others, giving complements, explaining problems, and defending their solutions (Chen, 2005). If learners do not have collaborative skills, groups cannot function effectively. Learners should therefore be taught good social skills to enhance collaboration in solving problems.

Group processing

According to Liao (2005), group processing entails reflecting on group sessions to describe what actions of the members were helpful and unhelpful and then decide which actions should be changed. The group processing serves to shed light on and improve the effectiveness of the members in contributing to the collective efforts towards attaining the group goals. Such processing enables the learning group to focus on group upholding, ensures that members receive feedback on their participation, facilitates the learning of interpersonal skills and encourages the use of these interpersonal skills. Interpersonal skills are instilled by the teachers through reminding learners to collaborate politely and humanely respect each other and their opinions.

Regnier (2009) suggests that teachers should first understand what cooperative learning is, be confident in the effectiveness of the cooperative learning approach and use know various ways of using cooperative learning approach. The teacher's role should include initiating group work, giving guidelines, preparing and introducing new materials, interacting with the groups, tying ideas together and evaluating the learners' performance (Regnier, 2009).

Benefits of cooperative learning

According to Regnier (2009), the proponents of Mathematics education reforms argue that Mathematics instructions should encourage learning of facts through learner involvement and not memorisation of facts presented by teachers.

Cooperative learning has many benefits such as promoting student learning and academic achievement; enhancing content retention and satisfaction with learning experiences; and developing learners' self-esteem and a positive attitude towards learning (Johnson et al., 2008). Promoting student academic achievement indicates a positive effect of cooperative learning on performance and the developing of learners' self-esteem. A positive attitude towards learning indicates motivation towards learning, raised through cooperative learning. Cooperative learning promotes mastery of the Mathematics content, while passive acceptance of information from an outside expert often promotes a sense of helplessness and reliance upon others to attain concepts (Johnson et al., 2008).

Johnson et al. (2008) further maintain that cooperative learning reduces classroom anxiety created by unfamiliar situations faced by learners. When a teacher calls upon a learner in a non-cooperative learning context, the learner becomes the focus of attention of the entire class. Any mistakes or incorrect answers become subject to scrutiny by the whole class. In contrast, when learners work in cooperative groups, the focus of attention is restrained among the group which then construct a solution which its members can review prior to presenting it to the whole class, thus diminishing prospects that mistakes will occur at all (Liao, 2005). When a mistake is made, it becomes a teaching tool instead of a public criticism of an individual student, asserts Liao (2005).

Cooperative learning also encourages the development of improved self-esteem in learners (Johnson et al., 2008). According to Johnson et al., some learners may have

been discouraged either by parents, teachers or friends who ridiculed their ideas in the past, however, with assurance and consistent encouragement from their peers during cooperative learning sessions they may become motivated to engage in learning.

Deficits of cooperative learning

Despites widely accepted benefits of cooperative learning and many recommendations for it to be used as a classroom instructional approach, cooperative learning has its limitations. Liang (2002) found cooperative learning to be time consuming given that most schools have prescribed content to be covered within a limited time frame. Another concern, according to Turco and Elliot (1990) cited in Liang (2002), was that the educational rationale for cooperative learning techniques tended to develop more of socialisation needs than the academic achievement needs. A study done by Carrol (1994) cited by Liang (2002) found learners to have negative perceptions of cooperative learning with a fear that other learners might think little of their opinions. According to Chen (2005), some teachers experienced frustrations from their learners as bright learners complained about being held back by their slower teammates; less assertive learners complained about being ignored in group sessions and resentment feelings when some group members did not deliver correct solutions to problems.

Critics of cooperative learning (Carroll, 1994; Turco & Elliot, 1990 all cited in Liang, 2002) maintain that cooperative learning widens the gap between high and low ability learners. If highly able learners are allowed to move ahead at their learning pace, the gap between them and the others in the class will widen to the point where grouped

cooperative learning situations will no longer be educationally beneficial for any of the learning involved (Carrol, 1994). Some learners poorly engage in group learning activities if the teacher does not engage in strict supervision. If interpersonal skills are not well reinforced, learners may resort to disagreements that may lead to resentments and eventually the failure of the group (Turco & Elliot, 1990).

Studies by Liao (2005), Regnier (2009) and Chen (2005) found cooperative learning approach to have great benefits for both learners and teachers. Liang (2002) claim that cooperative learning elements might not have been properly adhered to, in the cases were the concept showed no positive improvement in performance, confidence and motivation of learners. The deficits of cooperative learning could thus be reduced if the teachers had undergone through training on how to use the cooperative learning approach (Liang, 2002). The following section focuses on how using cooperative learning affects performance in the Mathematics classrooms.

Cooperative learning and academic achievement

Performance refers to the academic accomplishment of a given task measured against preset known standards of accuracy, completeness and speed (Cobb, 2005). In this study the word performance referred to the scores/marks of learners in Mathematics tests/examinations.

A number of studies that have investigated the effects of the cooperative learning method on the learners' achievement have been carried out (Malin, 2007). The results of

these studies indicated that cooperative learning instructions had an improvement effect on academic achievement in Mathematics. Learners who participated in cooperative learning activities had higher levels of academic performance than peers in the control groups (Chen, 2005). Malin (2007) and Chen (2005) highly reinforced the incorporation of the basic elements of cooperative learning whenever cooperative learning approach was to be used to ensure effectiveness.

Regnier (2009), Bawn (2007), and Liang (2002) found that cooperative learning enhances learners' performance. In their studies, students were placed in either the cooperative learning class or in the individual learning traditional class. These studies used a pre-test and post-test designs to compare the achievement of the control and experimental groups. They found a statistically significant mean improvement from pre-test to post-test for the students in the cooperative learning classes than the students who studied the same Mathematics activities individually.

Effandi & Zalton (2006) found that cooperative group instruction showed significantly better results in Mathematics achievement and problem solving skills. Effandi & Zalton recommended the use of cooperative learning instructional approaches in Mathematics classrooms.

Motivation

Many researches in the field of educational psychology have been interested in understanding students' motivation to improve their academic performance for the last

twenty years (Nwihim, 2007). Nwihim defines motivation is an internal process that activates, guides and maintains the behaviours of an individual. It involves a collection of closely related beliefs, perceptions, values, interests, and actions a human takes (Lai, 2011). Motivation plays an important role in the conceptual change process of learners by enhancing positive perceptions of value in learning activities, learning engagement and critical thinking, which then lead to learning achievement (Nwihim, 2007). According to Lai (2011), motivation can be intrinsic or extrinsic.

Intrinsic motivation refers to the motivation that comes from rewards inherent to a task or activity itself (Lai, 2011). It is the motivation which is animated by personal enjoyment, interest, or pleasure for example, the enjoyment of a puzzle or the love of playing games, (Lai, 2011). According to Deci et al. (1999) cited in Lai (2011), intrinsic motivation manifests in behaviours such as exploration, challenge and collaboration and has therefore been considered by educators as more desirable to result in better learning outcomes than extrinsic motivation.

Extrinsic motivation refers to the motivation that comes from outside of the learner. There are people who are motivated to complete a task because of the incentives that are attached to them such as prizes or promotions to the next grade (Lai, 2011). Learners might keep up performing higher only because their parents reward their consistent excellent performance. Sometimes though, instead of rewards, external coercion may force a person to engage in an action. This external factor may be seen as a punishment

or a necessary action that is forced on an individual. Threat of a punishment is a common form of extrinsic motivation in learners (Lai, 2011).

This intrinsic motivation can be distinguished from extrinsic motivation. With extrinsic motivated people, satisfaction does not come from the activity itself but rather from the extrinsic consequences such as tangible or verbal rewards, to which the activity leads (Lai, 2011). Intrinsic motivation comes from the human himself while extrinsic motivation is controlled from outside. "Extrinsic motivation refers to behaviour where the reason for doing it is something other than interest in the activity itself" (Lai, 2011, pg. 35).

Motivational theories

Cooperative learning is supported by motivational theories (Liao, 2005). The motivational theories this study reflected on are the self-efficacy theory, effectance motivation theory and the expectancy-value theory.

Self-efficacy theory

Self-efficacy is a state of system of belief and confidence level of oneself that he or she is able to perform a specific task (Bandura, 1997). This general understanding is rooted by Bandura's social cognitive learning theory. Self-efficacious learners feel confident about solving problem as they have developed an approach to problem solving that worked in the past. The development of learners' self-efficacy in successfully

completing a task is closely related to the effective use of learning strategies (Zimmerman, 1989). Zimmerman has been instrumental in tracing the relationships among self-efficacy perceptions, self-efficacy for self-regulation, academic self-regulatory processes, and academic achievement and found that self-efficacy mediated the influence on learning and was positively correlated to high academic achievement.

The Effectance motivation

White (1959) came up with the 'The Effectance Motivation' which states that people have an innate tendency to have control of their environment. This motivates people to learn about their environment and to be competent in given situations. In the empirical tradition, most psychologists refer to the non-drive-based motivation as 'Intrinsic Motivation', suggesting that the energy is intrinsic to the nature of the organism.

Expectancy-Value theory

Liao (2005) defines expectancy-value theory as the theory that conceive that learners' motivation to perform a learning task depends on the expectancy of success in the given task and the value attached to successfully performing the task.

The usefulness of Mathematics as in the views of learners is an important factor to consider as the expected value will highly influence motivation and self-efficacy (Hodges, 2004). Hodges adds that the learners will not be motivated to work hard if they believe that the outcome is of little value in their lives. Therefore, the views of the learners on the usefulness of Mathematics in their lives influences their academic ahievement.

The national commitment in improving Mathematics performance is based on the fact that the promotion of quality and effective Mathematics and Science education in schools will heighten the attainment of, particularly, the technical, scientific and economic development (Iyambo, 2010). These skills are fundamental for transforming the Namibian Nation into a knowledge based economy. The importance of Mathematics and science is equally emphasised by NIED (2010a, pg.18), "Mathematics is essential for success in scientific and technical education. Unless the foundations are secured, it will be extremely difficult to build Mathematical and scientific success at tertiary level".

The learners thus need to understand how useful Mathematics is in their own lives as well as the national views on the importance of Mathematics. Learners are then likely to perform well in solving problems when they attach significance to Mathematics (Hodges, 2004).

Cooperative learning and motivation

Learners' motivation towards learning is generally regarded as one of the most critical determinants of the success and quality of any learning outcome. According to Malin (2007), one of the key components to cooperative learning that consistently contributed to improved academic performance was the motivational aspect. Malin demonstrated that cooperative learning strategies significantly affect the learners' motivation when compared to control groups who learned the same materials independently (Malin,

2007). Zourez (2010) supported Malin (2007) that cooperative learning "raises students' motivation, fosters their self esteem and reduces their anxiety" (p. 131).

Johnson et al. (2008) found that, as learners become more engaged in their learning, their motivation increases. To be motivated to learn, learners need opportunities to interact with each other as well as steady encouragement from fellow learners through the support of their learning efforts (Johnson et al., 2008). By placing students in groups and giving them tasks that require interdependence, each member of the group becomes accountable for achieving a shared goal. Students are then motivated by the team effort as well as by seeing their own contributions accepted by the group (Deana, 2007). Correspondingly, the model of cooperative learning argues that the setting of group goals will trigger motivation to learn, motivation to encourage group members to learn and motivation to help group members to learn (Liao, 2005).

Lai (2011) suggests that teachers should attempt to give learners more autonomy over their own learning by allowing them to make choices as that may increase learners' motivation and boost up academic performance. In cooperative learning groups, learners receive peer support, guidance and assistance. This encourages them to get involved in their own learning and consequently motivates them to explore more concepts in the subject. Moreover, learners feel better about the learning process when they share their own work with the entire class (Lai, 2011). Teachers thus need to create a supportive classroom environment with respect to goal structures, attributions and external evaluations. According Peters (2006), children learn best when they are actively

involved in the learning process and the teaching approach should encourage and motivate learners to actively participate in their learning.

Conclusion

The reviewed literature support the use of cooperative learning in the Mathematics classrooms. The common element found in the many definitions of cooperative learning is a group of 3-4 learners working together to inquire about and engage in discussions to accomplish a goal. This practice enhances deeper understanding of concepts (Johnson et al., 2008). Literature also revealed that simply allocating learners to groups is not enough; the components of cooperative learning need to be included for maximum effectiveness.

In the Namibian context with low performance in Mathematics, it seemed little had been done on researching teaching methodologies which may be a major contributing factor to poor performance in Mathematics (NIED, 2010a; Nambira et al., 2009). Therefore this study sought to determine the effects of cooperative learning on the motivation and academic performance of learners in Mathematics. The next chapter discusses the methodology used in this study.

CHAPTER THREE: RESEARCH METHODOLOGY

This chapter outlines the methodology used in this study. It discusses the research design by discussing the investigational concept of a quasi-experimental design and how the research site and study participants were selected. It further explains the instruments and methods used in collecting data for the research. The data analysis procedures and research ethics are also discussed towards the end of this section.

Research design

This research was situated in the quasi - experimental paradigm and sought to determine the effects of cooperative learning on the learners' motivation and performance in Mathematics in the Oshana education region. In a quasi - experimental study, the researcher manipulates an independent variable, controls the other variables and then observes the effects on the dependent variables (Gay, Mills, & Airasian, 2009). An experimental group and control group were selected from the two Grade 11 higher level Mathematics classes at one school in Oshana education region. These two groups were given a pre-test, followed by different treatments and then the post-test in orders to measure the effects of cooperative learning (an independent variable) on performance and motivation (dependent variables).

The study used a non-equivalent control group design by studying the intact classrooms (Gay et al., 2009) because the class groups already existed in the school and the

researcher could not split up the classes. The researcher did everything possible to control other variables. All two groups were taught by the researcher, who gave the same teaching materials, homeworks and assignments during the experiment. The only difference that existed was in the teaching approach. The experimental group was taught using cooperative learning approaches and the control group was taught in a non-cooperative approach. These teaching approaches are discussed later in this section.

Population

The population of the study comprised all the grade 11 learners doing Mathematics on higher level in the Oshana education region in 2012. There were two schools offering Mathematics on higher level in the Oshana Education region in 2012.

Sample and sampling procedures

Sampling is the method used to select a given number of people or things from a population to represent the population in a study (Gay et al., 2009). The information from a subset of the population is generalised to the population in the context of the study.

One of the two schools offering Mathematics on higher level in the Oshana education region was purposefully selected to take part in the study. The sample consisted of two Grade 11 higher level Mathematics classes; one class was the control group and the other an experimental group. There were two Grade 11 classes doing Mathematics on higher level at the school and each class had 31 learners which gave a total of 62

learners. Random sampling was done to select the experimental and control group between the two classes. The two classes were assigned pseudonyms to protect the learners' identities. The control class was known as group C and the experimental class was known as group G. Twelve learners were residing outside the school hostel and were unable to come for classes in the evenings, and did not take part in the study. The experimental class reduced to 27 learners and the control class to 23 learners, totalling 50 learners altogether that participated in the study.

Research instruments

The instruments used in this study were Mathematics pre-test, Mathematics post-test and the Fennema-Sherman Mathematics Attitude scale.

Mathematics Pre-test and post-test

The Mathematics pre-test and post-test were used to determine whether a significant difference exist between the scores of the experimental group and the control group. The researcher set up the questions based on differential calculus, a topic in the higher level Mathematics syllabus. The topic was selected as it does not require basics from other topics in the syllabus and in this way the researcher controlled for maturation as a threat to the validity of this study (Gay et al., 2009). The researcher requested the Mathematics Advisory Teacher in the region to assess the construct validity of the tests. According to Phelan & Wren (2006), having a second evaluator enhances the reliability of the instruments and controls instrumentation as a threat to validity, which reflects the lack of consistency in measuring instruments (Gay et al., 2009).

The Fennema-Sherman Mathematics Attitude scales

The Fennema-Sherman Mathematics Attitude Scales was developed in 1976, and it has become one of the most popular instruments used in research over the last three decades (Marsh and Tapia, 2004). The complete Fennema-Sherman Mathematics Attitude Scale instrument is composed of nine subscales, each with 12 items. The nine scales include Attitude Towards Success in Mathematics; Mathematics as a Male Domain Scale; Mother, Father, and Teacher scales; Confidence in Learning Mathematics; Mathematics Anxiety scale; Effectance Motivation in Mathematics scale; and Usefulness of Mathematics scale.

Fennema and Sherman (1976) suggested that the scales can be used as a total package for measuring important attitudes related to Mathematics learning, or the sub-scales can be used individually. This study therefore, chose to use the three sub-scales; Confidence in learning Mathematics scale, Usefulness of Mathematics scale and the Effectance motivation scale in Mathematics. The three subscales were viewed relevant for this study as they were closely linked to the motivational theories in the theoretical framework of this study.

Confidence in learning Mathematics scale, considers an abstract indication to Mathematics self-efficacy and has consistently been found to predict both Mathematics performance and Mathematics anxiety (Marsh and Tapia, 2004). The importance of students' judgments about their capability or self-efficacy has been highlighted by social cognitive theorists such as Bandura (1997) and research has supported self-efficacy's

role as an important mediator for all types of achievement behaviour as well as many other types of behaviours (Marsh and Tapia, 2004). The Confidence in Learning Mathematics Scale was intended to measure the confidence in one's ability to learn and to perform well on mathematical tasks. The dimension ranges from distinct lack of confidence to definite confidence.

The Mathematics Usefulness scale was intended to measure students' beliefs about the usefulness of Mathematics currently, and in relation to their future education, vocation, or other activities (Fennema and Sherman 1976). According to Hodges (2004)'s expectancy value theory, learners are likely to perform well in school activities when they value a subject and when they know the importance of that subject in their own lives.

The Effectance Motivation in Mathematics scale is intended to measure effectance (or problem-solving) as applied to Mathematics. The dimensions range from lack of involvement in Mathematics to active enjoyment and seeking of challenge. The scale is not intended to measure interest or enjoyment of Mathematics; rather, it attempts to measure attitudes towards the enjoyment of Mathematics (Fennema and Sherman 1976).

The learners responded to items in the Fennema-Sherman Mathematics Attitude scale which were ranked on a Likert scale from one to five based on their level of agreement or disagreement with the item. Tapia & Marsh (2004) assert that the Fennema-Sherman Mathematics Attitude scale is an efficient and effective research tool because of its content validity and reliability.

Pilot study

The pilot study was designed to gather information on the appropriateness of the instruments and other administrative logistics prior to the conduct of the main study, in order to improve its quality and efficiency (Lancaster, Dodd, & Williamson, 2004). According to Lancaster, Dodd, & Williamson (2004), a pilot study can reveal deficiencies in the design of a proposed experiment or procedure and these can then be addressed before time, and resources are expended on a large scale study.

The pilot study was carried out with 45 Grade 11 learners doing higher level Mathematics at another school in Oshana Education Region which admits learners to Grade 11 from the same geographical backgrounds as the school where the main study was carried out. The researcher assumed that the higher level Mathematics learners at these two schools had the same characteristics (Lancaster, Dodd, & Williamson, 2004). There were three grade 11 higher level Mathematics classes at the school were the pilot study was conducted. A sample of 15 learners was randomly picked from each one of the three higher level Mathematics classes giving a total of 45 learners. A pre-test was administered to them and then the learners were randomly separated into two groups. The experimental and control group. The experimental group was taught *Differentiation,* a Mathematics higher level topic, using cooperative learning methods while the control group was taught the same topic and used a non-cooperative learning approach where by the teaching and physical classroom setup did not allow learner interactions. The post-test was then administered to both groups after the lessons. After the post-test, the

Fennema-Sherman Mathematic Attitude Scale was administered to the learners in both the control and experimental groups to complete.

The pilot study helped to point out flaws in the instruments, selection of the sample and the data collection procedure. The first problem discovered was in allocating pseudonyms, roman numerals to the control group and alphabetical letters to the experimental group. Both groups had; I, X, V and that was difficult to determine which learner belonged to which group. For the main study, the researcher opted to name the control group as group C and the experimental group as group G and give the participants codes starting with the letter of the group, e.g. the first participants in the control and experimental group were given the codes C01 and G01 respectively.

The second problem occurred in the pre-test; Question 3(d), Simplify $\frac{\sqrt[3]{9a^2}\times\sqrt{a^{\frac{2}{3}}b^{-1}}}{(3b^{\frac{3}{2}})^{-\frac{1}{3}}}$, was too long and unnecessary repetition of Question 3(c), simplify $8x^{\frac{2}{3}} \div 4x^{\frac{1}{2}}$, since these two expressions required knowledge of radicals and fractional exponents. Question 3 (d) was thus deleted. The instructions on the test were also clarified, for the learners to show all their working to earn full marks allocated for each question.

The third problem was found in the post-test Question 5, which required learners to find the values of x for which the gradient of the curve $y = 2x^3 + 3x^2 + 12x + 3$. The cubic differentiates to a quadratic $y' = 6x^2 + 6x + 12$, which has no real roots as its graph is a parabola with every point above the x-axis. This question was replaced. The

pilot study also helped the researcher in re-structuring the order in which the sub-sections of Differentiation were to be taught during the main study.

The pilot study also gave insight into the problems that could be encountered in coding, and tabulating the data from the Fennema-Sherman Mathematics Attitude Scale (1976). The scale was adopted using the sub-scales; confidence in learning Mathematics scale, effectance motivation scale in Mathematics and Mathematics usefulness scale. The learners responded to items ranked on a Likert scale from 1 to 5 based on their level of agreement or disagreement with the item. A 5 on the scale represents strongly agree, a 4: agree, a 3: Not sure, a 2: disagree and a 1: strongly disagree.

Each subscale had 12 items and these items were placed randomly in the main scale, which required a lot of time to code the items per subscale for descriptive statistics analysis. The items were then re-arranged according to sub-scales to lessen the analysis process. Space was also provided for learners to write in their participant codes on the Fennema-Sherman Mathematics Attitude Scale to lighten the analysis process by recording how much each participant scored from the sub-scales.

Reliability

The scores from the pilot study were used to test for the reliability of the version of the Fennema-Sherman Mathematics Attitudes Scales in a Namibian context using the Cronbach alpha, α. The internal reliability for the three sub-scales was generally good. The Confidence scale had 0.91, Usefulness scale had 0.92 and the Effectance motivation

scale had 0.86. These data indicate that the scores on the inventory and the subscales have high consistency, indicating high level of reliability of the scores on the subscales.

Data collection procedures

After securing permission from the Permanent Secretary of the Ministry of Education and the Director of Oshana Education Region as well as from the school principal at the school where the study was conducted, the researcher obtained the class lists of the two classes doing Mathematics on higher level at the school. He then randomly selected one class as the control group and the other class as the experimental group. The participants were given pseudonyms to protect their identity and use the codes during the study. The Mathematics pre-test was administered and the results of the individual participants were recorded.

Lessons

The experimental group and control group were taught separately, Differentiation.

Differentiation is a study of functions that do not change at a constant rate (Emanuel, 2005). The value of the function called the derivative is that varying rate of change. In the Namibia Senior Secondary Certificate (NSSC) higher level syllabus, the following objectives are listed under Differentiation;

1. Demonstrate understanding of the concept of limit of a function.
2. Use the notations; $f'(x)$, $\frac{dy}{dx}$, $f''(x)$ and $\frac{d^2y}{dx^2}$.

3. Recall and use the derivatives of ax^n (for any rational number *n)* , $a \ln x$, ae^x , together with sums, differences and composites of these.
4. Apply differentiation to gradients and tangents.
5. Locate stationary points, and distinguish (by any method) between maximum and minimum points and points of inflection.
6. Express rates of change in terms of derivatives, and use differentiation to solve problems concerning rates of change, especially involving displacement, velocity and acceleration (rates of change of connected variables are not included). (NIED, 2010b, p. 17).

The lessons were planned and taught according to the order of the objectives in the syllabus. The lessons were all structured in a learner-centred approach by making sure the starting point include learners' existing knowledge and link the lessons to learners' experiences thereby arousing the learners' curiosity and eagerness to investigate and make sense of the new concepts. Most problems were contextualised.

The sessions ran for five weeks as "short time period minimises the history threat to validity" (Gay et al., 2009, p. 243). The Mathematics post-test was written immediately after the treatment. The tests were written by all the participants at the same time and in the same room to minimise external influences on the performance of the learners by allowing them to write under the same environmental conditions. Two Mathematics teachers at the school were the study was conducted invigilated the Mathematics pre-test

and post-test. The adopted Fennema-Sherman Mathematics Attitude scale was administered before and after treatment parallel to the pre-test and post-test. The learners responded to items in the Fennema-Sherman Mathematics Attitude scale ranked on a Likert scale from strongly agree to strongly disagree.

Experimental group

The instructions in the experimental group were guided by the social constructivism theory whereby the researcher encouraged interactions among learners to enhance learning (Vygostky, 1978). Vygotsky argued that cooperation promotes learning because the process enables learners to operate within one another's "zone of proximal development" (p.86). The experimental group consisted of 27 learners. After all participants had been given a general introduction to the topic and the teaching approach that was used, the researcher gave out copies of group rulers and group commitments to every participant (see Appendix N). The researcher discussed the main purpose of group collaboration and encouraged participants to abide by the rules and commit themselves to tasks assigned to groups. The researcher explained to the learners how to collaborate effectively in groups, how to communicate and criticise constructively, how to participate in group tasks and act according to assigned individual responsibilities (See appendix N).

Each participant was then given a list of the learning objectives as prescribed in the grade 12 higher level Mathematics syllabus. The learning objectives informed the participants of what to expect in each section of the topic on Differentiation.

Each lesson was 60 minutes long. For the first ten minutes of each lesson, the researcher discussed with the entire group how to solve certain problems based on the learning objectives. After this the learners were randomly split into groups of four giving a total of seven groups in the class. The jigsaw and think-pair-share methods were used to teach the experimental group (Aronson, 2002). A jigsaw method is a cooperative learning approach were each member of the small groups is given a problem to solve for a certain amount of time and then later the members give each other turns of showing how they solved the problems they got (Aronson, 2002). This approach ensured every learner participated in the discussions of the group.

The jigsaw method was approached by giving out Mathematics problems to each group to solve and then later the groups had to choose representatives to present the group solutions to the entire class. During these presentations, the researcher encouraged questions from the class for clarifications and also encouraged the class to give other contributions relevant to the problem. This approach gave room to the entire class to hear and critique different methods of solving given problems.

In a think-pair-share strategy, the researcher gave a question to the class. The learners were then given a few minutes to work out solutions and then paired with partners to compare and discuss their solutions. The pairs then shared their ideas with the whole class. A learner is more comfortable presenting refined ideas to the group with the support of a partner (Njenga, 2010). The groups of four learners did not have to separate

during the think-pair-share strategy as two members in each group were already seated facing each other.

According to Njenga (2010), the think-pair-share approach provides opportunities to learners to learn higher-level thinking skills from their peers, raises self-esteem and accommodates everyone in the class. The groups were arranged differently everyday in order to give chances to learners to work and think with different partners. The pair discussions were monitored by the researcher and common misconceptions as well as unique ideas were discussed with the whole group.

The Control group

The learners in the control group were taught using various instructional strategies ranging from explanatory, demonstrations, discussions with teacher and question and answer method. The class desks were arranged in rows to minimise discourses and learners were encouraged to work on problems individually. Like for the experimental group, the learning objectives were stated prior to each lesson to create levels of expectations for learning. The control group was taught by linking to their prior knowledge and then building on their existing background knowledge and gradually extending to new concepts. The content was presented and discussed with the learners as the researcher consistently used the questions to actively involve learners in the learning process.

Mathematics Post-test

After the five week sessions, the Mathematics post-test was administered. The experimental and control groups wrote the same test at the same time in the same room. The test scripts were collected and later marked by the researcher. After the Mathematics post-test, the learners were given the Fennema-Sherman Mathematics Attitude scale to complete.

Fidelity of treatment.

In intervention research, treatment fidelity is defined as the strategies that monitor and enhance the accuracy and consistency of an intervention to ensure it is implemented as planned and that each component is delivered in a comparable manner to all study participants over time (Bellg et al., 2004). If treatment fidelity is not measured, researchers cannot ascertain with confidence whether the study outcomes were due to treatment or to factors incidental to the intervention (Laventhal & Friedman, 2004).

Laventhal and Friedman (2004) argue that experimental research in education should not only encompass whether an intervention has positive effects, but should also include determining the effectiveness of the dissemination of these interventions to ensure treatment integrity and treatment differentiation. Treatment integrity refers to "the degree to which a treatment condition is implemented as intended" (Bellg et al., 2004, p. 247), and treatment differentiation refers to "whether treatment conditions differ from one another in the intended manner such that the manipulation of the independent variable actually occurred as planned" (p. 248).

A critical factor in determining the efficacy, effectiveness, and successful dissemination of an educational practice is ensuring that the professionals who are responsible for its implementation deliver an intervention under study with accuracy and conformity (Bellg et al., 2004). To ensure fidelity of treatment, including treatment integrity and treatment differentiation, the researcher taught the lessons himself throughout the duration of the study to ascertain that both the experimental and the control groups follows the protocol they were meant to follow. In addition, the researcher videotaped some lessons while the learners were busy working on the assigned problems. The results of the procedures showed that the instructional programmes in both the experimental and the control groups were able to be carried out as intended by their individual protocols.

Data analysis procedures

A *t*-test was used to test for a significant difference in the Mathematics pre-test and post-test results for the Mathematics performance tests. Initially, a t-test calculation was carried out to test for significance in the Mathematics pre-test results of the control and the experimental group to determine whether the groups were equivalent prior to the study. Intra-group comparisons were then made in each group to compare the Mathematics pre-test results to Mathematics post-test results. Finally, t-test was calculated to test whether there was a significant difference in the Mathematics post-test scores of the control and experimental group.

For the Fennema-Sherman Mathematics Attitude scale that measured the motivation levels of the participants prior to and after the intervention, the t-test was also done to

test for significant differences like it was done for performance. Three subscales of the Fennema-Sherman Attitude scale were adopted. The sub-scales were 12-item instruments each, six items worded positively and six items worded negatively (Fennema & Sherman, 1976). The instrument uses a Likert scale with a range of Strongly Agree to Strongly Disagree. A total score is calculated by assigning a value of 1 (Strongly Disagree) to 5 (Strongly Agree) to each positively worded item, and 1 (Strongly Agree) to 5 (Strongly Disagree) to each negatively worded item and then adding the values. Possible scores range from 12 to 60 for each subscale.

The 50 participants were given the Fennema-Sherman Mathematics Attitude scale prior to intervention. The prior to intervention scores were recorded and a t-test was calculated to determine whether the groups were equivalent in terms of the motivation levels prior to the study. After the lessons on Differentiation, the Fennema-Sherman Mathematica Attitude scale was administered again to determine the motivation level of the learners after the study. Comparisons of the prior to and post intervention scores were carried out for the control and experimental group and then the t-test was calculated to compare the post-test scores of the control and experimental group for each of the three Fennema-Sherman Mathematics Attitude sub-scales used.

Ethical considerations

In this study, the researcher took into account ethical issues involved in studies that deal with human subjects (Bassey, 1999) by firstly seeking copy right to use the Fennema-Sherman Mathematics Attitude Scale from the author, the permission was obtained.

Secondly, the researcher obtained permission from the Permanent Secretary of the Ministry of Education and the Director of the Oshana Education Region to conduct the study. The study site was only entered after permission was secured from both Permanent Secretary and the Director of Oshana Education Region.

The names of the participants were not revealed anywhere as the participants were given codes and their views were only used for the purpose of this study. The researcher ensured that participants were fully informed of the purpose of the study before asking them to decide whether or not to participate in the study. According to Bassey (1999), informed consent is about individuals choosing whether or not to participate in the study after having been informed of all the facts, and hence, consent from the learners' parents by signing a letter of consent was obtained prior to the collection of data and the participants were told of their right to refuse to participate or to withdraw from participating anytime they felt like doing so without any penalty against them.

This chapter began with the statement of the research design and discussed the methodology of obtaining the sample from the population. The chapter also discussed the research instruments (Mathematics pre-test and post-test and the Fennema-Sherman Mathematics Attitute Scale) used in this study. The data collection procedures, data analysis and ethical considerations were also discussed in this chapter. The next chapter presents and discusses the collected data.

CHAPTER FOUR: PRESENTATION AND DISCUSSION OF RESULTS

Introduction

In this chapter, the data collected from the Mathematics pre-test and post-test and the Fennema-Sherman Mathematics Attitude Scale are presented and discussed. The study sought to determine the effects of cooperative learning on performance and motivation of the grade 11 higher level Mathematics learners at one secondary school in the Oshana Education Region.

PRESENTATION OF RESULTS

Participants

Fifty Grade 11 learners taking Mathematics on higher level at one school in the Oshana Education Region participated in this study. The control group comprised 23 participants while the experimental group had 27 participants.

The effects of cooperative learning on the performance of Grade 11 learners in higher level Mathematics.

For the measurement of learners' performance, two types of Mathematics tests were written, a pre-test and a post-test. The results of the Mathematics pre-test and post-test for the control and experimental groups are recorded in Appendix L. In order to find out whether a significant difference existed between the scores of the control group and the experimental group, the following hypothesis was tested.

H_o: There is no significant difference between the performance of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

H_1: There is a significant difference between the performance of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

The Mathematics pre-test was administered in order to determine whether the participating groups were at the same level of understanding of Mathematics so that the degree of change occurring in the post-test results of the treatment group could be attributed to the treatment (Gay et al., 2009). The Mathematics pre-test results yielded the mean scores of 35 for the control group and 33 for the experimental group. Table 1 presents the results of the Mathematics pre-test for the control and experimental groups.

Table 1. Results of the pre-test for the control and experimental groups.

	Mean	Number	Standard Deviation	$t_{calculated}$
Control	35	23	14.8	0.4759
Experimental	33	27	9.97	

The t-test for the Mathematics pre-test results with degrees of freedom, df = 48 at the significance level, α = 0.01 yielded $t_{calculated} = 0.4759$. The obtained $t_{calculated} = 0.4759$ is less than $t_{critical} = 2.000$. This result shows that there was no significant difference in the performance of the control and experimental groups at the

beginning of the study. Therefore, the control group and experimental group could be said to have been equivalent at the beginning of the intervention.

Control group

After the Mathematics pre-test, the control group was taught Differentiation, a topic on rates and derivatives in the higher level Mathematics syllabus, using the following instructional approaches; explanatory, demonstrations and question and answer method. The class desks were arranged in columns and rows to minimise discourses amongst the learners. The learners were encouraged to work on problems individually. The Mathematics post-test was administered at the end of the five week teaching period. A t-test was carried out to compare the Mathematics pre-test and post-test results of the control group (See Table 2).

Table 2. Control group pre-test and post-test results.

	Mean	Number	Standard Deviation	$t_{calculated}$
Pre-test	35	23	14.8	1.3767
Post-test	30	23	7.99	

Table 2 shows that at α = 0.01 and *df* = 22, the $t_{calculated} = 1.3767$ and is less than $t_{critical} = 2.819$. This result shows that there was no statistical significant difference between control groups' pre-test and post-test scores.

Experimental group

After the Mathematics pre-test, the experimental group was taught the same content as the control group using the cooperative learning approach. The Mathematics post-test was administered at the end of five weeks of instruction. The scores for the experimental group from the pre-test and post-test are given in Table 3.

Table 3. Experimental group's pre-test and post-test results.

	Mean	Number	Standard Deviation	$t_{calculated}$
Pre-test	33.4	27	9.97	2.8595
Post-test	42.1	27	15.9	

Table 3 shows the t-test for non-independent scores at $\alpha = 0.01$ and $df = 26$, yielded $t_{calculated} = 2.8595$ and the $t_{critical} = 2.779$. This result shows that there was a significant difference in the Mathematics post-test scores and pre-test scores of the experimental group.

Experimental versus control group comparison

In order to find out the effects of cooperative learning on the learners' performance in higher level Mathematics, the following hypothesis was tested:

H_o: There is no significant difference between the performance of the Grade 11 higher level Mathematics learners taught using cooperative learning and those who are not.

H_1: There is significant difference between the performance of the Grade 11 higher level Mathematics learners taught using cooperative learning and those who are not.

Table 4 shows the means of the experimental and control groups on the post-test after five weeks of instruction.

Table 4. Experimental and control groups' post-test results.

	Mean	Number	Standard Deviation	$t_{calculated}$
Control	30.0	23	7.97	3.306
Experimental	42.1	27	15.92	

The calculated t-test value was $t_{calculated} = 3.306$ greater than $t_{critical} = 2.660$ at $\alpha = 0.01$ and $df = 48$. The results indicate that there was a significant difference in the Mathematics post-test scores of the experimental and the control groups.

The Fennema-Sherman Mathematics Attitude Scales results

In addition to the examination of the effects of cooperative learning on the performance of learners, this study sought to find out the effects of cooperative learning on the higher level Mathematics learners' motivation towards the study of Mathematics. The Fennema-Sherman Mathematics Attitude Scale (F-SMAS) was used to measure the motivation of the learners prior to and after the intervention. The motivation was measured based on the learners' responses on the three subscales; Confidence in learning Mathematics (CM), Usefulness of Mathematics (UM) and Effectance Motivation (EM). Appendix M shows the results of the control and experimental groups.

The following hypothesis was tested in order to determine the effects of cooperative learning on the motivation of Grade 11 learners to study higher level Mathematics.

H_o: There is no significant difference in the motivation of the Grade 11 higher level Mathematics learners who are taught using cooperative learning and those who are not.

H_1: There is a significant difference in the motivation of the Grade 11 higher level Mathematics learners who are taught using cooperative learning and those who are not.

The F-SMAS prior to intervention was given with the purpose of determining whether the participants in the control and experimental groups had the same level of motivation so that the degree of change occurring in the post-test results can be attributed to the treatment. The *t*-test was used to compare the learners' motivation towards the learning of Mathematics. The F-SMAS result are presented according to the three subscales; Confidence in learning Mathematics, Usefulness of Mathematics and Effectance motivation in Mathematics.

Confidence in learning Mathematics Sub-Scale (CM)

Confidence in learning Mathematics scale considers an abstract indication to Mathematics self-efficacy and has consistently been found to predict both Mathematics performance and Mathematics anxiety (Marsh and Tapia, 2004). The Confidence in Learning Mathematics Scale was intended to measure the confidence in learners' ability to learn and to perform well on mathematical tasks prior to and after the intervention. The dimension ranged from distinct lack of confidence to definite confidence.

The *t*-test was used to compare the learners' motivation towards the learning of Mathematics. The CM results before the intervention yielded the mean scores of 48 and 49 for the control group and the experimental group respectively as shown in Table 5.

Table 5. The Confidence in learning Mathematics prior to intervention for the control and experimental groups

	Mean	Group size	$t_{calculated}$
Control	48	23	0.7777
Experimental	49	27	

$t_{critical} = 2.000$

The t-test calculations of the CM scores before intervention at $\alpha = 0.01$ and $df = 48$, yielded $t_{calculated} = 0.7777$. This result shows that there was no significant difference in the levels of confidence towards learning Mathematics for the control and experimental groups at the beginning of the intervention.

Control group results

The CM scores of the control group showed a significant difference at the 0.01 level of significance after the intervention (see Table 6).

Table 6. Prior to and post intervention CM scores for the control group

	Mean	Number	$t_{calculated}$
Prior to Intervention	48	23	5.7213
Post Intervention	36	23	

$t_{critical} = 2.819$

The drop in confidence levels of the control group may be explained by what Njenga (2010) said that when learners work alone, most of the problems they encounter are left unresolved which demoralises them and this may result in low confidence. The results in Table 6 appear to support Njenga's results.

Experimental group result

The confidence in learning Mathematics sub-scale prior to intervention and post intervention scores of the experimental group showed no significant difference at the 0.01 level of significance (see Table 7).

Table 7. Prior to and post intervention Confidence in learning Mathematics scores for the experimental groups

	Mean	Number	$t_{calculated}$
Prior to Intervention	49	27	0.3433
Post Intervention	50	27	

$t_{critical} = 2.779$

Table 7 shows the $t_{calculated} = 0.3433$ less than $t_{critical} = 2.779$ at 0.01 level of significance and $df = 26$. This result indicates there was no significant difference in the level of confidence in learning Mathematics prior to and after intervention in the experimental group.

Experimental versus control group

Table 8 shows the scores of the experimental and control groups on the Confidence in learning Mathematics Scale after the intervention.

Table 8. The Confidence in learning Mathematics post intervention scores for the control and experimental group.

	Mean	Group size	$t_{calculated}$
Control	36	23	6.5490
Experimental	50	27	

$t_{critical} = 2.660$

The t-test was calculated to determine whether the confidence in learning Mathematics post intervention scores of the control group and the experimental group were significantly different at $\alpha = 0.01$ and $df = 48$ (see Table 8). The t-test calculation yielded $t_{calculated} = 6.5490$ greater than $t_{critical} = 2.660$. The confidence in learning Mathematics post intervention scores indicated there was a significant difference in the confidence level of the control group and experimental group.

Usefulness of Mathematics sub-scale (UM)

The Mathematics Usefulness sub-scale was intended to measure the learners' beliefs about the usefulness of Mathematics currently, and in relation to their future education, vocation, or other activities (Fennema and Sherman, 1976). According to Hodges (2004), learners are likely to perform well in school activities when they value a subject and when they know the importance of that subject in their own lives. The usefulness of Mathematics sub-scale was administered prior to and after the intervention.

The usefulness of Mathematics sub-scale results before the intervention yielded the mean scores of 49 and 50 for the control group and the experimental group respectively as shown in Table 9.

Table 9. The Usefulness of Mathematics Scale prior to intervention results for the control and experimental groups.

	Mean	Group size	$t_{calculated}$
Control	49	23	0.2413
Experimental	50	27	
			$t_{critical} = 2.000$

The t-test calculation of the usefulness of Mathematics sub-scale scores before intervention at $\alpha = 0.01$ and $df = 48$, yielded $t_{calculated} = 0.2413$ less than $t_{critical} = 2.000$. The results show that there was no significant difference in the usefulness of Mathematics sub-scale scores of the control group and experimental group prior to the intervention.

Control group results

The usefulness of Mathematics sub-scale scores of the control group showed a significant difference at 0.01 level of significance between the prior to and post intervention scores (see Table 10).

Table 10. **Prior to and post intervention UM scores for the control group.**

	Mean	Number	$t_{calculated}$
Prior to Intervention	49	23	3.9665
Post Intervention	42	23	
			$t_{critical= 2.819}$

The $t_{calculated} = 3.9665$ is greater than $t_{critical} = 2.819$ at 0.01 significance level and $df = 22$. This result shows that there was a significant difference in the means of the usefulness of Mathematics sub-scale scores prior to and after intervention in the control group.

Experimental group results

The comparison of the usefulness of Mathematics sub-scale prior to intervention and post intervention scores of the experimental group was carried out using a t-test at the 0.01 level of significance and *df* = 26 (see Table 11).

Table 11. Prior to and Post intervention usefulness of Mathematics sub-scale scores for the experimental group.

	Mean	Number	$t_{calculated}$
Prior to Intervention	50	27	2.2732
Post Intervention	54.	27	

$t_{critical} = 2.779$

The $t_{calculated}$ = 2.2732 less than $t_{critical}$ = 2.779. This result indicates that there was no significant difference in the usefulness of Mathematics sub-scale prior to and post intervention scores of the experimental group.

Experimental versus control group comparison

Table 12 shows the scores of the experimental and control groups on the usefulness of Mathematics sub-scale after the intervention.

Table 12. The Usefulness of Mathematics post intervention scores for the control and experimental groups.

	Mean	Group size	$t_{calculated}$
Control	42	23	6.7236
Experimental	54	27	

$t_{critical} = 2.660$

The t-test was calculated to determine whether the Usefulness of Mathematics post intervention scores of the control group and the experimental group were significantly different at $\alpha = 0.01$ and $df = 48$. The t-test calculation yielded $t_{calculated} = 6.7236$ which is greater than the $t_{critical} = 2.660$, indicating a significant difference between the mean scores of the experimental and control groups.

Effectance Motivation in Mathematics sub-scale (EM)

The Effectance Motivation in Mathematics sub-scale (EM) is intended to measure effectance motivation (or problem-solving) as applied to Mathematics. The dimensions range from lack of involvement in Mathematics to active enjoyment and seeking of challenge. The scale is not intended to measure interest or enjoyment of Mathematics. Rather, it attempted to measure attitudes towards the enjoyment of Mathematics (Fennema and Sherman, 1976). The EM was administered prior to and after the intervention.

The *t*-test was used to compare the learners' motivation towards the learning of Mathematics. The effectance motivation results before the intervention yielded the mean scores of 45 and 46 for the control group and the experimental group respectively as shown in Table 13.

Table 13. The effectance motivation prior to intervention results for the control and experimental groups.

	Mean	Group size	$t_{calculated}$
Control	45	23	0.7804
Experimental	46	27	

$t_{critical} = 2.000$

The t-test calculations of the EM scores before intervention at $\alpha = 0.01$ and $df = 48$, yielded $t_{calculated} = 0.7804$. This result shows that there was no significant difference in the effectance motivation levels of the two groups at the beginning of the intervention.

Control group results

The Effectance motivation sub-scale results for the control group prior to and post intervention are given in Table 14.

Table 14. Prior to and post intervention EM scores for the control group.

	Mean	Number	$t_{calculated}$
Prior to Intervention	45	23	5.008
Post Intervention	36	23	

$t_{critical} = 2.819$

The EM prior to and after intervention mean scores of the control group were compared using a t-test to determine whether the difference was significant. The $t_{calculated} = 5.008$ is greater than $t_{critical} = 2.819$ indicating a significant difference at 0.01 level of significance (see Table 14). Deana (2007) and Njenga (2010) explains that when learners work alone, most of the problems they encounter are left unresolved which discourage them and this may result in low effectance motivation.

Experimental group results on Effectance motivation sub-scale

The comparison of the Effectance motivation prior to intervention and post intervention mean scores of the experimental group are given in Table 15.

Table 15. Prior to and post intervention Effectance motivation scores for the experimental group.

	Mean	Number	$t_{calculated}$
Prior to Intervention	46	27	1.6089
Post Intervention	48	27	

$t_{critical} = 2.779$

A t-test was calculated at the 0.01 level of significance and $df = 26$ and yielded $t_{calculated} = 1.6089$ less than $t_{critical} = 2.779$. This result shows that there was no significant difference in the effectance motivation prior to and after intervention mean scores of the experimental group.

Experimental versus control group comparison on Effectance motivation sub-scale.

Table 16 shows the scores of the experimental and control groups on the effectance motivation sub-scale after the intervention.

Table 16. The effectance motivation post intervention scores for the control and experimental groups.

	Mean	Group size	$t_{calculated}$
Control	36	23	7.3275
Experimental	48	27	

$t_{critical} = 2.660$

In order to test the null hypothesis that "There is no significant difference between the level of motivation of the Grade 11 Mathematics learners taught using cooperative learning and those who are not", a t-test was calculated to determine whether the Effectance motivation post intervention scores of the control group and the experimental group were significantly different at $\alpha = 0.01$ and $df = 48$. The t-test calculation yielded $t_{calculated} = 7.3275$ which is greater than $t_{critical} = 2.660$. Thus the null hypothesis was rejected in favour of the alternative. Table 16 shows that the post intervention mean score from the experimental group was higher than from the post intervention mean score of the control group.

Summary of the findings

The results of this study are summarised in Figure 1.

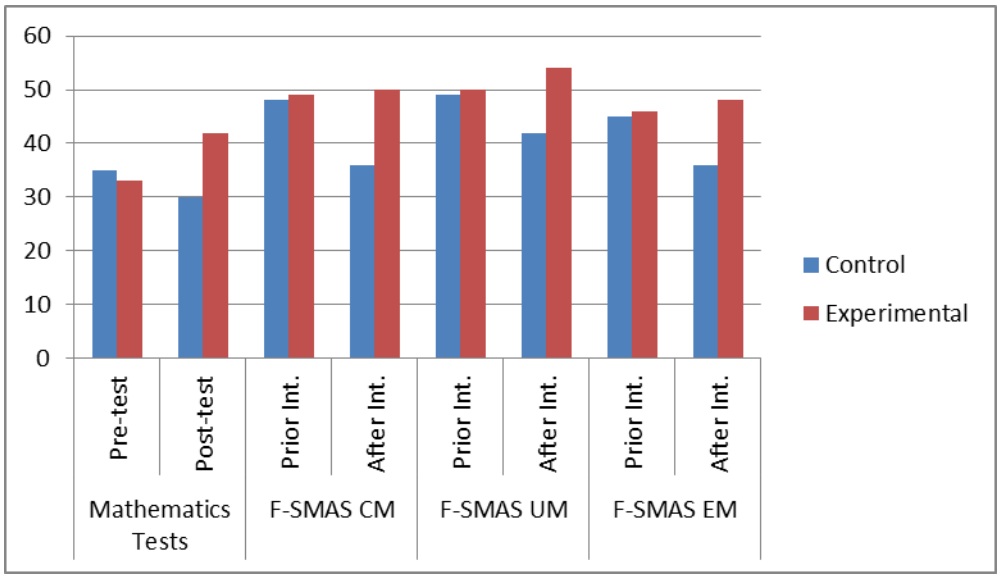

Fig.1 The summary of the results.

Figure 1 gives a graphical representation of the results of the study. It is worth noting that the experimental and control groups have close mean scores in the Mathematics pre-test and in the Fennema-Sherman Mathematics attitude scales (F-SMAS) prior to intervention for all sub-scales. This indication seems to suggest that the groups were equivalent before the intervention. After the intervention, the experimental group outperformed the control group (see Fig.1) in both the Mathematics post-test and the F-SMAS sub-scales post intervention. These results seem to suggest a positive effect of cooperative learning.

DISCUSSION OF THE RESULTS

This section discusses the findings presented in this chapter according to the following headings:

1. The effects of cooperative learning on the performance of Grade 11 learners in higher level Mathematics.
2. The effects of cooperative learning on the motivation of Grade 11 learners to study higher level Mathematics.

The effects of cooperative learning on the performance of Grade 11 learners in higher level Mathematics.

The comparison of the Mathematics pre-test of the experimental and the control groups reflected that there was no significant difference between the experimental group and the control group (Table 1). This means the experimental and control groups were almost equivalent with respect to mathematical knowledge at the beginning of the experiment.

The comparison of the Mathematics pre-test and post-test mean scores for the control group (Table 2) showed $t_{calculated} = 1.3767$ less than $t_{critical} = 2.819$. This result indicates that there was no significant difference between the pre-test mean and post-test mean of the control group at the 0.01 level of significance. On the other hand, the Mathematics pre-test and post-test scores (Table 3) showed a significant difference at the 0.01 level of

significance. The $t_{calculated}$ = 2.8595 greater than $t_{critical}$= 2.779. Indeed the mean score of the experimental group was better in the Mathematics post-test in comparison to the mean score in the Mathematics pre-test. The significant performance of the experimental group supports the views by Malin (2007) who warned that it is important to confirm that the intervention caused a significant change within the experimental group.

The experimental group performed significantly better than the control group on the Mathematics post-test. The $t_{calculated}$ = 3.306 greater than $t_{critical}$ = 2.660. The post-test mean scores of the experimental and control groups were significantly different at 0.01 level of significance (Table 4). Thus the null hypothesis that "There is no significant difference between the performance of the Grade 11 Mathematics learners taught using cooperative learning and those who were not" was rejected.

The experimental group mean score of 42.1 was greater than the control group mean score of 30.0. The results of this study seem to indicate that the cooperative learning approach resulted in higher achievement than the non-cooperative learning approach. The significant improvement in the performance of the experimental group supports the findings by Regnier (2009), Bawn (2007), Malin (2007), and Liang (2002) that cooperative learning enhances learners' performance. The possible reasons for the significant difference, found in the performance of the experimental group could be due to the learners' involvement in explaining and receiving explanations from fellow learners in which the concepts could be understood easily, and due to opportunities for students to solve problems collaboratively, create solutions, provide ideas and help each

other (Bawn, 2007). The results of this study suggest a positive effect of cooperative learning on the performance of the Grade 11 learners in higher level Mathematics compared to non-cooperative learning.

The Fennema-Sherman Mathematics Attitude Scales results

The second research question sought to determine the effects of cooperative learning on learners' motivation towards studying Mathematics. The motivation was measured using the Fennema-Sherman Mathematics Attitude Scale prior to and after the intervention alongside the Mathematics pre-test and post-test. Three sub-scales were used; the confidence in learning Mathematics, usefulness of Mathematics and effectance motivation in Mathematics. The results from these sub-scales are presented separately.

Confidence in learning Mathematics sub-scale (CM)

The Confidence in Learning Mathematics Scale was intended to measure the confidence in one's ability to learn and to perform well on mathematical tasks. The comparison of the CM prior to intervention mean scores of both the experimental (mean = 49) and control groups (mean = 48) reflected that there was no significant difference in the motivation level of the two groups prior to the intervention. This result seems to indicate that the control group and the experimental group were equivalent in terms of confidence in learning Mathematics prior to the intervention.

The comparison of the CM prior to intervention and after intervention mean scores in the control group (Table 6) showed a significant difference at the 0.01 level of significance. The learners scored lower in the CM after intervention. The drop in

confidence levels of the control group may be justified by what Njenga (2010) and Deana (2007) said that when learners work alone, most of the problems they encounter are left unresolved which discourages them to seek for more subject activities. It could be that the learners in the control group encountered some problems which they could not solve which may have resulted in the drop in their confidence to learning Mathematics.

Meanwhile, the comparison of the CM prior to intervention and after intervention scores (Table 7) in the experimental group showed no significant difference at the 0.01 level of significance. The prior to intervention mean was 49 and the post intervention mean was 50. The highest possible score was 60. This means the learners in the experimental group scored almost at the same level of confidence towards studying Mathematics prior to intervention and after intervention. The results may be due to the fact that the levels of motivation were already higher at the beginning and thus did not change significantly after the intervention.

The t-test calculated to test for significance in the after intervention CM results of the control and experimental groups (Table 8) indicated a significant difference between the control and experimental groups. The $t_{calculated} = 6.5490$ is greater than $t_{critical} = 2.660$. Njenga (2010) and Johnson et al. (2008) argue that the learners working cooperatively appeared to encourage each other to seek elective feedback to their responses and to practice content items during and after lessons and thus their confidence is mostly higher. High confidence may imply high self-efficacy to solve mathematical problems.

The development of learners' confidence in successfully completing a task is closely related to the effective use of learning strategies such as cooperative learning (Zimmerman, 1989). The post intervention Confidence in learning Mathematics results seem to indicate that learners who received the cooperative learning treatment displayed higher confidence in learning Mathematics than those who were taught in a non-cooperative learning classroom (Table 8).

The Usefulness of Mathematics sub-scale (UM)

The Mathematics usefulness sub-scale intended to measure students' beliefs about the usefulness of Mathematics currently, and in relation to their future education, vocation, or other activities (Fennema and Sherman, 1976). According to Hodges (2004) and Schunk (2005), learners are likely to perform well in school activities when they value a subject and when they know the importance of that subject in their own lives.

The mean scores from the usefulness of Mathematics prior to intervention was 49 and 50 for the control and experimental groups respectively. The t-test comparison of these scores of the experimental and control groups (Table 9) reflected that there existed no significant difference in the scores of the control and the experimental groups prior to the intervention.

The usefulness of Mathematics prior to intervention (mean = 49) and after intervention mean scores (mean = 42) in the control group (Table 10) showed a significant difference at 0.01 level of significance. The $t_{calculated}$ = 3.9665 is greater than $t_{critical}$ = 2.819. The mean scores indicate that learners in the control group scored lower in the usefulness of

Mathematics sub-scale after intervention. Liao (2005) argues that the lack cooperative learning model is likely to trigger low expectancy value resulting in a state where learners see no usefulness of the learning activities in their current or future lives.

The mean scores of the usefulness of Mathematics sub-scale prior to intervention and after intervention scores of the experimental group (Table 11) showed no significant difference at the 0.01 level of significance. The prior to intervention mean score was 50 and the after intervention mean score was 54. The experimental group scored almost at the same level on the Usefulness of Mathematics scale prior to intervention and after intervention. The highest possible score from the usefulness of Mathematics scale is 60. This indicates that the experimental group scores were higher in the Usefulness of Mathematics post intervention scale.

The t-test was calculated to test for a significant difference in the Usefulness of Mathematics post intervention mean scores of the control group and the experimental group (Table 12). The $t_{calculated} = 6.7236$ is greater than $t_{critical} = 2.660$, indicating a significant difference between the mean scores of the control group and the experimental group. The experimental group had a higher mean score (54) than the control group (42). This result corresponds to Liang's (2002) argument that cooperative learning helps learners to overcome Mathematics anxiety and realise how Mathematics is integrated in their daily lives, and thus viewing Mathematics as useful in their lives. Learners from the cooperative learning group attached more value to Mathematics compared to their

peers in the control group. These findings support those by Liao (2005) on the benefits of cooperative learning.

Effectance motivation in Mathematis sub-scale (EM)

The Effectance Motivation in Mathematics sub-scale (EM) is intended to measure effectance (or problem-solving) as applied to Mathematics. The dimension ranges from lack of involvement in Mathematics to active enjoyment and seeking of challenge. The scale is not intended to measure interest or enjoyment of Mathematics; rather, it attempts to measure attitudes towards the enjoyment of Mathematics (Fennema and Sherman, 1976). The EM was administered prior to and after the intervention.

The EM prior to intervention mean scores were 45 and 46 for the control group and experimental group respectively. The t-test calculation showed that there was no significant difference in the motivation level of the two groups prior to the intervention.

The comparison of the EM prior to intervention and after intervention mean scores in the control group (Table 14) showed a significant difference at the 0.01 level of significance. The learners scored lower in the EM after intervention with a mean score of 36 compared to the mean score of 45 prior to intervention. In non-cooperative learning settings, learners may have been discouraged by their fellow learners who laughed at their ideas and hence lost the desire to enjoy Mathematics activities (Jonson et al., 2008).

The EM prior to intervention and after intervention scores for the experimental group (Table 15) showed no significant difference at the 0.01 level of significance. The prior to

intervention mean of 46 was compared with the after intervention mean of 48 using a t-test. The calculations yielded $t_{calculated} = 1.609$ less than $t_{critical} = 2.779$. The learners in the experimental group scored almost at the same level in the EM prior to intervention and after intervention. Deana (2007) argues that during cooperative learning the learners encourage each other to work on more activities and acquire mastery, which in turn motivates them. In cooperative learning classrooms, learners get consistent encouragement from their peers which might enhance motivation (Johnson et al., 2008). Johnson et al. (2008) and Deana (2007) seem to explain the consistent higher effectance motivation sub-scale scores of the experimental group as due to the continuous encouragement.

The EM after intervention scores indicated that learners from the experimental group had significantly higher motivation levels than those in the control group (Table 16). The experimental group mean score of 48 was compared with the control group mean score of 36 using a t-test. The calculations found $t_{calculated} = 7.327$ greater than $t_{critical} = 2.660$. Thus the null hypothesis that "There is no significant difference between the level of motivation of the Grade 11 Mathematics learners taught using cooperative learning and those who are not" was rejected at 0.01 level of significance. Table 16 shows that the learners in the experimental group had higher motivation mean score than those in the control group.

This finding concurs with those by Liao (2005), Malin (2007) and Effandi & Zatlon (2006) that cooperative learning maintained a higher level of learners' motivation and

created a positive learning environment. They all found cooperative learning to have a positive effect on learners' motivation. The learners' motivation towards learning is retained with the use of cooperative learning due to a more enjoyable learning context, increased self efficacy and higher levels of learner accountability (Deana, 2007).

Conclusion

The results of this study revealed that using cooperative learning in teaching Mathematics at a higher level has a positive effect on the learners' achievement and motivation. The findings indicated that there were no statistically significant differences in the scores of the experimental and control group for both Mathematics pre-test and the Fennema-Sherman Mathematics attitude scales prior to intervention scores.

The Mathematics post-test scores and the Fennema-Sherman Mathematics Attitude Scale post intervention scores showed that the control and experimental groups' results were significantly different at $\alpha = 0.01$. The results of this study seemed to suggest that using cooperative learning to teach Mathematics enhanced learners' performance in Mathematics, raised confidence, boosted positive perceptions on the usefulness of Mathematics and enhanced the learners' motivation towards studying Mathematics.

CHAPTER FIVE: SUMMARY, CONCLUSION AND RECOMMENDATIONS

This chapter provides the summary of the study. The statement of the problem, the research methodology, and the findings are given. The chapter also provides the conclusion, recommendations and prospects for further research.

SUMMARY

This study sought to determine the effects of cooperative learning on learners' performance and motivation in Mathematics at one school in the Oshana Education Region.

The study addressed the following two questions:

3. What are the effects of cooperative learning on the performance of Grade 11 learners in higher level Mathematics in Oshana Education Region?
4. What are the effects of cooperative learning on the motivation of Grade 11 learners to study higher level Mathematics in Oshana Education Region?

Two hypotheses were also tested. These were:

1. There is no significant difference between the performance of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

2. There is no significant difference between the level of motivation of the Grade 11 Mathematics learners who are taught using cooperative learning and those who are not.

The study was carried out with 50 higher level Mathematics learners doing grade 11 at one senior secondary school in the Oshana Education Region. The school was purposefully selected and two Grade 11 classes doing higher level Mathematics were randomly selected to form the experimental group and the control group. There were 23 learners in the control group and 27 in the experimental group. The Mathematics pre-test and post-test on differentiation were used to test for the effect of cooperative learning on the learners' performance. After the Mathematics pre-test, the control and experimental groups were separately taught the same content on differentiation for five weeks by the researcher. The experimental group used a cooperative learning approach while the control group was taught in a non-cooperative learning approach.

In order to determine the effects of cooperative learning on the learners' motivation to study Mathematics, the Fennema-Sherman Mathematics Attitude Scale was used, specifically the three subscales; Confidence in learning Mathematics, Usefulness of Mathematics and Effectance motivation in Mathematics. The Fennema-Sherman Mathematics Attitude Scale was administered prior to and after the five week teaching intervention in order to determine whether cooperative learning had any effect on the motivation of the learners towards studying Mathematics.

The results of this study suggested a positive effect of cooperative learning on learners' performance in Mathematics compared to non-cooperative learning. The experimental group Mathematics post-test mean score of 42.1, was higher than that of the control

group mean score of 30.0. The performance of the control group and the experimental group was significantly different at the 0.01 significance level.

The Fennema-Sherman Mathematics attitude scales results also indicated that the post intervention mean scores of the learners in the control and experimental groups were significantly different at the 0.01 level of significance for all three sub-scales. The experimental group scored a mean of 50 compared to a mean score of 36 for the control group in post intervention mean scores of the confidence in learning Mathematics subscale. From the post intervention usefulness of Mathematics subscale, the experimental group scored a mean of 54 while the control group scored a mean of 42. The experimental group scored a mean of 48 compared to a mean score of 36 for the control group in post intervention scores of the effectance motivation sub-scale. These results seem to suggest that cooperative learning enhanced confidence, boosted positive perceptions of the usefulness of Mathematics and raised motivation level towards studying Mathematics. The findings of this study concur with the findings of other researchers such as Regnier (2009), Bawn (2007), Malin (2007) and Liang (2002) among others.

CONCLUSION

The cooperative learning approach seemed to have significantly improved the performance of the Grade 11 higher level Mathematics learners in the Oshana Education Region. The experimental group scored significantly higher (Mean 42.1) than the control group (Mean 30.0) in the Mathematics post test. The results seem to suggest that using the cooperative learning approach in schools might improve the results in the Oshana Education Region in Mathematics.

The Confidence in learning Mathematics post intervention mean scores of the experimental group (50) and control group (36) were found to be significantly different at the 0.01 level of significance. Similarly, the Usefulness of Mathematics post intervention mean scores of the experimental group (54) and control group (42) were found to be significantly different at the 0.01 level of significance. The Effectance motivation post intervention mean scores also yielded similar results with the experimental group (48) scoring significantly higher than the control group (36).

The results from the Fennema-Sherman Mathematics Attitude scales seem to suggest that cooperative learning may enhance the learners' Confidence in learning Mathematics, views on the Usefulness of Mathematics and motivation of the Grade 11 higher level learners in the Oshana Education Region towards studying Mathematics.

RECOMMENDATIONS

Based on the findings of this study the following recommendations are made:

1. Mathematics teachers should be encouraged to use cooperative learning to improve the academic achievement of their learners.
2. Cooperative learning strategies and materials which make the learning of Mathematics active, interactive, investigative and adventurous should be used in the teaching of higher Mathematics.

Further research

1. Further studies should be conducted to investigate the effectiveness of cooperative learning in other education regions and other subjects at both the ordinary and higher levels.
2. A study should be conducted on the teachers' perceptions towards the use of cooperative learning approach in Mathematics and other subjects.
3. A study may be conducted to investigate the effects of cooperative learning on the performance of small groups versus bigger groups.

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APPENDICES

Appendix A: Letter to the permanent secretary

Box 3714, Ongwediva

5th January 2011

To: Mr. A. Ilukena

The Permanent Secretary

Ministry of Education

Private Bag 13186, **Windhoek**

Dear Mr. Ilukena

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN OSHANA REGION ON THE TOPIC: *"THE EFFECTS OF COOPERATIVE LEARNING ON THE MOTIVATION AND PERFORMANCE OF GRADE 11 HIGHER LEVEL MATHEMATICS LEARNERS."*

I am a registered student for a Master's degree in Mathematics Education at the University of Namibia. In partial fulfilment to qualify for my Master's degree, I am required to conduct a research on the topic stated above. The goal of my research is to determine the 'effects of cooperative learning on the motivation and performance of higher level Mathematics learners', in the Oshana Education Region.

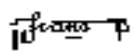
The Oshana region was lowly ranked in the 2010 national examinations rankings. Recent studies on performance in Mathematics suggest that the low performance can be

attributed to teaching methods (Nambira et al., 2009). This study therefore will investigate the effects of cooperative learning on learners' motivation and performance in higher level Mathematics in the Oshana Education region and the results might help the higher level Mathematics teachers to boost performance in the subject. Attached please find a copy of my research proposal. I would like to initially conduct a pilot study for a week in order to test for the validity and discrepancies in the research instruments before the main study.

I kindly request your good office to allow me to use one school in the Oshana region as my research site for the research project. I will conduct the study with two classes doing Mathematics on higher level. The data will be collected using observations, learners' pre-test & post-test and a motivation scale. The lessons will be based on Calculus, a topic in the higher level Mathematics syllabus. I hope to complete this study before the end of April 2012. The school and participants will be assured of confidentiality and anonymity in the final research report. A time table for class sessions with dates and times of visits will be provided, and will not interact with the normal class teaching time at the school.

For any clarifications, please contact me at +264 81 148 8183 or my Main Supervisor Professor Kasanda at +264 61 206 3726.

Yours Sincerely,



Frans Ndemupondaka Haimbodi

Appendix B: Permission from the permanent secreatary

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION

Tel: 264 61 2933200
Fax: 264 61 2933922
E-mail: mshimho@mec.gov.na
Enquiries: MN Shimhopileni

Private Bag 13186
Windhoek
NAMIBIA
9 February 2012

File: 11/2/1

Mr Frans Ndemupondaka Haimbodi
P. O. Box 3714
ONGWEDIVA

Attention: Mr F. N. Haimbodi

RE: REQUEST FOR PERMISSION TO CONDUCT A RESEARCH AT [redacted] SECONDARY SCHOOLS IN OSHANA REGION

Your letter requesting permission to conduct a research at the secondary school referred above to has reference.

Kindly be informed that the Ministry does not have an objection to your request to conduct a research at the school concerned.

However, you are kindly advised to approach the Regional Council Office, Directorate of Education, for authorization to enter and carry out your study at the school.

Kindly ensure that your research activities do not disrupt the normal school activities.

By copy of this letter the Regional Director of Education is made aware of your request

Yours faithfully

MINISTRY OF EDUCATION Office of the Permanent Secretary Private Bag 13186 Windhoek, Namibia 14 FEB. 2012

A Ilukena

PERMANENT SECRETARY

cc: Regional Director: Oshana

Appendix C: Letter to the regional director of education

P.O.Box 3714, Ongwediva

4th February 2012

The Regional Director of Education

Oshana Region

Private Bag 5518, OSHAKATI

Dear Mrs. Shinyemba

REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT ONE SCHOOL IN OSHANA REGION ON THE TOPIC: *"THE EFFECTS OF COOPERATIVE LEARNING ON THE MOTIVATION AND PERFORMANCE OF GRADE 11 HIGHER LEVEL MATHEMATICS LEARNERS."*

I am a registered student for a Master's degree in Mathematics Education at the University of Namibia. In partial fulfilment to qualify for my Master's degree, I am required to conduct a research on the topic stated above. The goal of my research is to determine the 'effects of cooperative learning on the motivation and performance of higher level Mathematics learners', at a School in the Oshana Education Region.

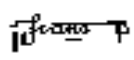
The Oshana region was lowly ranked in the 2010 national examinations rankings. Recent studies on performance in Mathematics suggest that the low performance can be attributed to teaching methods (Nambira et al., 2009). This study therefore will investigate the effects of cooperative learning on learners' motivation and performance in higher level Mathematics in the Oshana Education region and the results might help the higher level Mathematics teachers to boost performance in the subject. Attached

please find a copy of my research proposal. I would like to initially conduct a pilot study for a week at in order to test for the validity and discrepancies in the research instruments before the main study.

I kindly request your good office to allow me to use a school in the Oshana region as my research site for the research project. I will conduct the study with two classes doing Mathematics on higher level. The data will be collected using observations, learners' pre-test & post-test and a motivation scale. The lessons will be based on Calculus, a topic in the higher level Mathematics syllabus. I hope to complete this study before the end of April 2012. The school and participants will be assured of confidentiality and anonymity in the final research report. A time table for class sessions with dates and times of visits will be provided, and will not interact with the normal class teaching time at the school.

For any clarifications, please contact me at +264 81 148 8183 or my Main Supervisor Professor Kasanda at +264 61 206 3726.

Yours Sincerely



Frans Ndemupondaka Haimbodi

Mathematics Teacher, Gabriel Taapopi Secondary School, Oshana region.

Appendix D: Permission from the director of education

REPUBLIC OF NAMIBIA

OSHANA REGIONAL COUNCIL
DIRECTORATE OF EDUCATION
Aspiring to Excellence in Education for All

Tel: 065 – 230057
Fax: 065 – 230035
Email: wnewaka@yahoo.com

Private Bag 5518
Oshakati
NAMIBIA

Ref: 11/2/1
Enquiries: **Mr. Williams Newaka**

February 21, 2012

To: Mr. Frans Ndemupondaka. Haimbodi
Gabriel Taapopi Sec. School
Oshana Region

Dear Sir,

SUBJECT: REQUEST FOR PERMISSION TO CONDUCT RESEARCH AT [redacted] SCHOOL IN OSHANA REGION:

Your letter to the Ministry of Education requesting for permission to conduct a research at Mweshipandeka High School has references.

Kindly be informed that the Ministry of Education does not have an objection to your request to conduct a research study at the above mentioned school in Oshana Region.
Attached herein, is a copy of the authorization letter from the office of the Permanent Secretary that granted you permission for your research study.

However, you are kindly advised to ensure that your research study activities do not in any way disrupt the normal school activities.

Yours faithfully,

Shinyemba 21/02/12

MRS. DUTTE N. SHINYEMBA
DIRECTOR OF EDUCATION
OSHANA REGION

DIRECTORATE OF EDUCATION
OFFICE OF THE DIRECTOR OF EDUCATION
21 FEB 2012
PRIVATE BAG 5518 OSHAKATI
OSHANA REGIONAL COUNCIL
REPUBLIC OF NAMIBIA

Appendix E: Letter to the school principal

P.O.Box 3714, Ongwediva

21st February 2012

The Principal

…… School

P/Bag …….

Dear Sir/Madam

Re: RESEARCH TO BE CONDUCTED AT **…….. SCHOOL.**

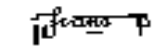
I am a registered student for a Master's degree in Mathematics Education at the University of Namibia. In partial fulfilment to qualify for my Master's degree, I am required to write a research report on the topic: *'effects of cooperative learning on the motivation and performance of higher level Mathematics learners in the Oshana region'.*

I therefore kindly, request your good office to allow me to carry out my research at your school. Attached please find the proof of permission to conduct the research in the Oshana Education Region, granted by both the office of the Permanent Secretary as well as the office of the Oshana Regional Director of Education, respectively.

I will conduct the study with two classes doing Mathematics on the higher level. The data will be collected using a pre-test & post-test and a motivation scale. The lessons will be based on Calculus, a topic in the higher level Mathematics syllabus. I hope to complete this study before the end of April 2012.The participants will be assured of confidentiality and anonymity in the final report. A time table for class sessions with dates and times of visits will be provided, and will not interrupt the normal class teaching time.

For any clarifications, please contact me at 081 148 8183 or my Main Supervisor Professor Kasanda at 061 206 3726.

Yours Sincerely,



Frans Ndemupondaka Haimbodi

Appendix F: Consent form: the school principal.

Consent form for the School Principal

Frans Ndemupondaka Haimbodi is hereby given permission to use ……School as the research site for the research study he is required to conduct in partial fulfilment for the Master's degree in Education of the University of Namibia.

I understand that:

- ✓ The data for analysis will be collected by means of employing the cooperative learning method in teaching the Grade 11 higher level Mathematics learners, administering the Mathematics pre-test & post-test and the motivation scales.
- ✓ The information from these instruments may be used in the final report of this study.

I have been assured that the school and the teachers will have anonymity in the final report and the information collected will be used for the sole purpose of the study.

_____ ____/ _____/ 2012

Principal's signature Date

Appendix G: Consent form: learners' parents

Instruction: Please fill out this consent form and return it.

I, _____, the parent of _____ a grade 11 learners at ……. hereby give consent for my child to be a subject in the study entitled "*effects of cooperative learning on the performance and motivation of high level Mathematics learners*" by attending the sessions, sit for the tests and completing the motivation scale.

I understand that:

1. My child is under no obligation to participate, and may withdraw from the study at any point prior to the publication or presentation of research results.
2. Anonymity will be maintained through the use of pseudonyms. The name of my child will not be reported.
3. The research will be used for academic and professional presentations and publications.

_____ **___/___/2012**

Signature **Date**

Appendix H: Mathematics pre-test

Participant code: … …

Instructions: **Do NOT** write your name on this paper.

Answer all questions and show your working.

Write your answers in the spaces provided after each question or part question.

Qn. 1 Each of *m, n, r, s* and *t* is a different number in the list 2, 3, 6, 7 and 8.
It is given that t $=s^3$, $m = 3s$, $s = t - m$, and $n = 3r - s$.

Find the values of *m, n, r, s* and *t.*

Answer; *m* = ...

n = ...

r = ...

s = ...

t = ... [5]

Qn. 2 A car left Rehoboth at 22:40 on the 18th February and arrived in Ongwediva the next day at 03:20. How long in hours and minutes, was the journey?

[3]

Qn. 3 Simplify

a) $-3m^3q^2 \times 4m^2q$

[2]

b) $3p^3(2p)^{-2}$ [2]

c) $8x^{\frac{2}{3}} \div 4x^{\frac{1}{2}}$ [3]

Qn. 4 Remove brackets and simplify: $2x - 3x(2x + 3)$ [3]

Qn. 5 Given $S = a + 4d$

a) Find S when $a = 17$ and $d = -5$.

Answer: S = ... [2]

b) Make d the subject of the formula $S = a + 4d$.

Answer: d = ... [3]

c) Find d when $S = 37$ and $a = 5$.

Answer: d = ... [2]

Qn. 6 Use the formula $P = \frac{V^2}{R}$ to calculate the value of P when $V = 6 \times 10^6$ and $R = 7.2 \times 10^8$.

Answer: $P =$ ..... [3]

Qn. 7 A straight line has a gradient of -3 and passes through the point (1, 3).

Find the equation of the straight line.

[4]

Qn. 8 Determine the gradient of the straight line that passes through the points K (1, 5) and L (-2, - 4).

[3]

Qn. 9 Find the equation of the straight line perpendicular to $y = 4x - 5$ and pass through the point (-2, - 4).

[5]

Total: 40

Appendix I: Mathematics post-test

Participant code: …

Instructions: **Do NOT** write your name on this paper.

Answer all questions and show your working.

Write your answers in the spaces provided after each question or part question.

Qn. 1 Find the derivative of

a) $y = -4x^5 - 8x^3 + 6$ [2]

b) $y = \sqrt{x}$ [2]

c) $y = \sqrt[4]{x^3}$ [3]

d) $y = 8x^4 - 6\sqrt{x} + \frac{5}{x}$ [3]

Qn. 2 Find the gradient function given,

a) $y = \sqrt[3]{4x^2 - 5}$ [3]

b) $y = e^{x^2 + 3x}$ [3]

c) $y = e^{x^2 + \ln x}$ [3]

Qn. 3 Find the equation of the tangent to the curve;

a) $y = x^3 - 3x$ at the point $x = -1$.

[4]

b) $y = \ln(x^2 + 1)$ at the point $x = 0$.

[5]

Qn. 4 Calculate the gradient of a tangent at $x = -2$ to the curve $y = 8 - x + 3x^2$.

[4]

Qn. 5 The line $y = 5x - 3$ is a tangent to the curve $y = kx^2 - 3x + 5$ at the point A. Find,

a) The value of k.

[5]

b) The coordinates of A.

[3]

Total: 45

Appendix J: Fennema-Sherman Mathematics Attitude Scale

Instructions. Participant code: … …

1. DO NOT WRITE YOUR NAME ON THIS PAPER.
2. Please indicate the extend of your agreement with each statement by ticking (√) in the column for: **strongly agree, agree, not sure, disagree and strongly disagree.**

Statement	Strongly agree	agree	Not Sure	Disagree	Strongly disagree
1. I am sure that I can learn math.					
2. Math is hard for me.					
3. I don't think I could do advanced math.					
4. I am sure of myself when I do math.					
5. Math has been my worst subject.					
6. I'm not the type to do well in math.					
7. I think I could handle more difficult math.					
8. I'm no good in math.					
9. Most subjects I can handle OK, but I just can't do a good job with math.					
10. I am sure I could do complex work in math.					
11. I know I can do well in math.					
12. I can get good grades in math.					
13. Math will not be important to me in my life's work.					
14. Knowing Mathematics will help me earn a living.					
15. I'll need Mathematics for my future work.					
16. I don't expect to use much math when I get out of school.					
17. Math is a worthwhile, necessary subject.					
18. Taking math is a waste of time.					
19. I will use Mathematics in many ways as an adult.					

20. I see Mathematics as something I won't use very often when I get out of high school.					
21. I'll need a good understanding of math for my future work.					
22. Doing well in math is not important for my future.					
23. I study math because I know how useful it is.					
24. Math is not important for my life.					
25. I am interested to learn new things in Mathematics.					
26. I don't understand how some people can get so enthusiastic about doing maths.					
27. I get a sense of satisfaction when I solve Mathematics problems.					
28. Having to spend a lot of time on a maths problem frustrates me.					
29. I plan to take as much Mathematics as I can during my education.					
30. I would like to avoid using Mathematics in college.					
31. I like to stick at a maths problem until I get it out.					
32. I can become completely fascinated doing maths problems.					
33. Learning Mathematics is enjoyable.					
34. If something about Mathematics puzzles me, I would rather be given the answer than have to work it out myself.					
35. The challenge of understanding maths does not appeal to me.					
36. I wish to avoid taking on Mathematics during my education.					

Appendix K: Permission to use the Fennema-Sherman Mathematics Attitude Scale

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Fennema-Sherman Mathematics Attitude Scale (1976)

Elizabeth Fennema efennema@wisc.edu 8:23 PM (12 hou ago)

to me

Dear Frans,

You have my permission to use the scales that you requested. But I think y ou will have to change the language of the various items so that they reflect your culture instead of the American culture in 1975. Just change the words in the items and try not to change the meaning. Perhaps you have a colleague who can compare your changes with the original items to see if the language is changed,

Best wishes to you and I hope you have success in attaining your masters degree.

Elizabeth Fennema

Appendix L: Mathematics pre-test and post-test scores

The raw scores of the control and experimental groups from the performance tests.

Group C (control group)			**Group G (experimental group)**		
Participant Code	Pre-Test	Post-Test	Participant Code	Pre-Test	Post-Test
C01	43	23	G01	40	50
C02	20	23	G02	40	68
C03	18	50	G03	40	35
C04	58	33	G04	43	38
C05	43	40	G05	33	30
C06	28	28	G06	23	20
C07	23	23	G07	38	45
C08	45	13	G08	25	48
C09	53	28	G09	30	53
C10	13	30	G10	28	38
C13	20	28	G11	18	15
C15	23	28	G13	40	70
C17	20	28	G14	10	15
C20	50	30	G15	38	48
C21	55	25	G16	25	45
C23	20	33	G17	38	68
C25	23	38	G18	35	13
C26	48	23	G20	53	20
C27	13	30	G22	28	40
C28	33	23	G23	25	53
C29	45	38	G24	40	45
C30	23	33	G25	18	48
C31	10	43	G26	38	40
			G27	43	63
			G28	45	40
			G29	43	50
			G30	25	40
Total	807	691		902	1138
Mean	35.1	30.0		33.4	42.1

The t-test for independent groups was calculated using the formulae;

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{s_1^{\,2} + s_2^{\,2}}{n_1 + n_2 - 1}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

And the t-test for non-independent scores; $t = \frac{\bar{D}}{\sqrt{\frac{\sum D^2 - \frac{(\sum D)^2}{N}}{N(N-1)}}}$.

Appendix M: The scores from the Fennema-Sherman Mathematics Attitude Scale

The table showing the pre-test and post-test results from the F-SMAS scale.

Control group									Experimental group								
Prior to intervention					Post intervention				Prior to intervention					Post intervention			
Code	C	UM	EM	Total	C	UM	EM	Total	Code	C	UM	EM	Total	C	UM	EM	Total
C01	46	50	43	139	50	46	36	132	G01	48	43	47	138	52	51	49	152
C02	48	54	40	142	45	43	39	127	G02	46	40	47	133	52	54	55	161
C03	51	59	55	165	38	41	28	107	G03	42	47	50	139	47	50	51	148
C04	44	53	46	143	42	47	41	130	G04	51	51	47	149	52	58	51	161
C05	49	47	49	145	43	41	42	126	G05	56	48	42	146	43	46	47	136
C06	44	46	42	132	36	47	41	124	G06	46	48	44	138	52	56	49	157
C07	49	55	48	152	27	55	45	127	G07	49	58	48	155	53	59	56	168
C08	43	40	40	123	41	38	28	107	G08	50	55	41	146	49	56	44	149
C09	47	40	40	129	38	48	39	125	G09	51	54	45	150	57	60	60	177
C10	44	52	46	142	20	52	39	111	G10	47	53	42	142	42	51	42	135
C13	47	35	43	125	39	43	42	124	G11	53	45	42	140	41	57	45	143
C15	51	52	43	146	39	40	28	107	G13	49	53	49	151	54	50	51	155
C17	48	55	49	152	39	41	38	118	G14	47	47	43	137	55	60	43	158
C20	44	45	38	127	21	30	32	83	G15	52	53	44	149	60	56	58	174
C21	52	51	49	152	26	39	24	89	G16	52	51	42	145	53	60	52	165
C23	40	44	33	117	35	41	33	119	G17	51	53	48	152	46	58	50	154
C25	51	55	44	150	29	35	18	82	G18	42	46	44	142	47	55	48	150
C26	44	53	53	150	34	50	38	122	G20	51	53	47	151	56	55	48	159
C27	36	42	39	117	45	44	38	127	G22	47	47	50	144	51	56	42	149
C28	56	49	46	151	39	42	41	122	G23	47	50	52	149	55	58	48	161
C29	57	56	46	159	38	35	45	118	G24	44	55	51	150	39	46	44	129
C30	60	60	58	178	41	46	33	120	G25	43	46	46	135	37	58	44	139
C31	55	56	50	161	25	28	35	88	G26	49	50	44	143	54	57	50	161
									G27	52	52	50	154	37	38	45	120
									G28	49	57	50	156	51	47	42	
									G29	51	54	48	153	44	46	39	
									G30	48	51	45	144	46	50	47	

Appendix N: Cooperative classroom rules

Basic classroom rules:

1. Listen to what others have to say.
2. Respect others and their ideas.
3. Take your responsibilities seriously.
4. Stick to the task at hand.

Actions of a cooperative group member:

1. Stays with the group, speaks quietly, and shares ideas and materials.
2. Addresses others by name, looks at the person speaking, and encourages others to participate.
3. Looks at the group's work and contributes ideas.
4. Allows each person to respond before speaking again.

Actions of an effective group member:

1. Criticizes ideas without criticizing people.
2. States the differences when there is a disagreement.
3. Pulls together all the ideas into a single position.
4. Asks others to verbalize how they would solve a problem or reach a decision.
5. Asks people to explain their reasoning.
6. Seeks elaboration by referring to other learning or knowledge.
7. Builds on others' ideas.
8. Listens to all ideas before reaching a conclusion.
9. Probes by asking in-depth questions that lead to deeper analysis.

NB: Well-constructed cooperative-learning exercises may be distinguished from simple group work by attention to four factors: 1. Careful distribution of students into groups; 2. Assignments of specific roles and responsibilities to each member of the group; 3. Specific and attainable objectives; and 4. A balance of emphasis on both group dynamic and individual accountability.

Appendix O: Lesson notes: Differentiation

DIFFERENTIATION

Differentiation is a study of functions that do not change at a constant rate. The value of the function called the derivative is that varying rate of change. In the Namibia Senior Secondary Certificate (NSSC) higher level syllabus, the following objectives are listed under Differentiation;

7. Demonstrate understanding of the concept of limit of a function.
8. Use the notations; $f'(x), \frac{dy}{dx}$, $f''(x)$ and $\frac{d^2y}{dx^2}$.
9. Recall and use the derivatives of ax^n (for any rational number *n)* , $a \ln x$, ae^x , together with sums, differences and composites of these.
10. Apply differentiation to gradients and tangents.
11. Locate stationary points, and distinguish (by any method) between maximum and minimum points and points of inflexion.
12. Express rates of change in terms of derivatives, and use differentiation to solve problems concerning rates of change, especially involving displacement, velocity and acceleration (rates of change of connected variables are not included). (NIED, 2010b).

Resources: 1.Text books - *Pure Mathematics 1* © 2005 by Rosemary Immanuel.

- Y= MX+C © 2012 by Karen D'Emilijo.

LESSON PLAN 1

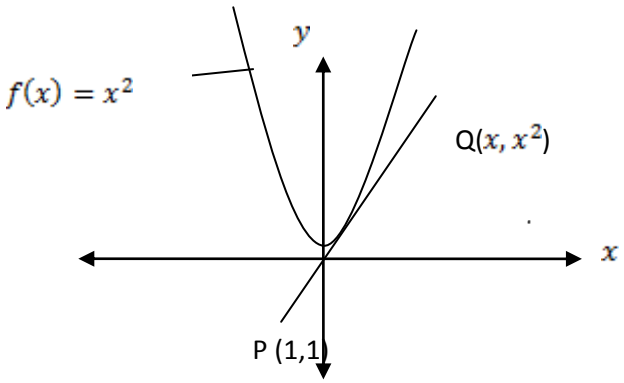
SUBJECT: *Mathematics*

TOPIC: *The limit concept and Derivatives* **GRADE:** 11

OBJECTIVES: By the end of the lesson, learners should be able to demonstrate an understanding of the concept of limit of a function. And use the notations; $f'(x)$, $\frac{dy}{dx}$, $f''(x)$ and $\frac{d^2y}{dx^2}$.

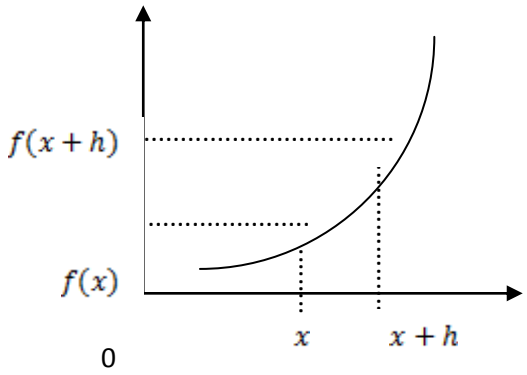
TEACHING RESOURCES:

Content	*Experimental group activities*	*Control group activities*
Introduction - State present lesson topic. Encourage learners to listen well and be active participants. - Mention application Derivates (differentiation) to daily life situations, e.g. in change and motion analysis especially involving displacement, velocity and acceleration and also in calculating of a gradient of a curve at a specific point. - Explain the concept limit and its relation to derivates. - Draw a graph of $f(x) = x^2$, ask learners how to find the gradient of the curve at point P (1,1) and another point Q (secant point) (x, x^2).	Listen and take notes	Listen and take notes.

 Gradient (M) = $\frac{\Delta y}{\Delta x} = \frac{x^2-1}{x-1} = \frac{(x-1)(x+1)}{x-1} = x + 1$		
<u>Teaching-Learning Phase</u> Explain how to calculate the gradient of the tangent line. Examples: $y = x^3$ at $x = 3$ Define differentiation as a technique used to calculate the gradient of a graph at different points. <u>Differentiation from first principles</u> Explain what first principle mean? - First principle means from basics Let $\Delta x = h$, if we take two points close to each other on the	Use the Jigsaw method	Work individually .

graph to calculate the of the line joining the two points, then we are approximating the gradient of the function $f(x)$.
$x-$ values are x and $x+h$ where h is some smaller number.
$y-$values corresponding to x *and* $x+h$ are $f(x)$ and $f(x+h)$,

The graph of $f(x)$:



Then the gradient is given by, $m = \frac{y_2 - y_1}{x_2 - x_1}$, the two points are $(x, f(x))$ and $(x+h, f(x+h))$

Therefore , $m = \frac{f(x+h)-f(x)}{x+h-x}$

$$m = \frac{f(x+h)-f(x)}{h}$$

As h becomes smaller and smaller the x and $x+h$ get closer to each other and m approximates the gradient of the function at the point $(x, f(x))$,$h \neq 0$ as it will lead to dividing with zero which is undefined. Instead we let h tend to zero $(h \to 0)$.

Hence: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$ is called the

Derivative

- Explain how to use the formula:
  Substitute the rule $f(x+h)$ and $f(x)$
  simplify to remove fractions that is when to substitute $h = 0$

This is <u>called differentiation from the First principle.</u>

Notations:

$f'(x)$ $(Prime\ notation)$ or $\frac{dy}{dx}$ (Leibniz notation)– The first derivative of $f(x)$

$f''(x)$ or $\frac{d^2y}{dx^2}$ - second derivative of $f(x)$

- Give examples how to differentiate using the definition of Derivates and how to find the gradient:
  Examples:

  1. (i) Find the derivates of $f(x) = x^2$ from the first principle

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$$

$= \lim_{h\to 0} \frac{2xh+h^2}{h}$ =

$\lim_{h\to 0} 2x + h = 2x$

(ii) Find the gradient of $f(x) = x^2$ when $x = 2$

$f'(2)=2(2) = 4$

2. (i) If $f(x) = 3x^2 - 4x$, find the derivative from first principle

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x+h)^2 - 4(x+h) - (3x^2 - 4x)}{h}$$

$$= \lim_{h\to 0} \frac{3(x^2 + 2xh + h^2) - 4(x+h) - (3x^2 - 4x)}{h}$$

$$= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 - 4x}{h}$$

$=\lim_{h\to 0}\frac{6xh+3h^2-4h}{h}=$ $\lim_{h\to 0} 6x+3h-4$ $==6x-4$ (ii) find $f'(-4)$ and state what this value indicates $f'(-4)=6(-4)-4$= -28 This value indicates the gradient of the curve at the point $x=-4$		
<u>Assessment:</u> 1. Find the derivates of the following functions using the first principle: $(a) f(x)=x^2$+2 $(b) f(x)=-3x^2$ 2. Given that $f(x)=4x^2+5$, find its gradient when $x=0$ 3. (i) If $f(x)=\frac{1}{2}x^2+3x$, determine $f'(x)$ from first principle. (ii) find the gradient of $f(x)=\frac{1}{2}x^2+3x$, where $x=\frac{1}{2}$	Qn. 1 and 2. Learners work in jigsaw metho.d. Qn. 3, learners work in think-pair-share method	Solve problems individually .
<u>Conclusion:</u> - Explain what learners are expected to know from		

presented topic. - State that **Derivative** is another name for gradient of a function, it is also known as a rate of change. Differentiation is a technique used to calculate the gradient. - Give feedback to the activity. - Give learners a chance to ask questions if any. <u>Next topic:</u> Rules for differentiation	Listen and take notes	Listen and take notes

<u>Lesson(s) Evaluation:</u> Learners seemed to have understood the topic well.

Frans N. Haimbodi

LESSON PLAN 2

SUBJECT: *Mathematics*

SUB-TOPIC: *Rules of Differentiation* **GRADE:** 11

OBJECTIVES: Recall and use the derivatives of ax^n (for any rational number *n)* , $a \ln x$, ae^x together with sums, differences of these.

TEACHING RESOURCES:

Content	*Experimental group activities*	*Control group activities*
Introduction - State present lesson topic. - Explain he link between previous lesson topic to the current one. - Ask learners how to differentiate using the first principle to recall learners' previous knowledge on the topic.	Listen and take notes. Discuss with teacher. Discuss in their groups and responds in turns of groups.	Answer individually.

Teaching-Learning Phase - Explain rules of differentiation : 1. The power Rule If $f(x) = x^n$ Then the derivate, $f'(x) = nx^{n-1}$ 2. The constant Rule $f(x) = c$ then $f'(x) = 0$ 3. The constant power rule $f(x) = cx^n$ then $f'(x) = cnx^{n-1}$ Note: *n* is a number. - Explain by giving problems how to differentiate using the above discussed rules. **Examples:** Differentiate the following functions 1. $f(x) = x^{-12}$ Answer : $f'(x) - 12x^{-13}$ 2. $f(x) = x$ Answer: $f'(x) = 1$ 3. $f(x) = -29$ Answer : 0 4. $y = \frac{1}{10}x^{-10}$ 5. Answer : $\frac{dy}{dx} = -x^{-11}$	Listen and take notes. Listen and take notes	Listen and take notes Listen and take notes

6. $h(x) = \frac{5}{6}x^{1\frac{1}{5}}$ Answer: = $\frac{5}{6}(\frac{6}{5})x^{\frac{6}{5}-1}$ $h'(x) = x^{\frac{1}{5}}$ - Give problems of functions involving roots and fractions, for the learners to discover how to differentiate them. **Problems: Write down the derivates of the following:** 1. $y = \sqrt{x}$ $(\frac{dy}{dx})$ 2. $S = \frac{2\pi}{5\sqrt{t^3}}$ $(\frac{ds}{dx})$		Solve problems individually.
Answers: 1. $y = \sqrt{x}$ $= x^{\frac{1}{2}}$ $\frac{dy}{dx} = \frac{1}{2}x^{\frac{1}{2}-1}$ $= \frac{1}{2x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$ **2.** $\frac{-3\pi}{5\sqrt{t^3}}$ - Explain and state the sum and difference rule by giving problems: i.e.	Solve the problems in think-pair-share methods.	

$if\ f(x) = h(x) \pm g(x)$ then $f'(x) = \frac{df}{dx} = \frac{dh}{dx} \pm \frac{dg}{dx}$ **Examples** (a) Determine $\frac{ds}{dx}$ if $s = xt\sqrt[3]{t} - \frac{2}{5t^5} + \frac{\sqrt[3]{t^5}}{4\sqrt{t}}$ (b) Write down $\frac{dv}{dt}$ for $v = \frac{6t^2+3t+1}{t^3}$ **Answers:** (a) $s = xt\sqrt[3]{t} - \frac{2}{5t^5} + \frac{\sqrt[3]{t^5}}{4\sqrt{t}}$ $= xtt^{\frac{1}{3}} - \frac{2t^{-5}}{5} + \frac{1t^{\frac{5}{3}}}{4t^{\frac{1}{2}}}$ $= xt^{\frac{4}{3}} - \frac{2t^{-5}}{5} + \frac{1}{4}t^{\frac{7}{6}}$ $\frac{ds}{dt} = \frac{4}{3}xt^{\frac{1}{3}} - \frac{2}{5} \times -5t^{-6} + \frac{1}{4} \times \frac{7}{6}t^{\frac{1}{6}}$ $= \frac{4}{3}x\sqrt[3]{t} - \frac{2}{t^6} + \frac{7}{24}\sqrt[6]{t}$ (b) $v = \frac{6t^2+3t+1}{t^3}$ $= \frac{6t^2}{t^3} + \frac{3t}{t^3} + \frac{1}{t^3}$ $= 6t^{-1} + 3t^{-2} + t^{-3}$	Listen and take notes	Listen and take notes

$\frac{dv}{dt} = -6t^{-2} - 6t^{-3} - 3t^{-4}$ $= -\frac{6}{t^2} - \frac{6}{t^3} - \frac{3}{t^4}$		
<u>Assessment</u> **Find the derivative of each of the following functions:** 1. $\sqrt{t} + 7\sqrt[5]{t^7} - \frac{1}{\sqrt[5]{t^2}}$ 3. $a = \frac{8+6x^2}{3x^4}$ $(\frac{da}{dt})$ 2. $y = 4t^{-8} + \frac{t}{\sqrt{t}}$	Solve questions in jigsaw methods.	Solve all questions individually
<u>Conclusion:</u> Summarises with reference to objectives of the lesson. Learners ask questions (if any). <u>Next topic:</u> The Chain rule		

<u>Lesson Evaluation(s):</u>
Learners understood the topic well.

Frans N. Haimbodi

LESSON PLAN 3

SUBJECT: *Mathematics*

TOPIC: *The Chain rule* **GRADE:** 11

OBJECTIVES: Recall and use the derivatives of the composite forms of ax^n (for any rational number n) , $a \ln x$, ae^x .

TEACHING RESOURCES:

Content	*Experimental group activities*	*Control group activities*
Introduction - Write present topic on the chalkboard. - Ask learners what they have researched on the topic. - Recap on composite functions $f(g(x))$. - Recap on differentiating functions such as $y = (x^2 + 3x)$ - State that the Chain rule is about differentiating composite functions e.g. $y = (x^2 + 3x)^{50}$.	Listen and take notes	Listen and take notes
Teaching-Learning Phase - State the Chain rule If we have $y = f(u)$ and $u = g(x)$ then the derivative of y is: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ - Give problems to explain how to	Discuss with teacher and fellow learners.	Discuss with teacher by responding to questions.

differentiate using the chain rule: **Examples:** 1. Find $\frac{dy}{dx}$ using the chain rule $y = (x^2 + 3x)^{50}$. **Solution:** Let $x^2 + 3x = u$ then $y = u^{50}$ $\frac{dy}{du} = 50u^{49}$ and $\frac{du}{dx} = 2x + 3$ $\frac{dy}{dx} = \frac{dy}{\cancel{du}} \times \frac{\cancel{du}}{dx}$ Therefore $\frac{dy}{dx} = 50u^{49} \times (2x + 3)$ $\frac{dy}{dx} = 50(x^2 + 3x)^{49} \times (2x + 3)$ $= 50(2x + 3)(x^2 + 3x)^{49}$ 2. Work out $\frac{d}{dx}\left(\frac{1}{\sqrt[3]{x^4+3x}}\right)$ $y = \frac{1}{\sqrt[3]{x^4 + 3x}}$ $y = \frac{1}{(x^4 + 3x)^{\frac{1}{3}}}$	Listen and take notes	Listen and take notes

$y = (x^4 + 3x)^{-\frac{1}{3}}$

$y = u^{-\frac{1}{3}}$ and $u = x^4 + 3x$

$\frac{dy}{du} = -\frac{1}{3}u^{-\frac{1}{3}-1}$ and $\frac{du}{dx} = 4x^3 + 3$

$$\frac{dy}{dx} = -\frac{1}{3}u^{-\frac{4}{3}}(4x^3 + 3)$$

$$\frac{dy}{dx} = -\frac{(4x^3+3)}{3\sqrt[3]{(4x^3+3)^4}}$$

- Explain how to differentiate logarithmic and exponential functions, state the rules.

Rules : 1. If $f(x) = e^x$ then $f'(x) = e^x$

2. If $f(x) = e^{ax}$ then $f'(x) = ae^{ax}$

3. If $f(x) = \ln x$ then $f'(x) = \frac{1}{x}$

Examples: <u>Differentiate</u>

1. $f(x) = 6e^{2x}$

Solution:

$f'(x) = 12e^{2x}$

2. $y = e^{-x}$ solution $\frac{dy}{dx} = -e^{-x}$ 3. $\ln(3x^2)$ 4. $\ln\frac{1}{\sqrt{x}}$ Answer: $y = \ln 3 + \ln x^2$ $= \ln 3 + 2\ln x$ $\frac{dy}{dx} = 2(\frac{1}{x}) = \frac{2}{x}$ Answer 4: $y = \ln\frac{1}{\sqrt{x}}$ $= \ln 1 - \ln x^{\frac{1}{2}}$ $= \ln 1 - \frac{1}{2}\ln x$ $\frac{dy}{dx} = -\frac{1}{2}\left(\frac{1}{x}\right)$ $= -\frac{1}{2x}$ - Explain another notation for derivatives D_x that its the same as one say $\frac{dy}{dx}$ or $f'(x)$	Listen and take notes. And discuss with fellow learners and teacher.	Discuss with teacher. Listen and take notes

Assessment: **Differentiate the following functions:** **1.** $y = \ln(2x^2)^3$ **2.** $\log_e(x^2 + 2x - 3)$ 3. $e^{x^4+\ln x^2}$ 4. $(\ln x + 4)^3$ 5. $D_t[-\frac{1}{2}t^{-5}+\frac{3}{t}-4t^4+5]$ 6. $s = \pi - \pi t^3$	Discuss in Think-pair-Share methods	Solve problems individually
Conclusion - Sum up main points with the reference to the objectives of the lesson. - Questions if any. Next topic: Gradients and Tangents	Listen and take notes	Listen and take notes

Lesson Evaluation(s):
Learners understood the topic well since they were all able to solve problems given.

Frans N. Haimbodi

LESSON PLAN 4

SUBJECT: *Mathematics*

TOPIC: *Gradients and tangents* **GRADE:** 11

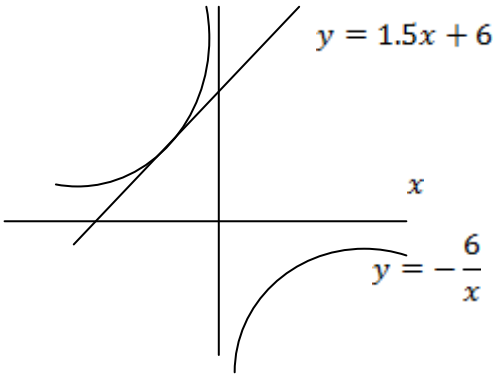
OBJECTIVES: Apply differentiation to gradients and tangents.

TEACHING RESOURCES:

Content	*Experimental group activities*	*Control group activities*
Introduction - Ask learners to recall applications of derivatives. **Applications:** Gradient calculation. - State present topic i.e. Gradients and tangents.	Groups take turns to respond to the teacher after a minute seconds of discussing.	Teacher calls for responses from different sectors of the classroom.
Teaching-Learning Phase - Explain how to calculate the gradient at a given point and how to calculate the y-coordinate at that given point. **Examples 1:** **1.** (a) Find the gradient of $y = 2x^3 + 6x - 5$ at the point where $x = -3$ **Answer** $f'(x) = 6x^2 + 6$	Teacher ask learners to relate to the *Graphs of functions* section and discuss with each other.	Teacher ask learners to relate to the *Graphs of functions* section.

$f'(-3) = 6(-3)^2 + 6$ $= 54 + 6 = 60$ (gradient at point $x = -3$) (c) Find the y −coordinates at the point. **Answer:** $y = 2x^3 + 6x - 5$ $= 2(-3)^3 + 6(-3) - 5$ $= -77$ **2.** Find the coordinates of teh point on the curve $y = -x^2 2x + 3$ at which its gradient is -2 **Answer:** $f'(x) = -2x + 2$ $-2 = -2x + 2$ $2 = x$ (x- coordinate) $y = -(2)^2 + 2(2) + 3$ $= 3$ ∴ Coordinates : $(2,3)$ - Give an exercise (see assessment exercise 1) - Ask learners what the tangent is? Tangent Is a straight line which touches the curve at a given point. - Explain by giving examples how to find the equation of the tangent. **Example 2:**	Listen and take notes.	Listen and take notes

(a) Find the equation of tangent to the curve $y = -\frac{6}{x}$ at the point where $x = -2$ **Answer :** $y = -\frac{6}{x} = 6x^{-1}$ $f'(x) = 6x^{-2} = \frac{6}{x^{-2}}$ $f'(-2) = \frac{6}{(-2)^2} = \frac{6}{4} = 1.5$(gradient at this point) $y = \frac{-6}{-2} = 3$ Coordinates of the shared point:$(-2,3)$ - Ask learners the general equation of a straight line; $y = mx + c$ $y = 1.5x + c$ Using the coordinates $3 = 1.5(-2) + c$ $-3 = 3 + c$ $c = 6$ Therefore the equation of tangent is: $y = 1.5x + 6$	Learners dialogue and respond to questions. Learners engage in discussions and respond to the questions.	Teacher calls for responses from individuals in the classroom. Teacher calls for responses from

 (b) Find the equation of tangent to the curve $y = 6 \ln 3x$ at the point where $x = 3$ **Answer:** $y = 6 \ln 3(3)$ $y = 6 \ln 9 = \ln 9^6 = \ln 531441$ To find the gradient of the tangent: $\frac{dy}{dx} = \frac{6}{x}$ (using the chain Rule) $f'(3) = \frac{6}{3} = 2$		individuals in the classroom.

Tangent equation; $y = 2x + c$ $\ln 5\,31441 = 2(3) + c$ $c = 7.18$ $\therefore$ Equation: $y = 2x + 7.18$		
<u>Assessment:</u> **Exercise 1** **1.** Find the gradient of the given curve at the given point on the curve and y −coordinates of the point. Hence give P(x, y) (a) $y = \sqrt{x} - 1$ where $x = 2$ (b) $y = \frac{(\sqrt[3]{x}-1)}{\sqrt[3]{x}}$ where $x = 27$ 2. Find the coordinates of the point(s) on the given curve at which its gradient has the given value. $y = \frac{9}{x}$ gradient -4 **Exercise 2** (a) Find the equation of tangent line to the curve $h(x) = \ln(x^2 - 2x + 1)$ where the curve crosses the positive x −axis. (b) Find the equation of tangent to the curve $f(x) = x^2 + 5x - 2$ at the point where the curve cuts the line $x = 4$.	Exercise 1 to be done using the jigsaw method. And groups present to the whole classgroup. Exercise 2 to be done in think-pair-share method.	Learners work alone and submit their work to the teacher.

<u>Conclusion</u> Sum up, stating how to find the gradient as well as the equation of the tangent line. <u>Next topic:</u> Stationery points and Rates of change		

<u>Lesson Evaluation(s):</u>

Learners understood the topic well as they well all able to solve problems involving gradients and tangents.

Frans N. Haimbodi

LESSON PLAN 5

SUBJECT: *Mathematics*

TOPIC: *Stationery points and Rates of change*

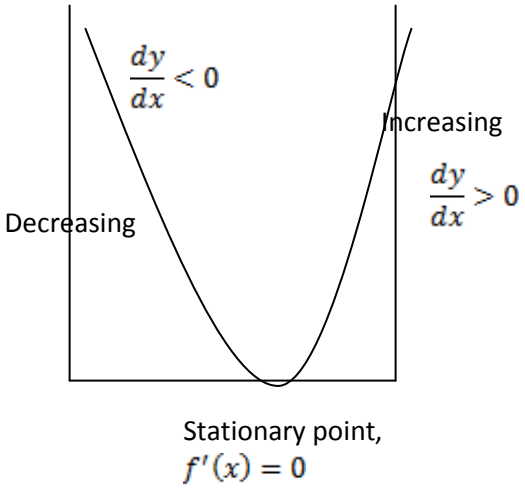
GRADE: 11

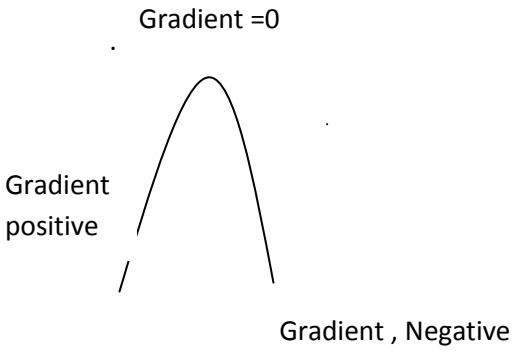
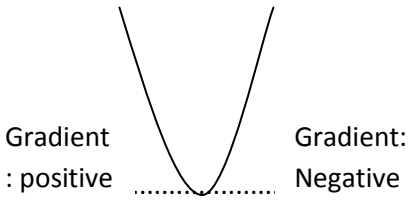
Objectives:

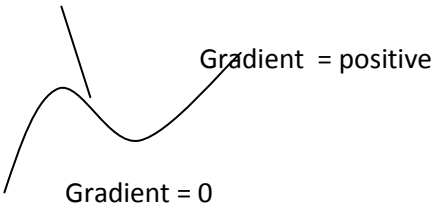
1. Express rates of change in terms of derivatives, and use differentiation to solve problems concerning rates of change, especially involving displacement, velocity and acceleration (rates of change of connected variables are not included).
2. Locate stationary points, and distinguish (by any method) between maximum and minimum points and points of inflexion.

TEACHING RESOURCES:

Content	*Experimental group activities*	*Control group activities*
Introduction - Ask if learners to recall applications of derivatives. - Encourage learners to listen well and be active during the lesson. - State present lesson topic and what is going to be discussed.	Groups take turns to respond to the teacher after a minute seconds of discussing.	Teacher calls for responses from different sectors of the classroom.
Teaching-Learning Phase - State that the derivative can be as well used to determine whether the function is **increasing** or **decreasing.** - Explain by demonstrating on a graph that If	Teacher ask learners	Teacher ask

$f'(x) = 0$ then the gradient is also zero(a horizontal line). The curve is said to have **a stationary point.**  -Explain how to find the stationary points by giving examples. **Examples 1:** Find the stationary values of $f(x) = x^3 - 12x + 1$ $f'(x) = 3x^2 - 12$ At stationary $f'(x) = 0$ $\therefore 0 = 3x^2 - 12$	to relate to the *Completing a square* section and discuss with each other. Listen and take	learners to relate to the *Completing a square* section. Listen and take

$3(x-2)(x+2)=0$ $x=2\ or\ x=-2$ $f(2)=-15$ and $f(-2)=17$ Therefore stationary points: $(2,-15)$ and $(-2,17)$ - Explain the 3 types of stationary points i.e. **maximum points, minimum points** and **point of inflexion** **Maximum TP**  **Minimum TP**  	notes. Listen and take notes	notes Listen and take notes

Gradient = 0 **Point of inflexion** Gradient =negative  see y=mx+c p.g. 269 the reverse **Example 2** Find the stationary points and determine the nature of the points. $y = 2 + 3x^2 - x^3$ $\frac{dy}{dx} = 6x - 3x^2$ $0 = 6x - 3x^2$ (turning point) $0 = 3x(2 - x)$ $\therefore x = 0$ or $x = 2$	Listen and take notes	Listen and take notes

$f(0) = 2 + 3(0)^2 - (0)^3$ $= 2$ $f(2) = 2 + 3(2)^2 - (2)^3$ $= 6$ Turning (stationary) points: $(0,2)\ and\ (2,6)$ Method 1: check the values of derived function either side of the x −coordinates. $f'(-1) = -9$ negative Gradient $f'(0) = 0$ minimum point $f'(1) = 3$ Positive gradient $f'(2) = 0$ Maximum point $f'(3) = -9$ Negative gradient ∴ Minimum stationary point:$(0,2)$ and Maximum stationary point: $(2,6)$ Method 2: Using the second derivative test. **Example 3** Find the stationary points and determine the nature of this point. $y = x^3$	Learners dialogue and discuss solution to example to with the teacher. Learners engage in discussions Discuss as they take notes.	Teacher calls for responses from individuals in the classroom. Listen and take notes Listen and take notes

Answer: $f'(x) = 3x^2$ $0 = 3x^2$ $x = 0$ $f(0) = 0$ Stationary points: $(0,0)$ <u>Determine the nature of this point</u> **<u>Method 1</u>** $f'(-1) = 3$ (Gradient positive) $f'(0) = 0$ (Point of Inflexion) $f'(1) = 3$ (Gradient positive) **Method 2: second- derivative test** $f''(x) = 6x$ $f''(0) = 6(0) = 0$ (point of Inflexion)		
<u>Assessment:</u> **Exercise** **1.** Find the stationary point(s) of the following functions and investigate their nature	Exercise 2 to be done in think-pair-share method.	

(a) $y = (2x + 1)(x - 3)$ (b) $y = 6x - x^2$ **2.** Find the coordinates of the stationary point(s) of the following and investigate their nature. (a) $y = x^2(3 - x)$ (b) $f(x) = 2x^3 + 7x^2 + 4x - 4$	Question 2 to be done using the jigsaw method. And groups present to the whole classgroup.	Learners work alone for all questions and submit their work to the teacher.
Conclusion Recap on lesson objectives. Learners ask questions if any. Next topic: INTERGRATION		

Lesson Evaluation(s):

Learners understood the topic very well.

Frans N. Haimbodi