INVESTIGATING THE GRADE 12 TEACHERS' SUBJECT AND PEDAGOGICAL CONTENT KNOWLEDGE OF MATHEMATICS IN SOME SELECTED PUBLIC SCHOOLS IN THE KHOMAS EDUCATION REGION, NAMIBIA

A THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE

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BY

JOSEPH JEULUKENI KANDJINGA

Student No: 200254758

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Main Supervisor: Dr Hileni M. Kapenda

APPROVAL PAGE

This research has been examined and is approved as meeting the required standards for partial fulfilment of the requirements of the degree of Master of Education.

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Internal Examiner   Date

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Dean of Education Faculty   Date

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External Examiner   Date

DECLARATIONS

I Joseph Jeulukeni Kandjinga, hereby declare that this study: *An investigation on Grade 12 teachers' Subject and Pedagogical Content Knowledge of Mathematics in some selected schools in the Khomas Education Region, Namibia* is my own work and is a true reflection of my research, and that this work or any part thereof has not been submitted for a degree at any other institution.

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DEDICATION

This thesis is dedicated to my wife, Tuuliki Ndakongele Kandjinga and my daughter, Blessing Tuuliki Kandjinga for their love, support and also for abandoning them sometimes during the research and writing of this thesis. The thesis is also dedicated to my father, Gabriel Mwashindange Kandjinga as well as my late mother, Rosalia Simon, for being there for me since childhood until I obtained my first degree, which was the stepping stone for other qualifications.

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ABSTRACT

Teachers' knowledge caught the attention of many educators in different disciplines such as Mathematics. In Namibia, some authors and organisations have claimed that Namibian Mathematics teachers at both Primary and Secondary School levels have insufficient Subject Content Knowledge (SCK) and Pedagogical Content Knowledge (PCK), which negatively impacted learners' academic performance over the past years. However, no study has been carried on the Grade 12 Mathematics' SCK and PCK in Namibia. Therefore, this study investigated the Grade 12 Mathematics teachers' SCK and PCK in some selected public schools in the Khomas Education Region.

A quantitative research design was utilised to collect the data of the Grade 12 Mathematics teachers in the Khomas Education Region. A stratified random sampling method was used to select 53 teachers from all four circuits in the Khomas Educational Region. Out of 53 Grade 12 Mathematics teachers, only 40 participated in the study; 13 teachers declined to take part. A closed-ended questionnaire and a test were used to collect data from the sample.

A Microsoft Excel spreadsheet was used to organise and present the data into graphs, figures and tables. Descriptive statistics was used to analyse the data obtained from the test. A correlation coefficient was used to describe the relationship between the teachers' knowledge (SCK and PCK), teaching experiences and their qualifications. A paired t-test was used to determine if a significant difference existed between the teachers' SCK and PCK. The findings of the study revealed the following: (i) Grade 12 Mathematics teachers have satisfactory Subject Content Knowledge (ii) Grade 12 Mathematics teachers have insufficient Pedagogical Content Knowledge of

Mathematics (iii) Grade 12 Mathematics teachers' SCK and PCK were strongly and positively correlated $(r = 0.681, p < 0.01)$ (iv) A significant difference existed between the Grade 12 Mathematics teachers' SCK and PCK with a *t*-test calculated value of 11.092 with a degrees of freedom of *39* at the significance level, α =0.01 (the $t_{critical} = 2.750$ value) (v) A strong positive relationship existed $(r = 0.87)$ between the Grade 12 Mathematics teachers SCK and their teaching experiences. (vi) No clear relationship was established between the Grade 12 Mathematics teachers' PCK and their teaching experiences $(r = 0.52)$ (vii) No clear relationship was established between the Grade 12 Mathematics teachers' knowledge (SCK and PCK) and their teaching qualifications.

Most of the Grade 12 Mathematics teachers (75%) acknowledged that some topics from the NSSCO Mathematics syllabus are difficult to explain to the learners. They also acknowledged that they require training on some topics such as Sequence and Series, Vectors, and Logarithms from the NSSCO Mathematics syllabus. Further, 55% of the participants indicated that they were not sufficiently prepared to teach Mathematics in terms of PCK, while 33% indicated that they were not adequately prepared in terms of SCK. The study recommends the following: The National Institute for Educational Development (NIED), in conjunction with the Ministry of Education, Arts and Culture should introduce an in-service training program in the Khomas Education Region to strengthen the Grade 12 Mathematics teachers' SCK and PCK. Further research should be conducted to determine the impact of Mathematics teachers' knowledge on the learners' academic performances of Mathematics in Namibia.

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LIST OF ACRONYMS

COACTIV	*Cognitively Activating*
DNEA	Directorate of National Examination and Assessment
MASTEP	Mathematics and Science Teachers Extension Program
MCK	Mathematics Content Knowledge
MBESC	Ministry of Basic Education, Sport and Culture
NBPTS	National Board for Professional Teaching Standard
NIED	National Institute for Educational Development
NMI	Namibian Mathematics Institute
NSSCO	Namibia Senior Secondary Certificate Ordinary level
PCK	Pedagogical Content Knowledge
SACMEQ	Southern and Eastern Africa Consortium for Monitoring Educational Quality
SCK	Subject Content Knowledge
TEDS-M	Teacher Education and Development Study in Mathematics
UNESCO	United Nations Educational, Scientific and Cultural Organization
UNICEF	United Nations International Children's Emergency Fund

CHAPTER ONE: INTRODUCTION

1.1 Background of the study

In its early history, teachers' knowledge has been a major focus among educators. Most emphases were placed on the teachers' Subject Content Knowledge (SCK), which entails, principles and concepts of the content (Saxena, 2015). However, in the mid-eighties, Shulman (1986) realised that there was a missing paradigm in teaching and coined the word Pedagogical Content Knowledge (PCK), which is the blending of pedagogical knowledge and content knowledge. This kind of knowledge (PCK) distinguishes between a qualified teacher and an unqualified teacher in the sense that qualified teachers are trained how to make the subject matter comprehensive to the learners through teaching methods, while the unqualified teachers are not. Shulman then proposed the model of teachers' knowledge that should be displayed by the teachers as they interact with the learners in the classrooms for effecting teaching and learners' understanding. Shulman's (1986) model of teachers' knowledge is divided into three main subcategories; Subject Content Knowledge, Pedagogical Content Knowledge, and Curriculum Knowledge.

Since the conception of the teachers' knowledge model by Shulman (1986), various researchers such as, DaniŞman and TaniŞli (2017); Tatto et al. (2012); An, Kulm and Wu (2004); Ball, Thames and Phelps (2008); and others have modified Shulman's original model of teachers' knowledge, by either making SCK as a subcategory of PCK or either adding new dimensions. Nevertheless, this study is based on Shulman's (1986) model of Teachers' Knowledge, with an emphasis on SCK and PCK. The reason for using Shulman's (1986) original model of teachers' knowledge in this study is because various researchers such Krauss, Baumert and

Blum (2008a) as well as Turnuklu and Yesildere (2007), validated that SCK and PCK are closely related kind of knowledge but different constructs of Knowledge.

 The importance of teachers' professional knowledge as an indicator of teachers' competencies is emphasised by different authors. For instance, Saxena (2015); Ding, He and Leung (2014) regard SCK and PCK as the crucial kinds of knowledge that lead to effective teaching and learners' understanding of the subject matter. According to Ding et al. (2014, p. 52), "SCK is grounded in the core teaching activities and influence teachers' decision making about content-specific instructions such as designing a task on posing a meaningful question for students' explorations. In contrast, PCK is regarded as a tool or vehicle" required for a teacher to deliver the content knowledge in the minds of the pupils in a comprehensive manner".

Echoing similar sentiments, Kathirveloo, Puteh and Matematik (2014), state that "the most important skill that a teacher should possess is the capacity to transform the knowledge to be taught to the students in a way that could easily be understood" (p. 2). That is, actual teaching should not only consist of the teacher demonstrating how to get the correct answer, but it also means that a teacher should find ways of how to make the subject better understood by the learners.

According to Ma'rufil, Ketu and Dwi (2016), teachers should master the subject content to be taught and they should also know how to transfer the content so that the learners can understand it with ease. A Chinese proverb quoted in An et al. (2004, p. 146) says "if you want to give the student one cup of water, you (teacher) should have one bucket of water of your own". This Chinese proverb implies that teachers

should know more than what they are to give to the learners in terms of the content of the subject.

On the same note, The National Board for Professional Teaching Standard ([NBPTS], 2002) in the USA states that teachers should understand the subjects they teach to the learners in the classrooms- that is, understanding the subject content and how the topics are linked to each other. For instance, Numbers as a topic is the foundation of other Mathematics topics. Therefore, before any other topic is taught, numbers should be taught first to the learners. Moreover, teachers' subject knowledge should be linked to pedagogical knowledge (French, 2005). Without a full grasp of PCK, teachers may not effectively deliver the content to the learners in a comprehensive manner, which may negatively impact the learners' academic performance. Similarly, Kapenda and Kasanda (2015) state that without enough subject knowledge little can be communicated to the learners, which may result in poor mastery and understanding of Mathematics. Thus, teachers should have sufficient SCK and PCK to maximise learning.

Literature has shown that there is little information about large-scale Mathematics projects that measured Secondary school Mathematics teachers' SCK and PCK. Up to date, the main projects carried out are the *Teacher Education and Development Study in Mathematics* (TEDS-M) conducted by Tatto et al. (2012), and the *Cognitively Activating* (COACTIV) conducted by Krauss et al. (2008b). TEDS- M comprised of 17 countries across all the continents and focused on Number and Operations, Geometry, Measurements, Algebra and Functions, while the COACTIV comprises of 198 teachers from Germany and focused on Mensuration and numbers.

Both the TEDS-M and COACTIV were quantitative projects and established that teachers with sufficient SCK had also adequate PCK. The COATIV project has also shown that teachers' SCK and PCK were correlated. Moreover, literature has also shown that there are few studies conducted that investigated the Secondary School Mathematics teachers' SCK and PCK as most studies focused on the Primary Mathematics teachers. However, such studies either focused on a particular topic or were carried out at particular schools (case studies). These content specific studies have established that many Mathematics teachers have shown insufficient SCK and PCK. Hurrell (2013) adds to this dispute that many teachers have an insufficient content knowledge and are not confident enough during the teaching and learning process of Mathematics.

In Namibia, literature has shown that no study has been conducted regarding the Grade 12 Mathematics teachers' SCK and PCK. However, few studies were conducted in Namibia that investigated the SCK of Primary school Mathematics teachers' SCK. These studies include The Southern African Consortium and Monitoring Education Quality [SACMEQ] (Ministry of Basic Education, Sport and Culture [MBESC], 2001, 2005: Miranda, Amadhila, Dengeinge & Shikongo, 2015); United Nations Educational Scientific and Cultural Organisation ([UNESCO], 2013); United Nations International Children's Emergency Fund ([UNICEF], 2011); and Haufiku (2008). Such studies revealed that Primary school Mathematics teachers have limited Subject Content Knowledge.

According to Haufiku (2008), few Mathematics programs were introduced to address the content knowledge of Mathematics teachers at primary and junior phases. These

programs include; the Mathematics and Science Teachers Extension Program (MASTEP) and the Namibian Mathematics Institute (NMI). Both MASTEP and NMI targeted the junior secondary teachers. Haufiku (2008) concluded that none of these programmes has addressed the content knowledge and skills of Mathematics teachers in Namibia.

The researcher is a Head of Department of Mathematics and Science and one of the core responsibilities is to conduct class visits. The purpose of the class visits is to identify teachers' weaknesses and strengths in relation to teaching and learning. During one class visit, a researcher witnessed a situation of a senior Mathematics teacher who was teaching *functions* and misled the learners. The instructions and questions about the problem were as follows: "*given two functions*; $f(x) = 2x + 3$ and $g(x) = 2x$, *find* $fg(2)$". *Instead of finding the value of* $g(2)$ *first, the teacher went on to find the value of* $f(2)$ *and substituted it into* $g(x)$, which showed insufficient SCK of functions in this particular teacher. Could the researcher not have intervened, then the teacher's misunderstanding of functions could have detrimental effects on the learners' understanding of functions.

Clegg (2006) claims that the greatest challenge in Namibia is to improve the competence of Mathematics teachers and this can be achieved by upgrading the formal qualifications and through in-service training. However, in-service training can only be implemented if the Ministry of Education, Arts and Culture is aware of teachers' characteristics that need to be addressed. Therefore, the need to carry out this study to investigate the Grade 12 Mathematics' SCK and PCK and also to fill the gap in terms of literature from the Namibian perspective.

1.2 Statement of the problem

Since teachers' SCK and PCK are the major kinds of knowledge required for effective teaching and learners' understanding, it is expected that Grade 12 Mathematics teachers should have sufficient SCK and PCK of Mathematics. However, throughout the researcher's experience of being a Subject Head of Mathematics for five years and Head of Department for Mathematics and Science for another five years running in the Khomas Education Region, he observed that some Mathematics teachers struggle to make the content accessible to the learners. Moreover, Mateya, Utete and Ilukena (2016) reported that the teaching of Mathematics in Namibian schools has been a concern since independence. Such concerns, according to Haufiku (2008), include poor subject content knowledge and knowledge of teaching Mathematics which contribute to poor performance among learners. Some organisations such as The Rossing Foundation (2015) speculate that poor performance in Mathematics among the Grade 12 learners is caused by poor content knowledge of the Grade 12 Mathematics teachers in Namibia. Additionally, Angula (2015) reported that some Mathematics teachers in the Otjozondjupa Region acknowledged that Mathematics is a difficult subject. Further, Clegg (2006) suggests that - both the pre-service and in-service teachers training in Namibia might not have equipped Mathematics teachers with sufficient PCK. He further observed that both the Bachelor of Education (B.Ed) and Basic Education Teacher Diploma (BETD) graduates have poor knowledge of fundamental Mathematics, inadequate skills of teaching problem-solving work, and most of their teaching is subject –centred and teacher – centred.

Based on the above-mentioned claims by different authors, it seems that most Mathematics teachers are experiencing some difficulties to teach Mathematics in Namibia. It also appears that Grade 12 Mathematics teachers have insufficient SCK as suggested by Haufiku (2008) as well as The Rossing foundation (2015). Kapenda and Kasanda (2015), as well as Mateya et al. (2016), advise that Namibian Mathematics teachers should possess the appropriate knowledge needed for teaching Mathematics. Therefore, this study investigated the Grade 12 Mathematics teachers' SCK and PCK in some selected public schools in the Khomas Education Region. The study also investigated the teachers' knowledge (SCK and PCK) in relation to their qualifications and teaching experiences.

1.3 Research questions

The following research questions were addressed in this study:

1. What is the Grade 12 Mathematics teachers` Subject Content Knowledge in some selected public schools in the Khomas Education Region?

2. What is the Grade 12 Mathematics teachers` Pedagogical Content Knowledge in some selected public schools in the Khomas Education Region?

3. What relationship exists between the Grade 12 Mathematics teachers' knowledge, teaching experience and qualifications in the Khomas Education Region?

1.4 Hypothesis

H_0 – There is no significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge.

H_1 – There is a significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge

1.5 The Significance of the study

Although this study only involved Grade 12 Mathematics teachers in the Khomas Education Region, the findings of the study might lay a strong foundation for policymakers and curriculum developers to formulate a policy that will strengthen SCK and PCK of Pre-service Mathematics teachers in Namibia. That is, Curriculum developers might ensure that modules that cater for both SCK and PCK at teachers training institutions should equip Pre-service Mathematics teachers with a deeper understanding of the PCK and SCK of Mathematics.

Moreover, since the findings of the study revealed that the Grade 12 Mathematics teachers' SCK and PCK are satisfactory and insufficient respectively, the NIED, in conjunction with the Ministry of Education, Art and Culture might plan and implement teachers' in-service training to strengthen Grade 12 Mathematics teachers' SCK and PCK. The Ministry of Education, Arts and Culture might also create an assessment criteria system that will evaluate the professional competence of Mathematics teachers before they enter the job market. It also might help Mathematics Education officers to organise workshops specifically meant to address the problem areas of PCK and SCK experienced by Mathematics teachers.

The Grade 12 Mathematics teachers in the Khomas Education Region might also benefit from the study, in the sense that they might become aware of their shortcomings with regard to Mathematics Content Knowledge. They might then opt to register for further courses that will improve their SCK and PCK. The Head of Departments (HODs) for Mathematics and Science in the Khomas Education Region might also benefit from these findings since they will be aware that their subordinates have insufficient PCK and satisfactory SCK.

The research instruments for the study might provide hints for assessing Mathematics teachers' SCK and PCK in Namibia. The results might also add to the field of research in the Namibian context since this is the first study of its kind that examined the Grade 12 mathematics teachers' PCK and SCK in Namibia.

1.6 Limitations of the study

The study was limited by some hindrances. Lack of finance was one the hindrances, in the sense that it made it difficult for the researcher to make copies for the research instruments on time, which resulted in the delay of the data collection process. Further, lack of finance also caused the participants to write the test unsupervised because the researcher could not afford to pay invigilators of 18 schools for the duration of 1hour and 30 minutes. Time was also one of the constraints, as the schools involved in the study are far apart and the researcher could not sample many schools in a day since the study was conducted after school hours. Furthermore, the use of interviews could have been used for triangulation purposes, however, due to the limited time, this was not possible.

Some of the participants declined to write the test which limited the findings of the study. Therefore, the researcher could not gather the amount of data he desired for the study. As indicated earlier in the introduction that no research had been carried out in Namibia regarding the Grade 12 Mathematics teachers' SCK and PCK, it limited the study in terms of national literature. The content of the research instruments of this study did not cover all the topics in the syllabus, which only limited the findings of the study to the topics that were assessed in the test. Additionally, teachers' SCK and PCK of Mathematics were measured through a written test and, as a result, the findings on the teachers' PCK cannot be compared to the practical PCK shown by the teachers during teaching in the classroom situations.

1.7 Delimitation of the study

The study only focused on the Grade 12 Mathematics teachers. Moreover, the content investigated was only restricted to Namibia Senior Secondary Certificate Ordinary level (NSSCO) Mathematics syllabus. The study was only carried out in the Khomas Education Region; therefore the results of this study cannot be generalized to other education regions in Namibia.

1.8 Definitions of terms

Pedagogical Content Knowledge (PCK): In this study, Pedagogical Content Knowledge refers to teachers' knowledge of Mathematics tasks, knowledge of identifying learners' misconceptions and difficulties, and knowledge of mathematics instructions (Kraus et al., 2008b).

Subject Content Knowledge (SCK): In this study, Subject Content Knowledge refers to the set of fundamental assumptions, definitions, concepts and problem- solving methods that constitute the ideas to be studied (Tatto et al., 2012).

Curriculum Knowledge: knowledge of the curriculum in this study refers to the teachers' awareness of a variety of instructional tools such as textbooks and visual aids to be used during teaching, instruments that can be used to assess the learners, and learning goals for the different grade levels (Bukava-Güzel, 2010) cited in Bukava-Güzel, E., Cantürk-Günhan, B., & Kula, S. (2013). It also includes the skill of interpreting the syllabus

Knowledge of learners' conceptions: knowledge of learners' conceptions in the study is defined as teachers' awareness of learners' pre-conceptions, misconceptions and difficulties to then correct any misconceptions they may have that can be used by the teacher during classroom teaching and lesson planning for effective teaching (Sibuyi, 2012).

Circuit: refers to the educational centre where Namibian schools in the region are clustered (Namupala, 2013).

Participants: In this study, participants refer to the Grade 12 Mathematics teachers that took part in the study.

1.9 Summary

In this chapter, the background of the study is discussed. Further, the statement of the study, research questions and hypotheses, the significance of the study, the limitation and delimitation issues are also discussed. The definitions of terms that are used in this study are also dealt with. Literature has indicated that various studies that focused on the teachers' content knowledge have been conducted and such studies are based on Shulman's (1986) model of Teachers' Knowledge. Literature has also indicated that teachers are required to have sufficient SCK and PCK needed for effective teaching. There are few large-scale Mathematics projects that investigated secondary schools' Mathematics teachers' SCK and PCK. The study might benefit educators, policy makers, curriculum developers, and professional development institutions to add value to the Grade 12 Mathematics teachers' SCK and PCK in the Khomas Region and Namibia at large. During the course of the study, there were some limitations that hindered the completion of the study as required. Such hindrances were: finances, time, lack of literature (Namibia context) and teachers' refusal to take part in the study. Further, the study was limited to the Grade 12 Mathematics teachers and Khomas Education Region, therefore the results cannot be generalised to other teachers and regions. In the next chapter, the theoretical framework and literature will be discussed.

CHAPTER TWO: THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

This chapter provides a theoretical framework on which this study is based. In addition, it also provides the literature review regarding Mathematics teachers' SCK and PCK. It also includes the relationship between SCK and PCK and their implications for learning.

2.2 THEORETICAL FRAMEWORK – TEACHER KNOWLEDGE

2.2.1 Shulman (1986) model of Teacher Knowledge

This study is underpinned by Shulman's (1986) model of Teacher Knowledge. As indicated earlier in the background of the study, Shulman (1986) identified three categories of Teacher Knowledge, namely; *Subject Matter Knowledge* (*Subject Content Knowledge), Pedagogical Content Knowledge an*d *Curriculum Knowledge.* These are the types of knowledge that teachers may need to display as they teach and interact with their learners in the classroom. Shulman then divided PCK into three sub-categories, namely; *knowledge of learning difficulties, knowledge of teaching strategies*, and *knowledge of conceptions and misconceptions*. Shulman's (1986) model of Teacher Knowledge is shown in Figure 1 below.
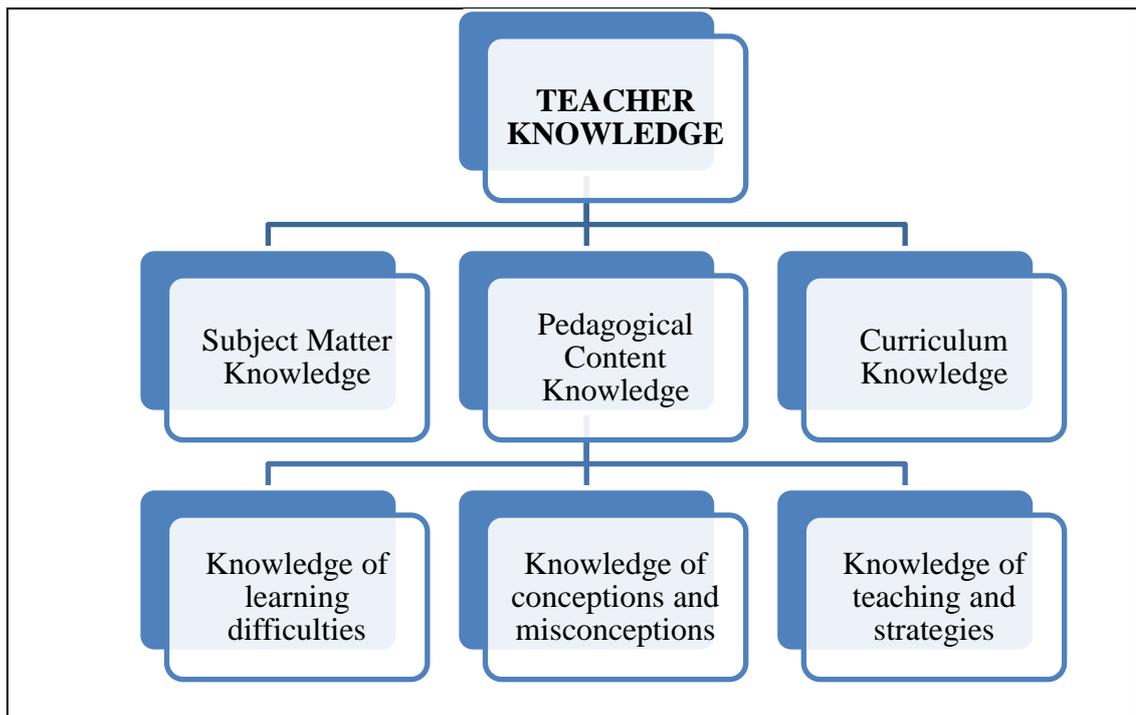
Figure 1: Model of Teacher Knowledge (Shulman, 1986)

2.2.2 Subject Matter Knowledge

Subject Content Knowledge has been variously defined. Shulman defines it as "the amount of organisation of knowledge…in the mind of teachers" (Shulman, 1986, p.9), which includes the understanding of the structure of the subject matter. For An et al. (2004, p. 147), Subject Content Knowledge refers to the "broad mathematics knowledge as well as specific Mathematics Content Knowledge at the level being taught". On the other hand, Mishra and Koehler (2006) define Subject Content Knowledge as the content to be taught to the learners. From the above-given definitions of SCK by different authors, it is clear that Subject Content Knowledge comprises of the principles, laws and concepts of a particular subject. Therefore, teachers should be knowledgeable about the subject content as indicated in the curriculum they are using.

Shulman (1986) adds that SCK also includes the structure of the subject which comprises of the *substantive* and *syntactic knowledge*. The *substantive knowledge* refers to "the variety of ways in which the basic concepts and principles of the discipline can be organised to incorporate its facts" (Shulman 1986, p. 9). It can be argued that *substantive knowledge* might also involve the identification of topics that are central to a specific discipline. For instance, in Mathematics, topics such as Numbers and Algebra can be considered to be central to the teaching of Mathematics because each topic needs an element of Numbers and algebra. Shulman seems to imply that teachers who understand the structure of a particular subject should be able to sequence topics of that particular subject in a way that shows how the topics are inter-related and how they complement each other.

"The *syntactic structure* of the discipline is a set of ways in which truth or falsehood, validity or invalidity, are established" (Shulman, 1986, p. 9). Kathirveloo et al. (2014) defined the *syntactic structure* as "the rules of evidence and proof within a discipline "(p. 2). On the same note, "teachers should comprehend the process of creative investigation and inquiry, whereby discoveries are made and new knowledge is formed, as demonstrated in the work of scholars" (NBPTS, 2002, p. 18). It can then be argued that the *syntactic structure* involves knowing how the knowledge of a particular discipline is generated and established. Thus, teachers should know the fundamental role that disciplinary study plays in the development of critical analysis and conceptual understanding (NBPTS, 2002).

Subject Matter Knowledge goes beyond theories, concepts, principles and facts about a particular subject. DaniŞman and TaniŞli, (2017) expanded that Subject Content

Knowledge requires teachers to know the prerequisite concepts or topics that are related to the learning content. For instance, a Mathematics teacher needs to teach Vectors as a topic first to the learners before attempting to teach topics such as Translation and Enlargement, simply because Mathematical shapes are translated and enlarged using vectors.

Teachers should also have knowledge of organising tasks for assessing learners' understanding of the subject matter (Krauss et al., 2008b). Tasks are important because many of Mathematics tasks of teaching Mathematical knowledge interact with the design of instructions (Ball et al., 2008). According to Ball et al. (2008), each task requires an interaction between specific Mathematical understanding and familiarity with learners and their Mathematical thinking. Ball et al. (2008) also advised that during the classroom discussions teachers should decide when to ask learners to clarify a point, when to use learners' remarks and when to ask a new question or give a new task to further learners' learning. Ball et al. (2008) then concluded that each of these requirements entails an interaction between Mathematical understanding and an understanding of the pedagogical issue that affects learners' learning. Thus, teachers should be in a better position to explain and clarify theories, concepts, theorems and principles of the subject that they are teaching to the learners in a comprehensive manner.

2.2.3 Pedagogical Content Knowledge

Shulman (1986) coined the term Pedagogical Content Knowledge after realising that ignoring one of the two types of knowledge (Pedagogical Knowledge or Content Knowledge) can make the other content useless as it will be content-free or

pedagogy-free. He then proposed an amalgamation – that is, the intersection of Pedagogical Knowledge and Content Knowledge to form what he terms "Pedagogical Content Knowledge" as displayed in Figure 2 below.
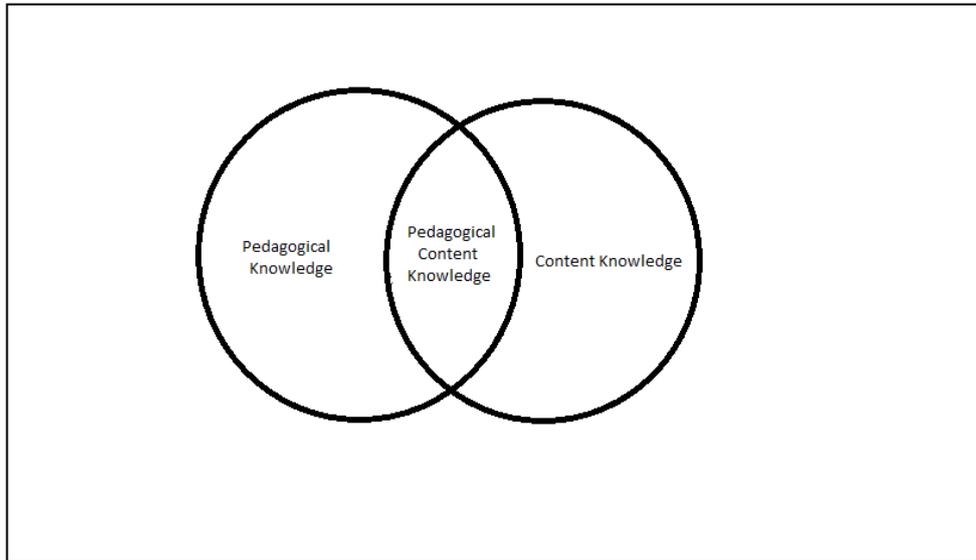
Figure 2: Pedagogical Content Knowledge as the intersection of Content Knowledge (CK) and Pedagogical (PK) - adapted from Mishra and Koehler (2006, p.1022).

Definition of PCK

Like Subject Content Knowledge, Pedagogical Content Knowledge is also variously defined. Shulman (1986) defines *Pedagogical Content knowledge* as the teachers' interpretation and transformation of Subject Content Knowledge in the context of making it comprehensive to the learners. It also includes "the most powerful analogies, illustrations, examples, explanations, and demonstrations, the ways of representing and formulating the subject that makes it accessible" (p. 9). Pedagogical Content Knowledge also comprises of "an understanding of what makes the learning of a specific topic easy or difficult: the conceptions and preconceptions that the learners of different ages and backgrounds bring with them to the learning of the

most frequently taught topics and lessons" (Shulman, 1986, p. 9). On the other hand, Tsafe (2013, p. 37) considers Pedagogical Content Knowledge as knowledge of "how to transform formal Subject Content Knowledge into meaningful learning outcomes for learners and it also involves an understanding of a particular topic and how teachers explained the topic or concepts to make sense to the learners in the classroom". Ball and Bass (2000, p. 88) defines Pedagogical Content Knowledge as a "special form of knowledge that bundles Mathematical knowledge with the knowledge of learner, learning and pedagogy".

Solis (2009) defines Pedagogical Content Knowledge as a special blending of "content and pedagogy that is uniquely constructed by teachers" (p.3). Further, PCK "comprises of integrated knowledge representing teachers' accumulated wisdom with respect to their teaching practice: pedagogy, students, subject matter, and the curriculum" (Solis, 2009 p. 3). Pedagogical content knowledge is an important type of knowledge that blends the subject matter with pedagogical strategies (Gess-Newsome, Carlson, Garder & Taylor, 2010).With reference to the definitions of Pedagogical Content Knowledge given above by different authors, it is evident that Pedagogical Content Knowledge involves the application of the subject content with a view of making it understood by the learners in the classroom.

Teachers are required to have a strong PCK of the subjects they teach as Morales, Anderson and McGowan (2003) attest that, having a strong PCK means that a teacher has a thorough understanding of Mathematics Content and knows the most developmentally effective ways of representing and explaining various topics and concepts to the learners. Echoing similar sentiments, Tsafe (2013, p. 37) states that

"teachers with a good Pedagogical Content Knowledge understand where learners may have trouble learning the subject matter and should also be able to represent Mathematical concepts in a way that their learners can comprehend its structure and avoid any difficulties". As indicated in Shulman's (1986) model of Teacher Knowledge in Figure 1, Pedagogical Content Knowledge is divided into three sub-categories of knowledge, namely: *knowledge of learning difficulties, knowledge of conceptions and misconceptions, and knowledge of teaching and strategies.* These sub-categories are described below.

(a) Knowledge of learning difficulties

DaniŞman and TaniŞli (2017) state that *knowledge of learning difficulties* expresses the problems that learners encounter or those they may experience when learning the content. Thus, teachers should use various forms of assessments to identify such difficulties and plan instructions required to overcome such learning problems. On the same note, teachers should anticipate what learners are likely to think through Mathematics tasks and what they will find confusing (Burke, 2013). Ball, Hill and Bass (2005) agree with Burke (2013), stating that a teacher should think from the learners' perspective and consider what it takes to understand a Mathematical idea for someone seeing it for the first time. An et al. (2004) noted that teachers need to attend to learners' Mathematical thinking- that is, preparing instruction according to learners' needs, and delivering instruction consistently with the learners' level of understanding.

According to DaniŞman and TaniŞli (2017), *knowledge of learning difficulties* requires teachers to show opinion about the conformance of the subject content to the

learners' level of development and their individual differences"(p. 25). DaniŞman and TaniŞli seem to imply that when preparing a task or the learning content, teachers should check if it meets the level of understanding required by the learners at a specific grade. Therefore, teachers should consider the individual differences and the cognitive development level of the learners when preparing learners' tasks.

(b) Knowledge of conceptions and misconceptions

Knowledge of conceptions and misconceptions involves teachers' ability to identify and interpret learners' emerging and incomplete thinking. Teachers are entrusted to enhance learners' academic learning; as a result, they are expected to be free of misconceptions and errors otherwise they will mislead learners during the teaching and learning process (Kathirveloo et al., 2014). According to Ma'rufi et al. (2016), a teacher should have knowledge of learners' mistakes and misconceptions about the subject matter to be taught so that he or she can adopt appropriate models, strategies and approaches meant to address such mistakes and misconceptions. DaniŞman and TanŞli (2017) suggested that, when a teacher is preparing the lesson, he or she needs to consider what the learners already know about the content to be learned. Moreover, teachers should ask probing questions at the introduction stage of the lesson in order to determine the prior knowledge of the learners which can also be used as a departing point of teaching. Further, teachers should detect the source of errors that learners make when completing tasks and this can be achieved by evaluating and analysing learners' reasoning (Ball et al., 2005). Ball et al. (2005) imply that when assigning a task, teachers need to anticipate how the learners are likely to do it and whether they will find it easy or difficult. For instance, if a teacher is preparing a task about solving linear equations, he or she should anticipate on how

learners are likely to solve a linear equation and errors that may arise in the process of solving a linear equation.

Additionally, teachers are required to be familiar with the structuring of examples during teaching so that they can build on the learners' prior knowledge in terms of difficulty (Burke, 2013). That is, teachers should decide on which example to start with and the one to end with so that they can take learners deeper into the content by using the previous examples as the foundation of the next examples. Such knowledge according to Hill, Ball and Schilling (2008) might help teachers to design instructions that will address issues such as, preconceptions, errors or misconceptions that the learners bring to the classroom. Hill et al. (2008) reasons as Shulman (1986), who states that a teacher needs an "understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that the learners of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons"(p. 9).

A natural question to ask is: "*What are the sources of the teachers' knowledge of learners' conceptions and difficulty?*" According to Li (2007), the teachers' knowledge of learners' conceptions and difficulties arise from the interactions between the learners and teachers, the learners and subject matter, other learners, the instructional technologies and other factors in the learning environment. Li implies that, as the learners ask questions, answer questions, do homework and exercises, and write examinations; they are indirectly providing information to the teachers to detect their conceptions, errors or misconceptions and difficulties that they bring to the classrooms. In agreement with Li (2007), Morales et al. (2003) stated that

"sometimes learners' prior knowledge contains misconceptions that conflict with developing an understanding of a current topic" (p. 44). This may suggest that, in the learning process, learners develop misunderstandings that may hinder their understanding of the subject matter. Effective teachers use various forms of assessment to identify the misconceptions and misunderstanding of their learners and then plan instructions to help their learners correct their learning (Morales et al., 2003). Therefore, teachers should ask learners to explain their Mathematical understanding in order to detect their misunderstanding and difficulties regarding the learning content and find ways to overcome such misunderstanding.

(c) Knowledge of teaching and strategies

This component of PCK is divided into two components, namely: association to the daily life, and use of representations. The two components are shown in Figure 3 below.
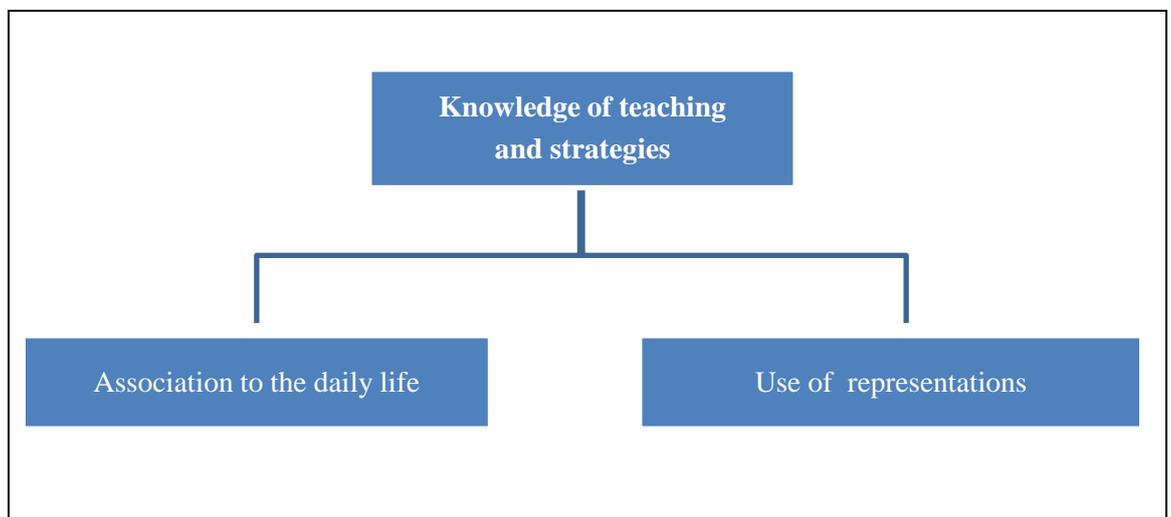
Figure 3: Sub-themes related to the knowledge of teaching and strategies. Source: DaniŞman and TaniŞli (2017).

Association to the daily life

DaniŞman and TaniŞli, (2017) state that this category requires teachers to relate the learning content to the real-life world – that is, finding the association between the learning content and real-life situations. The usage of daily life examples might contribute positively to learners' understanding of the topic since they will be able to relate these to what they have already experienced in the real-life setting. Liakopoulou (2011) adds that when planning the lessons, Mathematics teachers should ensure that the lessons are in relation to social issues and everyday life situations. Similarly, Ball et al. (2008) advise that, when choosing an example, teachers need to predict what the learners will find interesting and motivating. Thus, teachers need to select Mathematics examples that fit into the learners' lives (real-life examples) and relevant to the society because the degree of their motivation to learn the content might be based on how they value the learning content and its applicability thereof.

Use of multiple representations

This type of knowledge requires teachers to know how to evaluate and represent Mathematical ideas in multiple ways to help learners achieve mastery of specific concepts (Shulman, 1986). In favour of Shulman, Li (2007), states that teachers should use all sorts of examples, games, questions, explanations, worksheets, tools, models, tasks, diagrams and technologies during their lessons in their teaching. This is because using one instructional method might make the lessons boring and some learners might withdraw themselves from the lesson. This is mainly due to the fact that learners have different learning abilities. Both Li (2007) and Shulman (1986)

seem to suggest that using multiple representations of teaching will promote conceptual understanding of a particular content.

Being a Head of Department for five years, the researcher observed during the class visits that some Mathematics teachers only use textbooks in their teaching. Using textbooks as the only source of teaching aid might have a detrimental effect to learners' academic performance because textbook have limited information and some textbooks might not cover all the objectives and basic competencies as stipulated in the syllabus. Thus, teachers should use differential methods and various teaching aids during teaching and learning since learners learn differently.

Central to the usage of multiple representations is that teachers need to evaluate the instructional advantages and disadvantages of the type of representation to be used when teaching an idea (Ball et al., 2008). If a specific instructional strategy has many advantages that suit a particular content then it should be used to enhance learners' understanding. *But how do teachers acquire the knowledge of using multiple representations?* According to Das (2015), the knowledge of using multiple representations in teaching can be acquired through school education, teachers' preparation, professional development and teaching practices. Therefore, it is vital for Mathematics teachers' training and professional development institutions to ensure that pre-service teachers are equipped with skills on how to use various teaching aids in their teaching. It is important to promote the conceptual understanding of Mathematics among learners in the schools.

2.2.4 Curriculum knowledge

Shulman (1986, p. 9) indicates that the "curriculum is represented by the full range of programs designed for teaching particular subjects and topics at a given level with a variety of instructional materials available in relation to those programs". The teachers should, therefore, have an understanding of the *curriculum* which entails, knowing the breadth and depth at which it is appropriate to teach the topics for a particular grade level (Shulman, 1986). Kilic (2009) adds that curriculum knowledge includes knowing learning goals of different levels. In addition to the breadth and depth of teaching and learning goals, teachers are required to know what tools will aid their learners in discovering new ideas or comparing and connecting multiple representations of a specific topic (Burke, 2013). According to DaniŞman and TaniŞli (2017), knowledge of the curriculum constitutes the *vertical* and *horizontal curriculum* as shown in Figure 4 below.
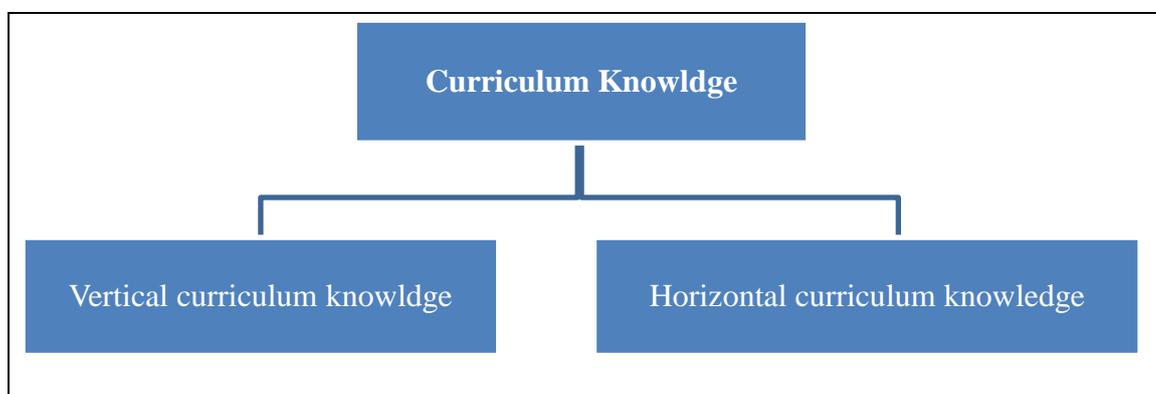
Figure 4: The sub-themes of the curriculum knowledge. Source: DaniŞman and TaniŞli, 2017

In agreement with Burke (2013), DaniŞman and TaniŞli (2017) define, the *vertical curriculum* as the knowledge of the topics taught in the other grade levels. This implies that when a teacher is introducing a particular topic, he or she should start from the basics principles that the learners were taught in the previous grade (s). On the other hand, the horizontal *curriculum* knowledge is considered as "the

Mathematical content knowledge of teachers at one grade level" (Danişman & Tanişli, 2017, p. 24). It implies that teachers should also explain to the learners how topics in a particular subject are connected to other domains. For instance, if a teacher is teaching Graphs as a topic in Mathematics, then the teacher should indicate how it can also be used in other subjects such as Physical Science, Biology and other subjects.

2.3 Teachers' training vs the development of teachers' SCK and PCK

Mogari (2014) says that quality Mathematics teaching is a function of teachers' performance which in turn depends on their abilities and competencies as well as the quality of teacher education programs. Thus, Mathematics teachers' training institutions should ensure that trainee teachers have what Shulman (1986) refers to as the Subject Content Knowledge and Pedagogical Content Knowledge required for effective teaching Mathematics. Baurmet et al. (2010), argue that teachers' programs that have limited SCK and PCK, have detrimental effects on the trainee teachers and consequently on the instructional quality in the classrooms. Guan, Khoh, Kin and Lam (2017) state that "Teacher preparation institutions normally focus on the pedagogical methods and expect the student teacher's prior university education to prepare him or her for the subject content" (p. 1). Kapenda and Kasanda (2015) advise that teacher preparation programs should ensure that Pre-service teachers are trained to an extent of bringing the learning content down to the level of learners that will enhance their understanding of Mathematics that they will be teaching.

In his study, Morgari (2014) reported poor SCK and PCK among South African Mathematics teachers and attributes it to Mathematics teachers' training institutions.

He indicates that teachers' programs are less rigorous in terms of the extent to which they prepare teachers. In the same vein, Shepherd (2013) claims that some teachers' training institutions only train teachers up to the grade level they would be teaching in terms of the content, which may not equip teachers with sufficient Content Knowledge required for effective teaching. Echoing similar sentiments, Ball et al. (2008) claimed that some courses are not related to the day – to – day reality of teaching. It seems that subject matter course in certain teachers' preparation programs tends to be irrelevant for classroom teaching in schools.

Similarly, Ball et al. (2005) report that many USA teachers lack sound Mathematical understanding and skills and attributed this to the fact that most of the teachers are graduates of the same system. The authors also argued that Mathematics teachers' own opportunities to learn Mathematics have been unequal and often inadequate, just like those of their non-teaching peers. Ball et al seem to suggest that teachers' preparation institutions are to be blamed for insufficient Mathematics knowledge among teachers for failing to equip Pre-service teachers with the necessary knowledge for teaching. They also seem to suggest that USA Mathematics teachers that took part in Ball et al. (2005) study have insufficient PCK since their understanding of how the content should be taught is similar to that of the unqualified teachers.

How do Mathematics teachers' SCK and PCK develop then? Shepherd (2013) claims that SCK is acquired by the teachers through pre-service training (in higher schools), while PCK is acquired through internships (teaching practice) or highly skilled training programs. For Baurmert et al. (2010), both SCK and PCK are obtained

through formal training at teachers' training institutions and supervised teaching practices, and the degree of knowledge attained depends on the intensity, duration and quality of the training and the supervised teaching practices. Krauss, Baumert and Blum (2008) argued that teachers' PCK increased by a strong base of their SCK, thus SCK is the only route to the development of PCK. Ball et al. (2005) recommend that teacher training institutions should train teachers about the Mathematics they will use in their jobs. Guan et al. (2017) concur with Ball et al, stating that teachers' courses should satisfy three basic requirements:

- The content and teaching approaches must clearly show how each topic is relevant to the curriculum to be taught in schools and hence the professional needs.
- The teaching approaches should role model a strong pedagogy and
- The course content and process should seek to develop in the potential teacher's familiarity with the practice of the disciplinary processes of Mathematics (p. 8).

In agreement with Guan et al (2017) views, Tirosh (2000) recommends that teacher education programs should familiarise prospective teachers with common, sometimes erroneous, cognitive processes used by learners in working out Mathematical problems and the effect of such processes. Guimarães, Sitaran, Jardon, Taguchi and Robinson (2013) also recommend that teachers' content knowledge should be increased through in-service training in order to improve learners' academic performances.

All in all, teachers training institution and professional development institutions should ensure that teachers are well equipped with an adequate depth of content

knowledge of the subject contained in the classroom, curricula and learners' minds. Additionally, curriculum developers should ensure courses offered to the Pre-service teachers are related to day-to-day reality. It is thus assumed that Pre-service teachers should be taught the Mathematics content that they will be teaching in high school during their final year so that they will be able to make the subject content understood by the learners; however, this is not a case in Namibia.

2.4 Mathematics teachers' status of SCK

Teachers' Mathematics knowledge plays an important role in their teaching of Mathematics (Hill et al., 2008). Thus, they need to know and understand the content that the learners need to master (Ball et al., 2008). Various researchers have investigated the Mathematical content of teachers at different Grades, however, most of the available findings focused on the primary phase and Pre-service teachers. Moreover, literature has indicated that there is little information about the SCK of in-service secondary school Mathematics teachers. The findings of Mathematics teachers' SCK are discussed below.

Turnuklu and Yesildere (2007) investigated 45 Pre-service Primary Mathematics teachers' competency of SCK in Turkey and their results revealed that Pre-service Primary Mathematics teachers lack sufficient content knowledge of Mathematics. They (Primary Mathematics teachers) experienced difficulty to understand the relationship between addition and subtraction. Pre-service teachers interpreted $5 - 3)$ as $+5 + (-3)$. If a teacher cannot explain the difference and similar features of $(+5) - (+3)$ and $+5 + (-3)$, then it is not possible for such a teacher to explain these operations and make them (operations) comprehensive to the learners.

Similarly, Olfos, Goldrine and Estrella (2014) studied the Content and Pedagogical Content Knowledge of 55 Grade 4 Mathematics teachers in Chile and found that the Grade 4 Mathematics teachers had little in-depth knowledge of fractions and instructions. The findings of Olfos et al. (2014) seem to suggest that such teachers cannot enrich learners with sufficient content knowledge of Mathematics. One should note that, at the primary phase, learners are supposed to get a strong foundation of Mathematics that will enable them to build on their future Mathematics courses. Additionally, a weak foundation of Mathematics at primary level may mean that learners might not pursue careers that require Mathematics.

Venkat and Spaull (2015) conducted a comparison study between American and Chinese Primary Mathematics teachers about solving "$1\frac{3}{4} \div \frac{1}{2}$". Their results revealed that only 9 out of 21 American teachers could answer the question correctly; whereas all (72) of the Chinese teachers were successful in getting the correct answer. Moreover, the American teachers who scored better in the test were much less successful than their Chinese counterparts in explaining why the process worked or in finding examples to exemplify the calculations. Surprisingly, American teachers had spent a longer period in higher education and have covered higher level courses before qualifying as teachers compared to their Chinese counterparts (Venkat & Spaull, 2015). The two authors appear to suggest that there is no connection between the higher level courses covered at teachers' preparation institutions and the teachers' competency in SCK. This is surprising because, in terms of SCK, teachers who have attended Advanced Mathematics courses are supposed to outperform their counterparts who attended lower-level courses.

Black (2009) assessed the Content and Pedagogical Content Knowledge of 65 Georgian secondary school Mathematics teachers and found that Georgian Secondary Mathematics teachers have a strong procedural knowledge of Mathematics. However, these teachers made errors in simplifying algebraic expressions, arithmetic errors, factorising, errors in substituting values into the quadratic formula ($-b \pm \frac{\sqrt{b^2-4ac}}{2a}$), and errors in writing inequalities. It is concluded that in spite of their strong procedural knowledge, Georgian Secondary Mathematics teachers that took part in the study have limited conceptual knowledge of Mathematics.

Similarly, Li (2007) investigated the Mathematical knowledge for teaching Algebra among 72 middle and secondary Mathematics teachers in Texas (United States of America). The results revealed that middle and secondary school Mathematics teachers have insufficient SCK of the concept of equivalent equations and properties of linear equations. Surprisingly, those teachers provided a wide range of identifying learners' misconceptions and difficulties as well as a variety of strategies for helping students to improve their understanding of how to solve linear and quadratic equations. Li's results are in line with Guan et al. (2017) who argue that "teacher preparation institutions normally focus on the pedagogical methods and expect student teachers' prior university education to prepare them for the subject content" (p. 1). Li's results seem to suggest that such teachers were adequately prepared in terms of PCK but inadequately prepared in terms of SCK.

Killic (2009) found that Georgian Pre-service Mathematics teachers' knowledge of Algebra was procedural and was grounded in their memorization of rules, facts and

procedures. Further, these teachers lacked a deep understanding of some Mathematics topics. In addition, Killic's (2009) findings indicated that Georgian Pre-service Mathematics teachers have limited knowledge of how Mathematical ideas are related to each other. Killic seems to imply that such Pre-service Mathematics teachers may not adequately transform the knowledge to the learners since their SCK is limited.

Morgari (2014) delivered an inaugural lecture at the University of South Africa (UNISA) entitled "A global perspective of Mathematics teaching: Implications for South Africa". Morgari states that several researchers indicated that Grade 11 Mathematics teachers in South Africa battled to teach Probability because of their poor content knowledge in Mathematics. Poor SCK among Mathematics teachers may mean that those teachers will not sufficiently explain the theorems, formulae, and principles of mathematics correctly to the learners and may consequently negatively affect their academic performances. In addition to poor academic performance, poor Mathematics foundation among the learners might also make learners develop a negative attitude towards Mathematics and consequently unwilling to pursue careers that require Mathematics courses.

Contrary to Morgari's (2014) work, Sibuyi (2012) investigated PCK of two Grade 12 Mathematics teachers in the Mpumalanga province (South Africa) and found that they (teachers) have adequate Subject Content Knowledge of quadratics functions topics. French (2005) advises that all stakeholders that are involved in Mathematics teachers' education should ensure that teachers are equipped with adequate sound understanding and skills of Mathematics. French believes that if the teachers are well

equipped with SCK, it will enable them to explain Mathematical content and tasks to the learners – that is, they will transform knowledge to the learners in a comprehensive manner.

2.5 Mathematics teachers' status of PCK

PCK allows teachers to make the learning content more or less accessible to the learners. One could thus conclude that PCK is the knowledge of teaching. Tsafe (2013) states that the way a teacher imparts knowledge to learners by using effective strategies during teaching is very important. Failure in this regard will lead to poor academic achievement among learners. A teacher with a good Mathematical Pedagogical Content Knowledge understands where learners may have trouble learning the subject, he/she should be able to represent Mathematical concepts in a way that their learners can comprehend its structure and avoid any difficulties (Tsafe, 2013). According to Liakopoulou (2011), teachers should plan their teaching and pedagogical actions according to the way students learn. That is, when teaching, teachers should consider learners' prior knowledge and experiences, the way they receive and organize new information, their cognitive needs and their motivation to learn (Liakopoulou, 2011). Echoing similar sentiments, Shulman (1986) advises that teachers need to identify learners' preconceptions and misconceptions that they bring to the classroom and they need knowledge of remedying such preconceptions and misconceptions.

The PCK of Mathematics teachers has been studied and various researchers have documented insufficient PCK of specific content among Mathematics teachers. For instance, Kaino and Moalosi (2013) investigated the Grade 12 teachers' pedagogical

content knowledge of solving linear equations with two variables. The results of their study revealed that many Grade 12 Mathematics teachers failed to explain thoroughly the characteristics of the equation system which indicated that they have insufficient PCK in teaching linear equations. Additionally, the Grade 12 Mathematics teachers also lacked the understanding of the nature of the problems that were to be solved and the meaning thereof.

Similarly, Sibuyi (2012) conducted a case study with two Grade 12 Mathematics teachers from the Mpumalanga Province (South Africa) and investigated the PCK of quadratic equations. He found that the two Mathematics teachers probed orally during teaching to identify learners' misconceptions about the quadratic functions; however, their questioning techniques were not effective. Sibuyi's (2012) findings can be interpreted that the two teachers have limited knowledge of identifying learners' misconceptions regarding the quadratic functions. Similar to Sibuyi's findings, Turnuklu and Yesildere (2007) found that Pre-service primary Mathematics teachers that took part in their study in Turkey had difficulty in determining learners' misconceptions about fractions and decimal fractions and they did not have sufficient knowledge of assessing learners during teaching.

According to Coe, Aloisi, Higgins and Major (2014), teachers must also understand the way learners think about the content, to be able to evaluate the thinking behind learners' own methods and identify learners' common misconceptions. Ball et al. (2008) advise that teachers do not only need to do Mathematics but they also need to unpack the elements of that Mathematics to make its features understood by the learners. On the same note, French (2005) advises that teachers should have the

knowledge and skills of identifying learners' misconceptions and the knowledge of identifying alternative ways of overcoming such misconceptions.

Coe et al. (2014) state that for a teacher to be effective, he/she needs to have a deeper knowledge of the subject she/he teaches, and when teachers' knowledge falls below a certain level, it negatively affects learners' understanding of the subject matter. Therefore, teachers training institutions should design programs that equip Pre-service teachers with a deep understanding of Mathematics so that they can make the subject content accessible to the learners during teaching.

2.6 The relationship between teachers' PCK and SCK

From the definition of both SCK and PCK stated earlier in parts 2.2.2 and 2.2.3 respectively, it is evident that SCK is more concerned with the principles, theories and concepts of a subject, while PCK is the knowledge of making the principles, concepts and theories understood by the learners. According to Li (2007), teachers' SCK is a prerequisite to their reasoning and decision making in specific contexts. Similarly, Guan et al. (2017) indicated that SCK defines the possible scope for the development of PCK.

Ball and Bass (2000) indicate that the debate among educators about which is the most important one between teachers' SCK and PCK remain unclear. However, Baurmert et al. (2010) in their study of Grade 10 German Mathematics school teachers wanted to find out if PCK is unimaginable without a substantial level of SCK and also to find if SCK can act alone. Their results revealed that "PCK is inconceivable without a substantial level of SCK but that SCK alone is not a

sufficient basis for teachers to deliver cognitively activating instructions that, at the same time provides individual supports for student learning" (p.164). They further found that PCK had greater power over SCK in explaining learners' progress. However, they contradicted themselves after they indicated that "these results do not imply that SCK has no direct influence on instructional features". In fact, "teachers with higher SCK scores are better able to align the material covered with the curriculum" (Baurmert et al., 2010, p. 164).

Similarly, Krauss et al. (2008b) researched the competency of teachers via the **Co**gnitively **Activ**ating (COACTIV) test of PCK and CK to address prospective Mathematics teachers' knowledge in German. The COACTIV test was meant to establish the construct validity of CK and PCK. The sample of the COACTIV comprises of the practising Mathematics teachers (n=198), prospective secondary school Math teachers (n=30) and school students in advanced Grade 13 Mathematics courses (n=30) that were used to validate the research instruments. The result of their study revealed that PCK is deeply interrelated with CK and CK is a prerequisite for PCK. The COACTIV study implies that teachers should have a thorough understanding of SCK since it facilitates the construction of PCK in the teachers' minds. Moreover, it is not possible for a teacher to make the subject content accessible to the learners if he or she doesn't understand the basic strands of the subject content.

On the same note, the NBPTS (2002) is of the opinion that both SCK and PCK are important in teaching stating that "teachers draw on pedagogical and subject matter understandings to respond to common misconceptions with content areas; address

challenging aspects of learning acquisition; and accommodate prior knowledge, experience, and skills that students at different developmental levels typically bring to the classroom" (p. 20). Looking at the analysis of the arguments above, it can be argued that both the SCK and PCK are vital for effective teaching because a teacher needs to understand the subject content first before making it comprehensive to the learners.

A question arises whether SCK is enough to teach Mathematics? Various researchers argued that a thorough understanding of SCK is important but not adequate to make the subject matter comprehensive to the learners (Turnuklu & Yesildere, 2007; French, 2005; Kilic, 2009; Goos, 2013; Mewborn, 2001; Ball et al., 2008; Raychandhuri, 2013; Olfos et al., 2014; Das, 2015; Baurmert et al., 2010). Goos (2013) clarifies that SCK is insufficient for effective teaching because teachers also need to know how to transform Mathematical content to make it comprehensive to the learners. Thus, it can be argued that PCK is required for effective teaching of Mathematics which is in line with Shulman (1986) who states that "a teacher need not only to understand that something is so, the teacher must further understand why it is so"(p. 9). In agreement with Shulman's argument, Ball et al. (2008) state that teachers need to show that the answer is correct and also explain what the steps of getting a correct answer mean and why they make sense. Central to this point is not what teachers need to teach, but what they themselves need to know and be able to do in order to carry out any responsible form of the teaching (Ball et al., 2008). The arguments of Shulman (1986) and Ball et al. (2008) are vital because some teachers might have passed Mathematics courses at universities with exceptional grades but

may not necessarily make the content accessible to learners and the opposite is also true.

Furthermore, Ding, He and Leung (2014) examined the connection between SCK and PCK of the three-term ratio among six Chinese pre-service Mathematics teachers. Their findings suggest that Pre-service Mathematics had limited conceptual understanding of ratio, which influenced their presentation of the concept of three-term ratio. Hill et al. (2008) argue that there is a powerful relationship between what a teacher knows, how he/she knows it and how he/she can do it in the context of instructions. This argument is supported by Killic's (2009) findings who found that Georgian Pre-service secondary Mathematics teachers' repertoire of teaching strategies was limited by the robustness of their subject-matter knowledge. Therefore, the findings above appear to imply that both SCK and PCK complement each other and are important in the effective teaching of Mathematics and any slip-up in one area will negatively affect learners' understanding of Mathematics.

Luna and Aclan (2015) investigated the SCK of Grade 5 Mathematics teachers in the Philippines and the influence of Mathematics teachers' PCK on the learners' achievement. Their study revealed that some Grade 5 Mathematics teachers did not experience difficulties in solving Mathematical problems. In addition, these Grade 5 Mathematics teachers demonstrated procedures for getting correct answers, which can be interpreted that they have sufficient content knowledge of Mathematics. However, they could not explain why the procedure is mathematically correct or why is it not applicable in some other situations. It implies that these teachers might have understood Mathematical concepts during training but were not sufficiently exposed

to the teaching methods of such Mathematical concepts. Therefore, teachers' programmes should cover adequate depth, breadth and thoroughness of Mathematics to be taught at schools with built-in PCK on top of the SCK foundation (Guan, et al., 2017).

The natural question to ask is: "*How do teachers' Subject Content and Pedagogical Content Knowledge vary with teachers' qualifications"?* There is little information documented regarding teachers' performance in both SCK and PCK questions in relation to their qualifications. French (2005) and Olfos et al. (2014) indicate that high-level Mathematics qualifications are much less important than the depth of teachers' understanding and abilities to make connections within the school Mathematics curriculum. That is, some teachers might have passed their subjects well at university but might not sufficiently make the content accessible to the learners. This is simply because they were probably not sufficiently exposed to the correct teaching methods during their training or that they might probably not have grasped the teaching methods well during training.

Morgari, Stols and Ogbonnaya (2009) cited in Morgari (2014) found that Grade 11 Mathematics teachers with the higher quality qualifications of Mathematics in South Africa were more effective in their teaching than their counterparts with lower quality qualification. Similarly, Li (2007) found a linear relationship between teachers' performances in Mathematics and the number of advanced Mathematics courses they were taught at teachers' training institutions.

On the same note, the COACTV (Kraus et al., 2008) study found that Mathematics teachers with an in-depth Mathematical training outscored their counterparts on both SCK and PCK. Furthermore, German secondary Mathematics teachers that received an in-depth Mathematical training also exhibited a high degree of cognitive connectedness between SCK and PCK. Morgari; Li; and Kraus et al seem to imply that in-depth Mathematical training consists of courses that are rich in both SCK and PCK. Further, if a teacher is taught a high-level Mathematics complemented by a high level of the methodology of teaching, then such a teacher is more equipped with both SCK and PCK required for the effective teaching of Mathematics.

How do teachers' Subject Content and Pedagogical Content Knowledge vary with their teaching experience? There is a limited number of research information in the literature about the relationship between teachers' performance in both SCK and PCK questions with regard to their teaching experiences. Sibuyi (2012) conducted a case study at one of the performing schools in the Mpumalanga province (South Africa) and found a positive relationship between teachers' content knowledge of quadratic functions and Mathematics teachers' teaching experience among the Grade 11 Mathematics teachers. However, despite their teaching experience of 11 years, their knowledge of learners' conceptions and misconceptions was low. Sibuyi's (2012) findings seem to suggest that the more a teacher is experienced in teaching Mathematics, the richer he or she is in terms of SCK. However, teaching experience does not have adequate effects on the development of the teachers' PCK of Mathematics. Contrary to Sibuyi (2012), other researchers' findings such as Kilic (2009) and Olfos et al. (2014) reveal that the teachers' SCK and PCK increase with teaching experiences. It implies that, as the teachers interact with the learners during

teaching and learning, they familiarise themselves with the learners' thinking, misconceptions and learning difficulties that they experience about a particular topic which becomes part of their PCK.

2.7 The implications of teachers' SCK and PCK on learning

The impacts of teachers' Subject and Pedagogical Content Knowledge on the learners' academic performance have been studied. However, literature has indicated that there is little information on this subject. Few researchers reveal that both SCK and PCK have a positive impact on learners' academic achievement; but, PCK has a greater influence on the quality of instructions than SCK does (Guerreiro, 2016; Goos, 2013; Kleickmann et al., 2013; Baumert et al., 2010; Hill et al., 2005). Olfos et al. (2014) conducted an exploratory study meant to find the relationship between the PCK of 55 Grade 4 Mathematics teachers and their learners' understanding of fractions (n=1532). Their findings revealed that Subject Content Knowledge is not associated with learners' gain and performance. However, PCK was significantly associated with learners' gain and achievement, although not at a higher intensity.

Hill, Rowan and Ball (2005) found a significant relationship between Mathematical Knowledge for teaching and learners' achievement in the first and second Grade in the USA. In agreement with the above-mentioned authors, Tsafe (2013) states that teachers' Mathematical knowledge, pedagogical competence and reasoning are a key to improving learners' Mathematical achievement. However, the success or failure in the process of teaching a particular concept in Mathematics lies in the pedagogical approach adopted by the teacher. Shulman (1986) believes that both SCK and PCK are integrated parts of effective teaching. That is, for teachers to help learners to

construct Mathematical concepts in their minds, Pedagogical Content Knowledge, as well as Mathematical Content Knowledge (MCK), should be displayed by the teachers during teaching.

Moreover, Gess-Newsome et al. (2010, p. 2) argue that "PCK is the key to effective instruction and ultimately student learning". Shepherd (2013) adds that PCK has the greatest ties to effective teaching and it also directly influences a teacher's ability to develop a curriculum. Shepherd implies that PCK might have a large impact on learning than SCK because it involves the process of making SCK accessible to the learners; the knowledge of explaining the content to the learners in a comprehensive manner. If a teacher has a thorough understanding of Mathematics content, it may not mean that such a teacher knows how to explain the content in a comprehensive manner. The NBPTS (2002) recommends that teachers should have a rich understanding of the subjects they teach and appreciate how knowledge in their subjects is created, organised, linked to other disciplines and applied to a real-world setting in order to improve the academic achievement of the learners. Hence, it is important that both subject content and teaching methods courses should be reinforced at teachers training institutions to ensure that Pre-service teachers are equipped with the necessary skills on how to deliver content in a comprehensive manner.

2.8 Summary

This chapter presents the theoretical framework that this study is based on which indicated the type of knowledge that Mathematics teachers should possess in order to effectively teach the learners in the classrooms. According to Shulman (1986), there are three main kinds of knowledge necessary for teaching – *Subject Content Knowledge, Pedagogical Content Knowledge and Curriculum Knowledge*. Each of the above-mentioned knowledge is divided into subcategories that teachers need to display when teaching the subjects. Various studies have indicated that both Primary and Secondary Mathematics teachers have limited or insufficient SCK and PCK. Some researchers blamed teacher preparation institutions, by stating that teachers are not well prepared to teach Mathematics that they are teaching at the secondary school. The next chapter will focus on the methodology used to carry out this study.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter introduces the methodology used to carry out the study by the researcher. It also comprises of the research design, population, sample and sampling procedures used, research instruments, data collection procedures, data analysis, and ethical considerations.

3.2 Research design

The study employs a quantitative approach method. A quantitative research is used to quantify the problem by way of generating numerical data or data that can be transformed into usable statistics (Best & Kahn, 2006; Wyse, 2011). Although the questions of the test were open-ended, a quantitative research was used because the researcher was interested in the participants' scores per question rather than the difficulties that they experienced per question. In addition, the researcher also wanted to collect the data that can be generalised to the population of the Grade 12 Mathematics teachers in the Khomas Education Region; as Creswell (2012) notes that the intent of a quantitative research is to generalise the results of the sample to the population.

In this study, a survey and pre-experimental designs were used to collect information from the Grade 12 Mathematics teachers in the Khomas Education Region. A survey design was used for the closed-ended questionnaire while the pre-experimental design was used to test teachers' SCK and PCK.

3.3 Population

A population refers to a large group of individuals with the same characteristics from which a researcher draws a sample which is usually stated in theoretical terms (Creswell, 2014; Neuman, 2014; Best & Kahn, 2014). The population of this study comprised of all 79 Grade 12 Mathematics teachers of all 27 public secondary schools in the Khomas Education Region.

3.4 Samples and sampling procedures

Mugoh (2002) defines sampling as the act, process or technique of selecting a suitable representative part for the purpose of determining the characteristics of the whole population. A sample is a set of respondents (people) selected from a larger population for the purpose of the survey (Cohen, Manion & Morrison, 2007). In this study, a stratified random sampling method was used to select Grade 12 Mathematics teachers who took part in the study. According to Creswell (2014), a stratified sampling is a probability sampling method whereby the researcher divides a population of the same characteristics into strata and then uses a simple random sampling method to get the sample from each stratum. The reason for using a stratified sampling method is to ensure a fair representation of all the circuits (strata) in the Khomas Education Region. The Khomas Education Region consists of four circuits, and in each circuit; five schools were randomly selected using the lottery method. Each school was given a unique counting number. The numbers were put in the box and shaken thoroughly. Consequently, the targeted sample schools were drawn randomly. As a result, all the Grade 12 Mathematics teachers at the sampled schools took part in the study. The number of schools and participants that took part in the study per circuit is indicated in Table 1 below.

Table 1: Number of schools and participants per circuits

Circuit	Number of participants (Grade 12 teachers)	Number of schools
A	9	3
B	12	5
C	9	5
D	10	5
Total	**40**	**18**

Out of 53 teachers that were supposed to take part in this study, only forty (75%) Grade 12 Mathematics teachers agreed to take part in the study. In circuit A, two schools (4 teachers) refused to be part of the study, and that is the reason why only 3 schools in circuit A participated as indicated in Table 1. One teacher from one of the schools in circuit A also refused to be part of the study. In both circuit B and C, three teachers refused, while in circuit D, two schools (one teacher from each school) refused to be part of the study. In total, 13 teachers refused to take part in the study.

3.5 Research instruments

(a) Closed-ended questionnaire

According to Neuman (2014), a closed-ended questionnaire is a type of research instrument that has both questions and fixed responses from which the participant will choose. Each participant completed the closed-ended questionnaire without any guidance from the researcher. Each close-ended questionnaire had two sections (A and B). Section A gathered biographic information of the Grade 12 Mathematics teachers (participants) while section B consisted of general questions regarding the teaching of the Grade 12 Mathematics NSSCO syllabus. All items of the

questionnaire were closed-ended. A closed-ended questionnaire was used in this study due to the following reasons as indicated by Neuman (2014):

- It is easier and quicker for respondents to answer since they needed to write the test afterwards.

- The answers of the respondents are easier to compare.

- Answers of the participants are easier to analyse.

- There are fewer irrelevant or confusing answers to questions.

- Respondents are more likely to answer about sensitive topics (p.205).

(b) Test

A test is a type of research instrument that consists of a series of questions meant for determining the participants' content knowledge of a given subject. In this study, a nineteen (19) – item test was used to assess the participants' SCK and PCK based on the Grade 12 Mathematics curriculum (NSSCO syllabus). The test consists of Section A and Section B. Section A consisted of nine items testing for Mathematics teachers' SCK, while Section B comprised of ten items testing for PCK. Section B covered three subscales: Knowledge of Mathematics tasks, Knowledge of learners' misconceptions and difficulties, and Knowledge of Mathematics instructions. The knowledge of task for teachers was tested to check teachers' awareness of different ways of solving the tasks (Krauss et al., 2008a). The knowledge of learners' misconceptions and difficulties was tested by presenting teachers with seven scenarios and asking them to detect, analyse (e.g. give cognitive reasons for comprehension problem), or predict a typical student error or a particular comprehension difficulty knowledge (Krauss et al., 2008a). The knowledge of subject-specific instructional strategies was assessed by items requiring teachers to

explain Mathematical situations (e.g., to provide useful representations analogies, illustrations, or example to make Mathematical content accessible to students).

The questions in Section A of the test were set by the researcher with exception to question one that is adapted from Krauss et al. (2008b). All the questions in Section B of the test were adapted from Krauss et al. (2008a); Li (2007); and French (2005). All nineteen (19) items of the test covered relevant content areas where learners usually didn't perform well as per the Examiner reports of 2013, 2014 and 2015 academic years. Hence, the topics covered in the test are indicated in Table 2 below.

Table 2: Main topics and subtopics covered in the test

Kind of Content Knowledge	Main Topic	Subtopic
Subject Content Knowledge	Numbers	Estimation
	Probability	The Probability of combined events
	Geometry	Angle properties
	Algebra	Transformation of formula
	Logarithms	Logarithmic equations
	Sequence and series	Geometric sequences
	Vectors	Vectors in two dimensions
Pedagogical Content Knowledge	Numbers and Operations	Division of fractions, and Directed number, Estimation
	Algebra	Linear equation, including Simultaneous equations
	Mensuration	The Area

In summary, as indicated in Table 2, the test constituted of the following topics: Limit of Accuracy, Algebra, Geometry, Trigonometry, Mensuration, Probability, Vectors, Sequence and Series. Teachers' responses were scored using answers that were established after consulting different kinds of literature. Participants were awarded marks in the test in the following manner:

Scores for Subject Content Knowledge (SCK):

- Each question scored Three (3) marks (correct answer and correct method used).

- A correct answer only without showing the methods of getting the answer scored one (1) mark.

- Questions without any response scored zero (0).

- The total marks for questions testing for SCK were 27 (9×3).

Scores for Pedagogical Content Knowledge (PCK):

- Each correct answer scored one (1) mark, for questions that require multiple answers; the sums of correct answers were calculated.

- Questions without any responses scored zero (0).

- Total marks for questions testing for PCK were 32 marks.

A test was used during this study because of the two reasons; firstly, tests cover a vast range of topics (Cohen et al., 2007), hence its usage because the researcher investigated the teachers' SCK and PCK scores in different Mathematics topics. Secondly, tests are often valid and reliable than perceptions or opinions (Bouffard & Little, 2004).

3.6 Pilot study

To ensure the reliability and validity of the test, a pilot study was conducted in two schools from Education Regions adjacent to the Khomas Education Region. The reason for piloting the study in another Educational Region rather than the Khomas Education Region was to prevent the leakage of information for the instruments used during the study. Specifically, piloting of the study was conducted due to the following reasons as indicated by Ormrod (2014):

- To determine whether the items in both the closed-ended questionnaire and the test are clear and understood by the participants,

- To determine if both instruments solicit the desired information,

- To check whether both the closed-ended questionnaire and the test truly measure what they were intended to measure, and

- To determine if there are items on the instruments which need to be removed or added.

Four Grade 12 Mathematics teachers were tested during the piloting of the instruments and the following changes were made to the instruments: Section A, question 6 - The degree of accuracy for the cup of water was changed from two significant figures to the nearest hundred because 300 represent three significant figures. In question 9 - vector $\overrightarrow{AB} = \frac{1}{4}AB$ was correctly changed to $\overrightarrow{AX} = \frac{1}{4}AB$. The signature of the participant and date were added onto the consent letter as these were accidentally omitted.

3.7 Data collection procedures

After being granted permission by the relevant authorities, the researcher then got permission from the school principals through the office of the Director of Education in the Khomas Education Region. Further, the researcher got permission from the participants through the office of the principals of the sampled schools. Appointments for data collections were then made between the participants and the researcher.

The study was carried out during after-school hours to ensure that it does not interfere with the academic activities of the sampled schools. Fortunately, the study was carried out at the beginning of the learners' examinations where schools closed earlier, and this made it easy for the participants to stay behind to complete the research instruments. The researcher collected data from sampled schools in the same vicinity on the same day and this was done to ensure that there was no leakage of information in the research instruments. Participants completed the research instruments unsupervised at their respective classes for 1 hour and 30 minutes as the researcher had to drop off the research instruments to the nearby sampled schools. Participants were allowed to read the letters of consent. The Grade 12 Mathematics teachers that showed interest in the study then completed the letter of consent. After the participants had completed the letter of consent, the researcher then gave them both the closed-ended questionnaire and a test to complete unsupervised in their respective classes. After one hour and 30 minutes, the researcher went back to the sampled schools and collected the research instruments.

3.8 Data analysis

The Microsoft Excel spreadsheet was used to sort out the data into graphs, figures and tables. The data were analysed based on the research instruments used in the study. Data from the questionnaire were analysed in the following manner:

- *Distribution of Grade 12 Mathematics teachers per circuit*

- *Grade 12 Mathematics teachers' age and gender*

The data from the questionnaire are presented in the frequency table and charts while data from the test were analysed using frequency tables, charts and descriptive statistics (e.g. means, percentage). The data collected by the use of a test were analysed as follows:

Analyses of SCK scores:

- *Analysis of individuals' SCK scores*

- *Analysis of participants' SCK scores per question*

- *Analysis of participants' SCK scores per circuit*

- *Analysis of participants' SCK scores by age group*

- *Analysis of participants' SCK scores by teaching experience*

- *Analysis of participants' SCK scores by qualification*

Analyses of PCK scores

- *Analysis of individuals' PCK scores*

- *Analysis of participants' PCK scores per question*

- *Analysis of participants' PCK scores per circuit*

- *Analysis of participants' PCK scores by age group*

- *Analysis of participants' PCK scores by teaching experience*

- *Analysis of participants' PCK scores by qualification*

A correlation coefficient (r) was used to determine if a relationship existed between the Grade 12 Mathematics' knowledge (SCK and PCK), teaching experiences and qualifications. A paired- t-test was used to determine if there is a significant difference between the Grade 12 Mathematics teachers' PCK and SCK. A paired -t-test was used due to the following reasons as given by (Cohen et al., 2007):

- A paired t-test is appropriate to use when a researcher wants to measure the performance of the same group in different factors, thus why it was used because this study explores the SCK and PCK of the same participants.

- Ease of calculation- in the modern world, the t-test is calculated with the aid of the computer; however, the formula for the t-test is simple and easy to understand.

3.9 Ethical consideration

According to Creswell (2014, p. 160), "data collection should be ethical and it should respect the site". Therefore, researchers should gain access to organisations by seeking permission from those in charge (Creswell, 2014; Cohen et al., 2007; Resnik, 2010). As a result, the researcher got the ethical clearance from the University of Namibia (UNAM) Research Ethics Committee and later got permission from the Permanent Secretary of the Ministry of Basic Education, Arts and Culture, and the Director of Education of Khomas Education Region respectively before carrying out the study. Ormrod (2014) recommends that when carrying out a research, the researcher must consider the right to privacy, and honesty with the participants. Furthermore, "protecting the anonymity of individuals by assigning numbers to

returned instruments and keeping the identity of individual confidential offers privacy to participants" (Creswell, 2014, p. 160). Thus, anonymity was ensured-the names of the participants are not appearing on either instrument and names of schools were substituted with codes.

Creswell (2014) advise that researchers are responsible for ensuring that the confidentiality of the participants and the data are protected. Therefore, the data in the form of hard copies are kept in a locked cupboard, while the soft copies are kept in a private laptop for five years. Afterwards, these soft copies of data will be deleted from the laptop while the hard copies will be shredded. Participants were also informed that their participation in the study is voluntary and can withdraw anytime (Creswell, 2014; Cohen et al., 2007) as indicated in the letter of consent that they completed before taking part in the study.

3.10 Summary

This chapter focused on the methodology used to collect data from the Grade 12 Mathematics teachers in the Khomas Education Region. Under the methodology, the research design, population of the study, samples and sampling procedures, research instruments, piloting of the study, data collection and analysis, and ethical considerations are discussed. A quantitative design was used in this study. A survey and pre-experimental designs were used to collect information from the participants. In this study, a stratified random sampling method was used to select participants who took part in the study. A closed-ended questionnaire and test were used to collect data. The research instruments were piloted at two schools adjacent to the Khomas Education Region. The data from both the research instruments were

presented in the frequency tables and charts. A correlation coefficient was used to describe the relationship between the participants' knowledge of Mathematics, teaching experiences and qualifications. A paired t-test was used to determine whether a significant difference existed between the Grade 12 Mathematics' SCK and PCK. Before conducting the study, the researcher got permission from relevant authorities such as the Permanent Secretary of the Ministry of Education, Arts and Culture, the Director of the Khomas Education Region, relevant School Principals and participants. The next chapter will focus on the results and discussions of the study.

CHAPTER 4: PRESENTATION AND DISCUSSION OF THE RESULTS

4.1 Introduction

This chapter presents the findings and discussion of the Grade 12 Mathematics teachers' Subject and Pedagogical Content Knowledge in the Khomas Education Region. The presentation and discussion of the findings centres at two main headings, namely;

- Findings from the questionnaire and

- Findings from the proficiency test

The results of the study were used to answer the following research questions and test the hypotheses:

1. What is the Grade 12 Mathematics teachers` Subject Content Knowledge in some selected public schools in the Khomas Education Region?

2. What is the Grade 12 Mathematics teachers` Pedagogical Content Knowledge in some selected public schools in the Khomas Education Region?

3. What relationship exists between the Grade 12 Mathematics teachers' knowledge, teaching experience and qualification in the Khomas Education Region?

Hypotheses

H_0 – There is no significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge.

H_1 – There is a significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge.

To answer the above-mentioned questions and test the hypotheses, the results were analysed according to five main themes: *Grade 12 Mathematics teachers' biography information*, *analysis of teachers' SCK scores, analysis of PCK scores, the relationship between participants SCK and PCK, and teachers' views regarding the NSSCO Mathematics syllabus.*

4.2 THE GRADE 12 MATHEMATICS TEACHERS' BIOGRAPHIC INFORMATION

4.2.1 Distribution of the Grade 12 Mathematics teachers per circuit

A total of 40 Grade 12 Mathematics teachers participated in this study as indicated in Figure 5. The teachers that participated in this study were randomly selected from all four circuits in the Khomas Education Region as indicated in the methodology under samples and sampling procedures.
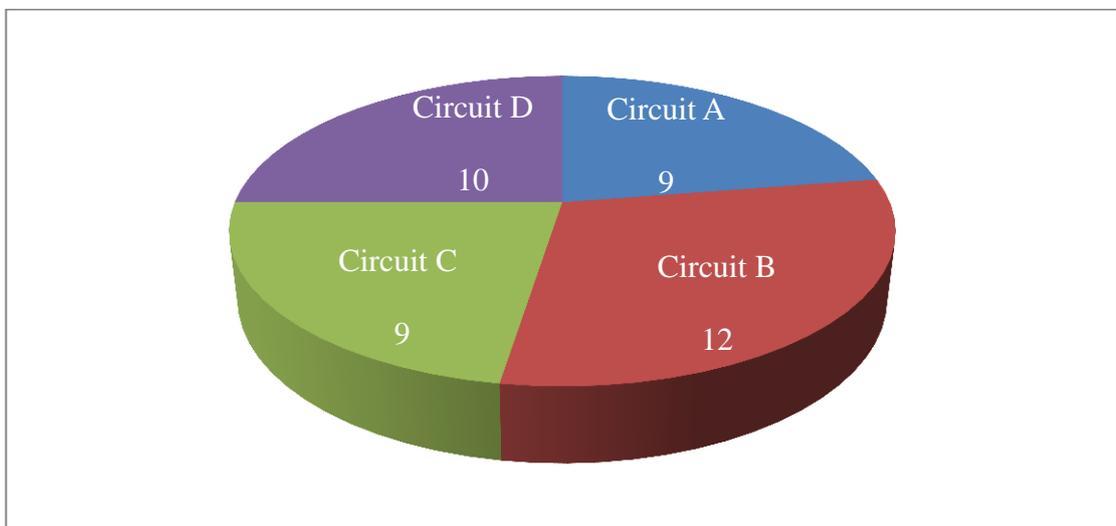
Figure 5: Distribution of Grade 12 Mathematics teachers per circuit

Figure 5 shows that the Khomas Education Region consists of four circuits. Circuit A and C had nine participants each that took part in this study, while circuit D was represented by ten participants. Circuit B was the most represented with 12 participants.

4.2.2 The Grade 12 Mathematics teachers' age and gender

The ages of all the participants ranged from 21-55 years old as shown in Table 3 below. The number of males that participated in the study was more than their female counterparts as indicated in Table 3 below.

Table 3: Teachers' age and gender

Age group in years	Male (s)	Female (s)	Total	%
21-25	2	5	7	17.5
26-30	6	3	9	22,5
31-35	6	3	9	22.5
36-40	8	3	11	27.5
41-45	1	1	2	5
51-55		2	2	5
Total	**23**	**17**	**40**	**100**

The results in Table 3 show that 27.5 % of the participants were in the age category of 36-40. The age groups of 26-30 and 31-35 were each represented by 9 (25.5%) participants, while the age group of 21-25 was represented by 17% of the total participants. The age group of 41-45 and 51-55 were each represented by 5%.

4.3. ANALYSIS OF TEACHERS' SCK SCORES

4.3.1 Analysis of individual teachers' SCK scores

Out of 53 Grade 12 Mathematics teachers, 40 teachers wrote the test. As indicated early in the methodology under research instruments, the test consists of Section A and B. Section A investigated the participants' Subject Content Knowledge, while Section B focused on the participants' Pedagogical Content Knowledge. The aim of Section A was to investigate the participants' conceptual understanding of Mathematics. For anonymity sake, each teacher was given a unique code, and the codes ranged from T1 up to T40 since there were 40 participants that took part in the study. Letter T in the codes represents "Teacher". The SCK questions were marked out of 27 marks. The results are shown in Figure 6 below.
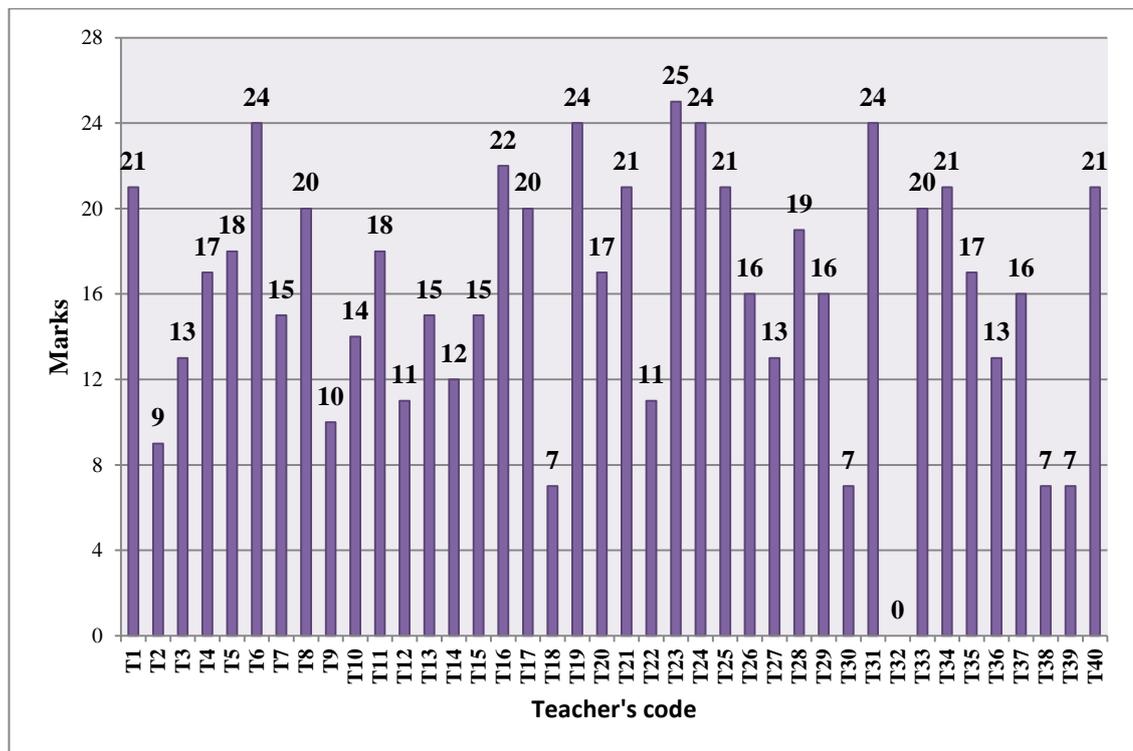
Figure 6: Individual teachers' SCK scores

Figure 6 shows that Teacher 32 (T32) scored zero marks out 27 marks in the SCK questions, while T18, T30, T38 and T39 scored seven marks each. T10 scored 14 marks, while T6, T19, T24, T31 got 24 marks each in the SCK questions. T23 scored the highest marks with 25 out of 27 marks. The overall scores look satisfactory because 27 teachers scored more than 50%. However, the scores are not good enough since the test was based on the basic competencies that are indicated in the NSSCO Mathematics syllabus that they teach to the learners. If teachers failed some questions based on what they teach to the learners, then it can be argued that their SCK is limited. Thus, it can be further argued that the Grade 12 Mathematics teachers in the Khomas Education Region have a fair Subject Content knowledge.

4.3.2 Analysis of teachers' SCK scores per question

In this section, the average % scores of SCK per question for all the participants were calculated. The individual participants' scores per question are added together, and the resulting answer is divided by the total marks of each question and then converted to percentages. The results are shown in Figure 7 below.
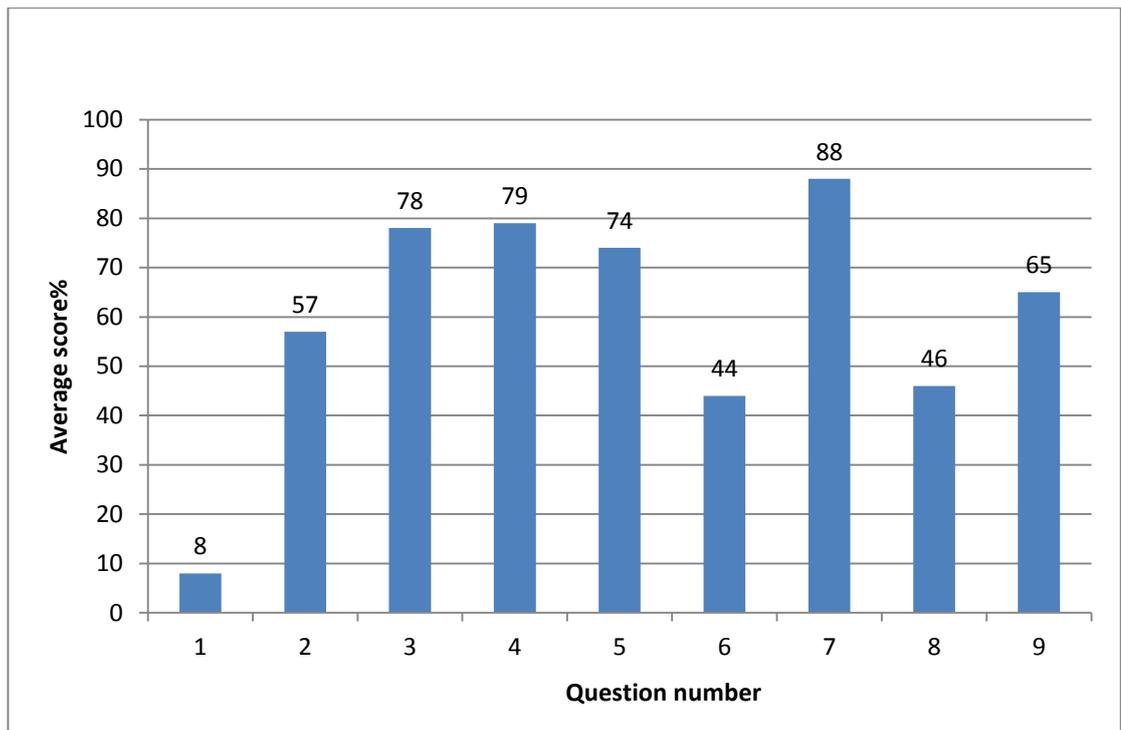
Figure 7: Grade 12 Mathematics teachers' SCK average scores % per question

In Figure 7, it is evident that question 1 was not well answered because on average, participants only managed to score 8%. Question 1 required teachers to give detailed reasons for this statement "Is it true that $0.9999999\ldots = 1$". Questions 6 and 8 were also not well answered because on average, the participants scored below 50%. In questions 2, 3, 4, 5 and 9, participants scored more than 56% on average in each question. Question 7 was well answered with 88% on average. In question number seven, teachers were asked to express m in terms of t and z in: $t + m = \frac{mz}{t}$. Although the results look better to the researcher, it is not good enough based on the same reason given earlier in 4.3.1 that the questions of the test were based on the basic competencies of the NSSO Mathematics syllabus that they teach to the learners.

4.3.3 Analysis of teachers' scores in SCK questions per circuit

Under this section, the sum of teachers' scores was divided by the total possible maximum marks per circuit and converted to percentages. The participants from different circuits scored differently, which resulted in different average scores as shown in Table 4.

Table 4: Teachers' SCK average scores per circuit

Circuits	Average teachers' SCK Scores (%)
A	49
B	69
C	63
D	54
Total average %	**60**

Table 4 illustrates that participants in Circuit B outperformed their counterparts in respective circuits with an average score of 69 %. The participants that represented Circuit C scored 63 % on average, while those from Circuit D got an average score of 54 %. Participants from Circuit A performed poorly compared to other circuits with an average score of 49 %. The overall teachers' scores in SCK questions stood at 60%.

4.3.4 Analysis of teachers' SCK scores by age group

In this section, participants' average SCK scores were analysed based on their age groups. The sums of individual scores were calculated, and the resulting answer was divided by the total scores per age group and converted to percentages. Teachers' ages were classified into eight groups; however none of the participants fell in the

age group of 46 -50 and 56 – 60, and that is the reason why there are only six age groups in Table 5 below.

Table 5: Teachers' SCK average scores per age group

Age group	Number of teachers	SCK average score (%)
21-25	7	57
26-30	9	60
31-35	9	69
36-40	11	59
41-45	2	63
51-55	2	57
Total/average	**40**	**61**

Table 5 shows that the majority of participants (11 out of 40) were between the ages of 36 – 40. The age group of 26-30 and 31-35 were each represented by nine participants; while seven of the participants were in the age category of 21-25. The age group of 41-45 was the least represented with only two participants. It is also evident from Table 4 that participants whose ages ranged from 31-35 performed better compared to the other age groups with an average score of 69%. The age group of 41-45 scored 63% on average, while the age group of 26-30 got an average score of 60%. The participants in the age group of 21-25 and 51-55 were the least performing category with an average score of 57% each. The results appear to suggest that there is no clear relationship between the teachers' SCK and their ages because of the following reason: firstly, participants in the age group of 21-25 and 51-55 obtained the same average scores. Secondly, participants in the age category of 51-55 were outperformed by the younger age groups. Finally, participants under the

age of 31-35 scored better than their counterparts from the age group of 36-40 and 41-45 respectively.

4.3.5 Analysis of teachers' SCK scores versus teaching experience

In this section, the participants' average percentage scores in SCK questions were compared to the number of years of teaching Grade 12 Mathematics. The teaching experience of the participants is grouped into nine categories as shown in Table 6 and Figure 8.

Table 6: Participants' average SCK scores obtained per number of years of teaching Grade 12 Mathematics

Teaching experience for Grade 12 (years)	Number of teachers	SCK average score (%)
1-2	14	50
3-4	4	54
5-6	4	59
7-8	3	69
9-10	5	67
11-12	5	57
13-14	1	78
15-16	2	81
17+	2	78
Total/Average	**40**	**61**

Table 6 shows that participants who taught Grade 12 Mathematics for 15-16 years performed better (81%) compared to all the other experienced groups. These results appear to show a positive relationship between the participants' average scores and the number of teaching experience as from 1-8 years of teaching Grade 12 Mathematics. However; there is also a negative relationship from 8-12 years of teaching experience. The patterns of the teachers' scores and their teaching experience are clearly shown by the line graph in Figure 8 below. In Figure 8, **TE** stands for teaching experience, e.g. TE1-2 means teaching experience for 1 to 2 years.
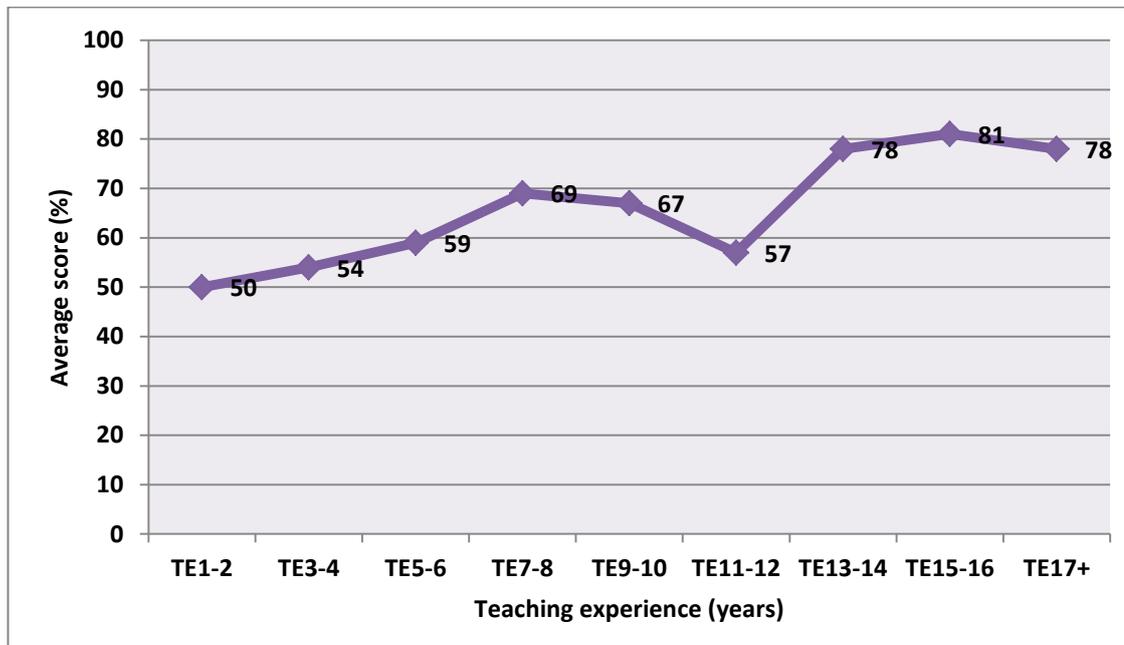
Figure 8: Teachers average SCK scores vs teaching experience (years) of Grade 12 Mathematics

Looking at the patterns in Figure 8 and the common variance ($r^2 = 0.76$), these results can be interpreted to indicate that there is a strong positive relationship (correlation coefficient $(r) = 0.87$) between the Grade 12 Mathematics teachers'

SCK and the number of years they have taught Grade 12 Mathematics (teaching experience). This strong positive relationship accounts for $\frac{76}{100}$ of the total variance.

4.3.6 Analysis of teachers' SCK scores versus teachers' qualifications

In this section, participants' average scores were compared with their qualifications. The main aim was to check if participants that possess higher qualifications scored better than their counterparts with lower qualifications.

Table 7: Teachers' SCK average scores by qualification

Qualification	Number of teachers	Percentage	Average score (%)-SCK
BETD + Mathematics and Science Teachers Extension Program (MASTEP)	1	2.5	78
BETD + Further Education Diploma (FED)	2	5	52
BETD + Bachelor of Education (B.Ed)	3	7.5	58
National Diploma in Education or Diploma in Education	2	5	31
B.Ed	11	27.5	60
Bachelor of Education Honours degree (B.Ed (Hon))	11	27.5	58
B.Ed +B.Ed (Hon)	2	5	76
Bachelor of Science (B.Sc) + Post-Graduate Diploma in Education (PGDE)	2	5	56
B.Sc, or B.Sc (Hon)	6	15	76

The results in Table 7 show that 85% of the participants are qualified to teach Grade 12 Mathematics and 15% are unqualified teachers. The results also show that 27.5% of the participants are Bachelor of Education holders and that the Bachelor of Education Honours degree was also represented by 27.5% of the participants. The results also show that 5 % of the participants were unqualified teachers with either a Bachelor of Science or a Bachelor of Science Honours degree. BETD + FED and National Diploma/Diploma in Education holders were each represented by 5% of the total participants. BETD+MASTEP holder was the least represented by one (2.5%) participant. A BETD+MASTEP candidate scored the highest percentage (78%). B.Ed (Hon) holders scored 58%, while NDE and DE scored the lowest percentage of 31%. These results seem to suggest that there is no clear link between teachers' qualification and SCK. This is mainly due to fact that the B.Ed (Hon) holders were outperformed by teachers with lower qualifications such as B.Ed, and BETD + MASTEP holders. This is also supported by the study findings which indicate a correlation coefficient ($r = 0.21$) representing a weak relation between the teachers' SCK scores and their teaching qualifications.

4.4 ANALYSIS OF TEACHERS' PCK SCORES

4.4.1 Analysis of individual teachers' PCK scores

The aim of Section B of the test was to investigate the Grade 12 Mathematics teachers' PCK: Knowledge of Mathematics tasks, Knowledge of student misconceptions and difficulties, as well as Knowledge of Mathematics instructions. In this subsection, individual teachers' scores in the PCK questions are analysed. The PCK questions were marked out of 32 marks. The results are shown in Figure 9 below.
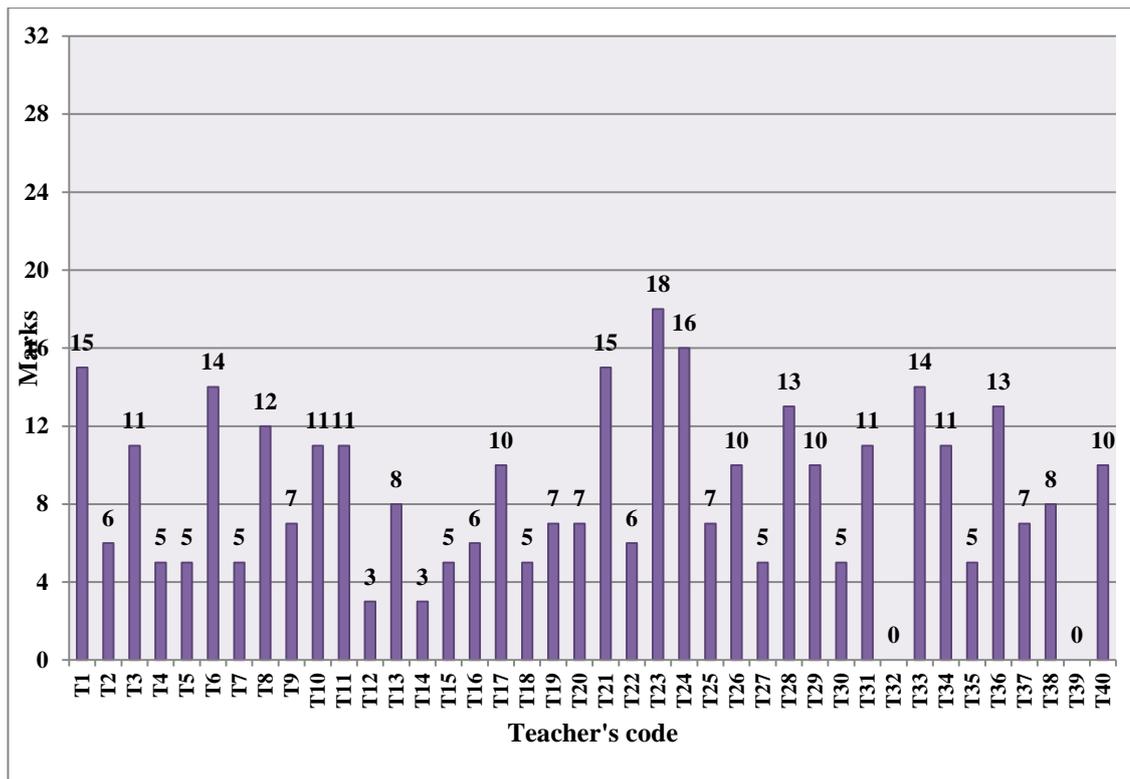
Figure 9: Individual teachers' PCK scores

Interesting enough, it is evident from Figure 9 that Teacher number 32 (T32) scored the lowest marks in PCK questions with a zero mark; while T12 and T14 scored three marks (out of 32 marks). T1 and T21 scored 15 each, while T24 was the second highest with 16 marks out of 32. T23 obtained the highest marks of 18 out of 32 (56%). The overall performance of participants in PCK is not good because out of 40 participants, only two teachers (T23 and T24) managed to reach half of the total marks of the PCK questions. This result seems to suggest that the participants have insufficient PCK required to teach the Grade 12 Mathematics NSSCO syllabus.

4.4.2 Analysis of teachers' PCK scores per question

Under this subsection, participants' average scores in PCK per question were calculated. Like in section A, the individual participants' scores per question were added together, and the resulting answer was divided by the total marks of each question and then converted to percentages. The results are shown in Figure 10 below.
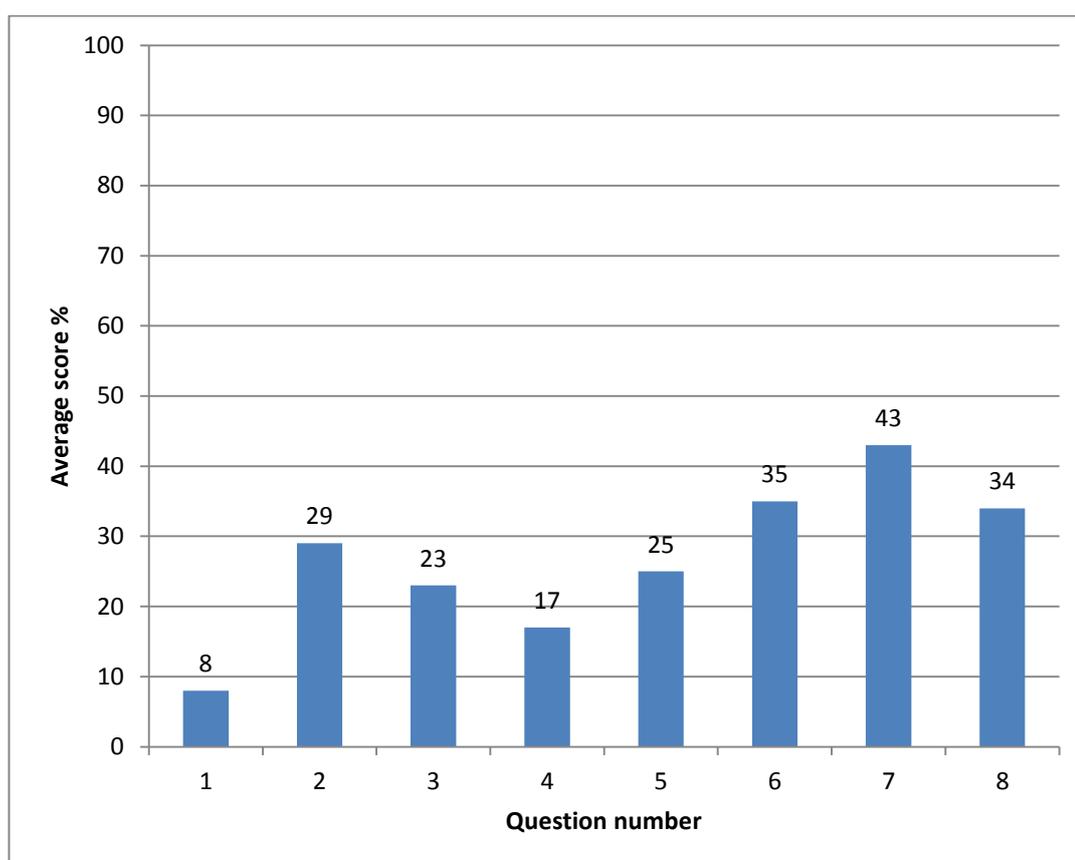
Figure 10: Average teachers' PCK scores per question

The results in Figure 10 above show that question 1 was the most poorly answered question by the participants with an average score of 8%. In question 1, participants were required to suggest ways of helping learners to understand that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$. The majority of the participants misunderstood the question and instead of suggesting

ways that will make the learners understand the problem, they just worked out the problem. Question 4 was also poorly answered with an average score of 11%. The rest of the questions (2, 3, 5, 7 and 8) were also not well answered as participants only managed to get average scores below 44%. These results shown in Figure 9 seem to indicate that the Grade 12 Mathematics teachers in the Khomas Education Region have inadequate PCK of teaching the NSSCO Mathematics syllabus.

4.4.3 Analysis of teachers' scores in PCK questions per circuit

In this section, the sum of teachers' scores was divided by the total maximum marks per circuit and converted to percentages. Participants from different circuits scored differently, which resulted in different average scores as shown in Table 8.

Table 8: Teachers' average PCK scores per circuit

Circuit	Average % scores
A	20
B	33
C	32
D	23
Average total %	**28**

Table 8 shows that the participants in all four circuits performed poorly with an average score of 28%. Circuit A and B obtained average scores of 20% and 23% respectively. Circuits B scored 33% on average, while C scored 32%.

4.4.4 Analysis of teachers' PCK scores by age group

Under this subheading of PCK, the Grade 12 Mathematics scores in PCK questions were analysed based on their age groups. Teachers' ages were classified into eight groups; however, none of the participants fell in the age category of 46 -50 and 56 – 60 as indicated earlier in 4.3.1 (c).

Table 9: Participants' average PCK scores per age group

Age group	Number of teachers	SCK average score (%)
21-25	7	32
26-30	9	26
31-35	9	27
36-40	11	21
41-45	2	36
51-55	2	38
Total/average	**40**	**30**

Table 9 illustrates that participants in the age category of 21-25 outperformed those that are in the age group of 26-30, 31-35 and 36-40. However, participants in the age group of 51-55 scored better than all the other age groups with 38%, but still not good enough. Based on the above analysis, these results appear to suggest that there is no clear relationship between the teachers' scores in PCK questions and the teachers' age.

4.4.5 Analysis of teachers' PCK scores versus teaching experience

In this section, participants' average scores in PCK questions are compared with the number of years they taught Grade 12 Mathematics (NSSCO) syllabus.

Table 10: Participants' average PCK scores obtained per number of years of teaching Grade 12 Mathematics

Teaching experience for Grade 12 (years)	Number of teachers	PCK average score (%)
1-2	14	22
3-4	4	33
5-6	4	27
7-8	3	28
9-10	5	21
11-12	5	29
13-14	1	22
15-16	2	36
17+	2	42
Total/Average	**40**	**29**

The results in Table 10 show that participants who taught Grade 12 Mathematics for 17 years or more, scored better than their counterparts on average with 42% in PCK questions. The participants that taught Grade 12 Mathematics for 1-2 years and 13-14 years shared the same average score of 22%. The patterns of teachers' scores are clearly indicated by the line graph in Figure 11 below.
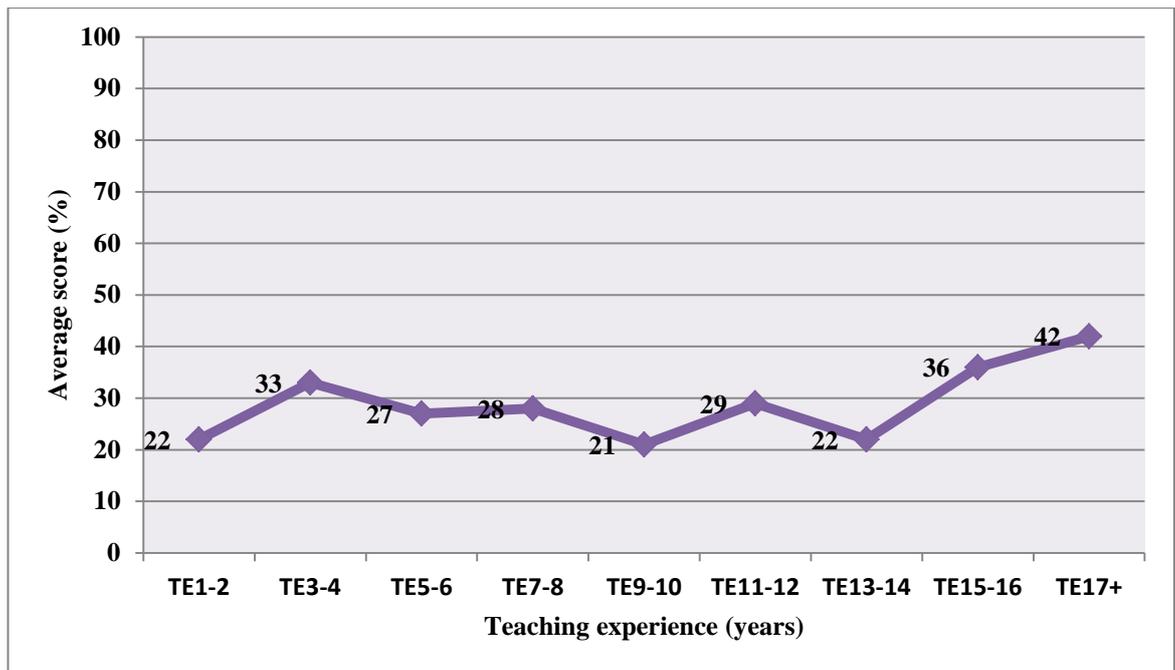
Figure 11: Teachers' average PCK scores obtained per number of years of teaching Grade 12 Mathematics

Looking at the patterns of participants' PCK average scores as compared to their teaching experience of Grade 12 Mathematics in Figure 11 and the common variance ($r^2 = 0.27$), these results can be interpreted that there is no clear relationship between teachers' PCK and their teaching experience ($r = 0.52$).

4.4.6 Analysis of teachers' PCK scores versus teachers' qualifications

In this part, the Grade 12 Mathematics teachers' average scores in PCK questions were compared to their qualifications of teaching Grade 12 Mathematics. It emerged that the participants that took part in the study had different qualifications of teaching Grade 12 Mathematics as shown in Table 11 below.

Table 11: Teachers' average PCK scores by qualifications

Qualification	Number of teachers	percentage	Average score (%) - PCK
BETD + Mathematics and Science Teachers Extension Program (MASTEP)	1	2.5	47
BETD + Further Education Diploma (FED)	2	5	28
BETD + Bachelor of Education (B.Ed)	3	7.5	23
National Diploma in Education or Diploma in Education (NDE or DE)	2	5	11
B.Ed	11	27.5	23
Bachelor of Education Honours degree (B.Ed (Hon))	11	27.5	24
B.ED +B.Ed(Hon)	2	5	34
Bachelor of Science (B.Sc) + Post-Graduate Diploma in Education (PGDE)	2	5	39
B.Sc, or B.Sc(Hon)	6	15	35

Table 11 shows that a BETD + MASTEP holder performed better (47%) compared to the other participants. Surprisingly, on average, unqualified teachers (B.Sc or B.Sc (Hon) holders) outperformed the qualified teachers (BED, B.Ed (Hon), NDE or DE, and BETD +FED). It is beyond comprehension for qualified teachers to be outperformed by unqualified teachers in PCK questions because, qualified teachers have covered teaching methods modules that cover topics on how to make the content comprehensive to the learners; while unqualified teachers did not. Furthermore, the common variance between the participants' PCK and their qualifications was $r^2 = 0.0074$, which accounts for a very weak correlation coefficient ($r = 0.086$). Therefore, based on these analyses, it can be argued that there is no clear relationship between teachers' PCK and their qualifications.

4.5 The relationship between the participants' SCK and PCK.

To find out if there is a significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge, the following hypothesis was tested.

H_0: μ_1 - μ_2 = 0 ("The difference between the paired population means is equal to 0")

H_1: μ_1 - $\mu_2 \neq 0$ ("The difference between the paired population means is not 0")

Whereby: μ_1 is the population mean of variable 1 (SCK), and μ_2 is the population mean of variable 2 (PCK). According to Kent State University Libraries (2018, January 9), when running data in SPSS, a paired sample t-test results in three tables, namely; *Paired Samples Statistics, Paired Samples Correlations*, and *Paired Samples Test. Paired Samples Statistics* gives univariate descriptive statistics (mean, sample size, standard deviation, and standard error) for each variable entered. One should note that the sample size of the participants in this study was 40. *Paired*

Samples Correlations shows the bivariate Pearson correlation coefficient (with a two-tailed test of significance) for each pair of variables was entered. *Paired Samples Test* gives the hypothesis test results. The results tables described above are shown in Table 12, 13 and 14 below.

Table 12: Paired sample information of teachers' SCK and PCK

		Mean	n	Std. Deviation	Std. Error Mean
Pair 1	SCK	16.03	40	5.842	.924
	PCK	8.50	40	4.291	.678

The results in Table 12 show that participants' mean score in SCK questions was 16.03 while that of PCK stood at 8.50 which indicates that the participants have performed better in the SCK questions than the PCK questions. The standard deviation for SCK was 5.842, while the figure for PCK was 4.29. This implies that teachers' SCK scores were more close to the mean than the PCK scores which were spread out from the mean. The standard error for SCK and PCK were 0.924 and 0.678 respectively which are relatively smaller and show that the sample fairly represented the population of the teachers in the Khomas Education region.

Table 13: Paired teachers' SCK and PCK Correlations

		n	Correlation	Sig.
Pair 1	**SCK & PCK**	40	.681	.000

Although the primary interest when you run a paired *t*-test in the SPSS software is to find out if the means of the two variables are significantly different, it is also

important to consider how strongly the two variables (SCK & PCK) are associated with one another (Kent State University Libraries, 2018, January 9). Thus, it is clear from Table 13 that SCK and PCK scores were strongly and positively correlated ($r = 0.681$, $p < 0.01$), which implies that SCK and PCK are a closely related.

Table 14: Paired teachers' samples (SCK and PCK) t-test results

		Paired Differences							
					95% Confidence Interval of the Difference				Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper	t-test	df	
Pair 1	SCK and PCK	7.525	4.291	.678	6.153	8.897	11.092	39	.000

It is clear from Table 14 that the mean difference between SCK and PCK scores is 7.525 ($\mu_1 - \mu_2$), and on average, SCK scores were 7.5 points higher than PCK scores (95% CI [6.153, 8.897]). The t-test for the SCK and PCK scores results with a degree of freedom df = 39 at the significance level, $\alpha = 0.01$ yielded $t_{calculated} = 11.092$. The $t_{calucated}$ value is greater than the $t_{critical} = 2.750$ value, which falls in the rejection of the null hypothesis (**H_0**). Thus, reject the null hypothesis and accepted the alternative hypothesis (**H_1**) which states that the paired population means are different ($\mu_1 - \mu_2 \neq 0$). Hence, it is concluded that there is a significant mean difference between the SCK and PCK scores.

4.6 TEACHERS' VIEWS REGARDING THE NSSCO MATHEMATICS SYLLABUS

4.6.1 Teachers' views regarding their training of teaching the NSSCO Mathematics syllabus

In the closed-ended questionnaire, participants were asked if they think that they were sufficient, moderately or insufficiently prepared to teach the NSSCO Mathematics syllabus in terms of both the SCK and PCK. Their responses are indicated in Table 15 below.

Table 15: Teachers' extent of training to teach the NSSCO Mathematics syllabus

	SCK	PCK
Extent of preparation	Frequency	Frequency
Sufficiently prepared	20	11
Moderately prepared	11	17
Insufficiently prepared	2	5
	n=40	**n=40**

Table 15 shows that, in terms of SCK, the majority of the teachers felt that they were sufficiently prepared to teach the NSSCO Mathematics syllabus; while in PCK, only 11 out of 40 indicate that they are sufficiently prepared. Further, 11 participants felt that they are moderately prepared in terms of SCK, while in terms of PCK, only 17 felt that they are moderately prepared. Additionally, two participants felt that they are not sufficiently prepared in terms of SCK, while five felt that they are not sufficiently prepared in PCK to teach the NSSCO Mathematics syllabus.

4.6.2 Teachers' views on the topics that are difficult to teach in the NSSCO Mathematics syllabus

This section presents teachers' views regarding the topics that are difficult to teach in the NSSCO Mathematics syllabus. Their responses are indicated in Table 16.

Table 16: Participants' views on the difficult topics to teach learners

Topic	Frequency
Vectors	8
Probability	10
Geometry	7
Trigonometry	7
Mensuration	7
Algebra	6
Sequence and series	12
Limit of accuracy	5
Functions	1
Estimation	3
Logarithms	16
Locus	4
Money and Finance	1
Transformation	1
Numbers	1
Measurement	1
Graphs in a practical situation	1
None	3

Table 16 illustrates that 16 participants find Logarithms difficult to explain to the learners, while 12 participants considered Sequence and series as a difficult topic. Money and finance, Transformation, Numbers and Graphs in practical situations are the least topics considered difficult to explain to the learners as indicated in Table 16. Three of the participants indicated that none of the topics was difficult to explain to the learners, which is interesting because these three particular teachers did not perform well in these topics and the test in general.

4.6.3 Teachers' view regarding the topics they need further training on

In the closed-ended questionnaire, participants were also asked if they required further training in the topics of the NSSCO Mathematics syllabus. Their responses are shown in Table 17 below.

Table 17: Participants' views regarding Mathematics topics they require training in

Topic	Frequency
Vectors	11
Probability	6
Linear programming	2
Geometry	6
Statistics	5
Trigonometry	5
Mensuration	4
Algebra	3
Sequence and series	10
Limit of accuracy	4
Estimation	4
Logarithms	14
Locus	3
Graphs	3
Transformation	2
Construction	2
None	12

Table 17 illustrates that many Grade 12 Mathematics teachers require training in different topics of the NSSCO Mathematics syllabus. For instance, the majority of the teachers (14) have indicated that they require training in Logarithms, while 11 require training in Vectors. Further, six participants require training in Probability

and Geometry, while two participants require training in Linear Programming, Transformations, and Constructions.

4.6.4 Professional development training that teachers have attended

This section presents the results of the type of training that the participants have attended with regards to the NSSCO Mathematics syllabus. Their responses are summarised in Table 18 below.

Table 18: Types of training that the Participants attended

	Responses		
Duration of training	**Workshops**	**Conferences**	**Professional development courses**
Once in a each term	0	0	0
Once in two terms	1	0	0
Once a year	2	0	0
Once in two years	0	0	0
Once in three years	3	2	2
Never attended any sort of training	26	26	30

It is evident from Table 18 that Grade 12 Mathematics teachers in the Khomas Education Region hardly attended Mathematics professional development training. It is unbelievable to the researcher as indicated in Table 18 that, in total, only 10 participants have attended professional development training e.g. workshops and conferences.

4.7 Discussion

This study investigated Grade 12 Mathematics teachers' SCK and PCK in some selected public schools in the Khomas Education Region, Namibia. The investigation was carried out to validate claims from various authors and organisations that Namibian Mathematics teachers have poor Mathematics content knowledge and Pedagogical Content Knowledge that may be negatively affecting learners' academic performance of Mathematics. The findings of the study are discussed in the order that they appear in 4.3 and 4.4, 4.5, and 4.6

Teachers' Subject Content Knowledge

Teachers' SCK scores look satisfactory as indicated in Figure 6 and 7 but not sufficient since the SCK questions in the test were set based on the basic competencies of the NSSCO Mathematics syllabus that they teach to the learners. The researcher anticipated all the Grade 12 Mathematics teachers who took part in the study to score better than the results in Figure 7. Therefore, it can be argued that Grade 12 Mathematics teachers in the Khomas Education Region have satisfactory SCK but a limited conceptual understanding of Mathematics - that is, they had not sufficiently mastered the Mathematics content that they teach to the learners.

Insufficient SCK among the participants should be considered as a concern because, teachers with limited content knowledge will not be in a position to simplify aspects of content in a manner that learners can learn them with ease (Morgari, 2014). Moreover, teachers might also not sufficiently make the content interesting and relevant by relating it to real life contexts. Jadama (2014) states that teachers are required to fully understand the subject matter that they teach to the learners so that

they can enable them to select the best pedagogy required to help learners understand the subject matter.

According to Ball et al. (2008, p. 404), "teachers must know the subject well". Further, teachers who know the content well are likely to have a good knowledge of learners and help them understand the subject content. Reinforcing Ball et al's argument, Jadama (2014) states that teachers' ability to understand the subject content thoroughly means they are able to impart the subject content in an efficient manner. However, if a teacher has inaccurate information about the subject content, he/she might keep on passing on inaccurate information to the learners, which might negatively affect their academic performances. Black (2009) adds that teachers should be knowledgeable in the content areas that they are teaching to the learners. He further argues that "it will be difficult for the teacher to teach learners about a subject if the teacher does not know the content himself/herself" (p. 2). It is then easy to conclude from the views of Black (2009); and Ball et al. (2008) that teachers need to know the principles, concepts and procedures of getting correct answers and transfer such knowledge to the learners.

In terms of teachers' SCK scores per age groups, the findings show that there is no clear relationship between teachers SCK scores and age, which is consistent with Haufiku's (2008) study which established that "teachers' knowledge of subject matter was not commensurate with his or her age"(p. 56). Thus, it can be argued that it does not matter how old the teacher is, if he/she has a good conceptual understanding of the subject matter then he/she can perform better in the subject content regardless of the age.

Table 4 showed that participants in different circuits have scored differently in SCK questions, which is not surprising as Altinok (2013) notes that teachers' subject content knowledge differs among countries as well as among clusters in countries. Echoing similar sentiments, Venkat and Spaull (2015) state that teachers differ in terms of subject content knowledge for teaching, both in different aspects of concepts they emphasized and in the use of a representation repertoire to structure learning activities. Differences in the teachers' scores can be attributed to the opinion of Goos (2013) that teachers' professional knowledge vary as they studied at different institutions of higher learning offering different teaching programs.

One of the research questions aimed at finding out if there is any clear relationship between participants' SCK and their teaching experiences of Grade 12 Mathematics. The results of the study as indicated in Table 6 and Figure 8 seem to suggest that there is a strong positive relationship between teachers' SCK and their teaching experiences ($r = 0.76$) which contradicts some authors such as Das, 2015; and Kraus et al, 2008a. Das (2015) established that secondary school Mathematics teachers that took part in his study in the Kamrap district (India) have inadequate SCK in spite of their teaching experiences. Similarly, Krauss et al. (2008a) found that there was no relationship between the Grade 12 Mathematics teachers' SCK and their teaching experiences. It implies that there might be a possibility of a teacher to have misunderstood a particular subject content/concept during his/her training and continues to pass on inaccurate information to the learners for years. Therefore, it does not matter how many years of teaching experience a teacher has. If his or her subject content knowledge is not sufficient, it will remain insufficient and will be

continued to be passed on to the learners, up until the teacher upgrades his/her subject content knowledge through in-service training, workshops and conferences.

The study also aimed to find out if there is any relationship between Grade 12 Mathematics teachers' SCK and their qualifications. In this regard, the results appear to suggest that there is no clear relationship between the Grade 12 Mathematics teachers' SCK and their qualifications ($r = 0.21$). It is assumed that teachers that have covered higher level courses of Mathematics should be in a better position to outperform their counterparts who have covered lower-level courses of Mathematics as they have covered detailed subject content. However, it may also depend on what they have scored in higher level courses. Some teachers might have passed such courses with lower grades which by implications might mean that they did not fully grasp the higher level Mathematics content. Another possibility is that the higher level courses teachers have covered at Universities and other institutions of higher learning might not be related to what they were assessed on in this study and hence on what they teach. These results contradict Baurmert et al. (2010) findings, which established that Grade 10 German Mathematics teachers who possess advanced qualifications outscored their counterparts with less advanced qualifications.

Teachers' Pedagogical Content Knowledge

One of the research questions of the study was to determine the Pedagogical Content Knowledge of the participants. The findings are shown in Figure 9 and 10 which seem to suggest that the participants have insufficient PCK of teaching the NSSCO Mathematics syllabus. These findings appear to be in accordance with Kaino and Moalosi (2013); and Black's (2009) findings. Kaino and Moalosi (2013) conducted a

study in South Africa to test teachers' pedagogical content knowledge on answering linear equations with two variables. Their results revealed that many teachers lacked the understanding of the nature, characteristics and interpretations of the problems that were to be solved. Similarly, Black (2009) also documented limited PCK among Georgian secondary school Mathematics teachers. His study revealed that Georgian Secondary School Mathematics teachers who took part in the study had difficulties in assessing learners' errors and providing different representations for Mathematical situations. Further, instead of responding to errors or statements made by the learners, secondary school Mathematics teachers generally ignored them or responded with how the learners should have worked out the problem by using a set of algebraic manipulation, rather than providing alternative explanations or models.

According to Ball et al. (2008), teachers need to understand the different interpretations of Mathematical operations in ways that learners do not. Adding to the view of Ball et al. (2008), Tsafe (2013) indicates that teachers should have a good Pedagogical Content Knowledge to enable them to unpack Mathematics into "discrete elements and explain a concept or procedure at a level that includes the steps necessary for the learners to make sense of reasoning" (p. 37). In agreement with Tsafe's view, Morales et al. (2003) state that teachers should have a strong PCK – that is, understanding the subject content, the most effective ways of representing and explaining various topics and concepts of detecting learners' misconceptions about a particular content.

Insufficient PCK among the Grade 12 Mathematics teachers in the Khomas Education Region is not surprising. This is mainly due to the fact that, in the

questionnaire, the majority of participants indicated that they were moderately prepared in terms of PCK as shown in Table 15. If teachers feel that they are not sufficiently prepared to teach what they teach, then this should be considered as a concern because by implication it means that they will not adequately make the content comprehensive to the learners. Thus, managers and curriculum developers at institutions of higher learning should ensure that their programmes are designed to adequately prepare teachers to impart accurate subject content to the learners in a comprehensive way.

When teachers were asked to indicate the extent to which they are trained to teach the NSSCO Mathematics syllabus, 11 out 40 participants indicated that they were sufficiently prepared in terms of PCK, while 20 felt that they were sufficiently prepared in the SCK. Teachers responses are a reflection of Goos' (2013) view that, "although the university-level study of Mathematics may be beneficial in developing secondary school Mathematics Content Knowledge, it does not have a significant influence on PCK and is therefore inadequate on its own for preparing effective teachers of secondary school Mathematics" (p. 1). It would be then easy to conclude from Goos' view that institutions of higher learning adequately prepare teachers in terms of SCK and not well enough when it comes to PCK. Ball and Bas (2000) are of the opinion that Mathematics Content Knowledge and Pedagogical Mathematical Content Knowledge that teachers are taught at universities and colleges are frequently not the knowledge most useful for teaching Mathematics. Similarly, Kaino and Moalosi (2013) claim that teachers training institutions do not sufficiently equip teachers with PCK required for teaching in schools. However, teachers' PCK can be improved during professional development courses (Kaino & Moalosi, 2013).

Das (2015) acknowledges that pedagogical skill development is a big challenge for Mathematics teachers in the secondary level, even though they are trained, they are faced with challenges in teaching Mathematics that needs an urgent drastic modification for effective pedagogical outputs through redefining and redesigning the entire teachers' training courses.

Some Grade 12 teachers acknowledged that they find some topics difficult to explain to the learners as indicated in Table 16, which may be one of the contributing factors to poor PCK among the participants. If teachers themselves acknowledged that they are struggling to teach and explain some topics to the learners as indicated in Table 16, then by implication it may suggest that they are perhaps not sufficiently prepared to teach the NSSCO Mathematics syllabus. This argument is supported by their responses in Table 17 as the majority indicated that indeed they require training in various topics.

As indicated earlier that many teachers acknowledged that they are struggling to teach specific topics, it emerged that there is a correlation between the topics considered to be difficult by the participants and the topics they need training in. For instance, many teachers have indicated that they find topics such as Sequence and Series, Logarithms, Probability, Vectors and Geometry difficult to teach and explain to the learners and, at the same time, they have also indicated that they require training in these topics.

In the closed-ended questionnaire, participants were also asked if they have attended any sort of training /workshops regarding the teaching of the Grade 12 NSSCO

Mathematics syllabus. Their responses as shown in Table 18 indicated that 26 teachers have never attended any sort of training, be it a workshop, conferences or any type of professional development training. Non-attendance of workshops or conferences by the Grade 12 Mathematics teachers could be one of the contributing factors to their poor scores in PCK questions because Solis (2009) notes that, professional development courses develop expertise in teaching, which consequently leads to the academic success of the learners. It implies that, as the curriculum reforms, new knowledge is introduced and it is during workshops and professional development training that the new knowledge is explained to the teachers. Therefore, it is the responsibility of the professional development institutions such as the National Institute for Educational Development (NIED) and education officers to organise and conduct workshops and conferences meant to strengthen teachers' PCK.

In terms of the relationship between teachers' PCK and teaching experiences, the findings show that no clear relationship was established ($r = 0.52$), which is contrary to the researcher's assumptions that experienced teachers should have accumulated substantial PCK over the years. These results are in accordance with Das (2015) who discovered in his study that Grade 12 secondary school Mathematics teachers at the Kamrup district (India) have inadequate PCK regardless of their teaching experience. Krauss et al. (2008) note that expertise does not increase simply by doing the job, rather teachers need to be motivated, identify their weaknesses and improve such weaknesses through professional development. The results also support the claim of Kaino and Moalosi (2013) that teaching experience is not enough to improve teachers PCK and that PCK can only be significantly improved through professional development. On the other hand, the results appear to contradict

Metzler and Woessmann (2010) view that the information that teachers acquire about learners in the course of instruction subsequently becomes part of their PCK.

This study also aimed at establishing the relationship between teachers' PCK and their qualifications. Based on the patterns of the findings in Table 11 and the correlation coefficient ($r = 0.086$), it can be argued that there is a very weak relationship between the participants' PCK and their qualifications. These findings support Goos (2013) argument that formal qualification of teachers is an important indicator for their knowledge and competence in teaching; however, it has limited utility in analysing how well teachers are prepared for what they have to teach in schools.

On the other hand, these results contradict Morgari, Stols and Ogbonnaya (2009) cited in Morgari (2014) who found that the Grade 11 Mathematics teachers with higher qualifications of Mathematics in SA are more effective in their teaching than their counterparts with lower quality qualifications. Thus, it can be argued that most of the B.Ed, B.Ed (Hon), and NDE holders are not sufficiently prepared to teach the NSSCO Mathematics syllabus or they might not have adequately grasped the teaching methods of Mathematics during their training at their respective institutions of higher learning.

The relationship between the participants' SCK and PCK

The results in Table 14 show that a significant difference exists between the teachers' SCK and PCK with the $t_{calucated= 11.092}$ value greater than the $t_{critical= 2.750}$ value at 39 degrees of freedom. The results also confirmed Shulman (1986) view that SCK and

PCK are empirically distinguishable types of knowledge. Apart from the significant difference, the results show that the participants' SCK and PCK were strongly and positively correlated ($r = 0.681$, $p < 0.01$), which seems to support Goos's (2013) view that PCK cannot exist alone without the foundation of SCK. These results of a strong correlation between teachers' SCK and PCK will add to the ongoing debate among educators whether the two types of knowledge are one entity or close but different kinds of knowledge. The results of the significant difference and positive correlation between participants' SCK and PCK seem to be in accordance with the COACTIV test (Kraus et al., 2008b) which was meant to determine the construct validity of SCK and PCK of German prospective Mathematics teachers and established that PCK is deeply interrelated with SCK and SCK is a prerequisite of PCK.

4.8 Summary

The findings of the study reveal that Grade 12 Mathematics in the Khomas Education Region has a satisfactory SCK but limited to a certain extent, while their PCK of Mathematics is insufficient. The findings of the study also revealed that there is no clear relationship between the participants Subject Content Knowledge and the number of teaching experience of the NSSCO Mathematics syllabus. Similarly, no relationship was established between the Grade 12 Mathematics teachers' PCK and teaching experience. The findings of this study also showed that there is no clear link between the teachers' SCK and qualifications neither between the teachers PCK and qualifications. Finally, the findings of the study also established significant differences between the Grade 12 Mathematics teachers' SCK and PCK. The next chapter focuses on the summary, conclusion and recommendations of the study.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

5.1 Introduction

In this chapter, the summary of the findings of the study is discussed. The conclusion and the recommendations for improving Grade 12 Mathematics teachers' SCK and PCK are also brought forth. Further recommendations are made for future studies to expand on this study.

5.2 Summary

The study sought to investigate the Grade 12 Mathematics teachers' Subject and Pedagogical Content Knowledge in the Khomas Education Region. It also sought to investigate if there was a significant difference between Grade 12 Mathematics teachers' SCK and PCK. Further, the study sought to find the relationship between the Grade 12 Mathematics teachers' knowledge (SCK and PCK) and their teaching experience. Finally, the study explored if any relationships existed between participants' knowledge (SCK and PCK) and their qualifications.

The study addressed three questions and one hypothesis as indicated below.

1. What is the Grade 12 Mathematics teachers' Subject Content Knowledge in some selected public schools in the Khomas Education Region?

2. What is the Grade 12 Mathematics teachers` Pedagogical Content Knowledge in some selected public schools in the Khomas Education Region?

3. What relationship exists between the Grade 12 Mathematics teachers' knowledge, teaching experience and qualifications in the Khomas Education Region?

Hypothesis:

H_0 – There is no significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge.

H_1 – There is a significant difference between the Grade 12 Mathematics teachers' Subject Content Knowledge and Pedagogical Content Knowledge.

In order to find answers to the research questions and to prove the hypothesis as stated above, a quantitative design was used because of the nature of the research problem and hence the data it requires. A stratified random sampling method was used to select 53 participants from all four circuits in the Khomas Education Region. In this study, a closed-ended questionnaire and a test were used to collect data from the participants. A Microsoft Excel spreadsheet was used to organise and present the data into graphs, figures and tables. Descriptive statistics were used to analyse the data obtained from the test. A correlation coefficient was used to describe the relationship between the teachers' knowledge, teaching experiences and their qualifications. Furthermore, a paired t-test was used to determine if the significant difference existed between the teachers' SCK and PCK. The findings of the study in accordance with the research questions and the hypothesis revealed the following results:

- The Grade 12 Mathematics teachers in the Khomas Education Region have a satisfactory but limited Subject Content Knowledge because many of them did not perform well in some questions such as question numbers 1, 2, 6 and 8. Moreover, on average, the Grade 12 Mathematics teachers scored 60% on SCK questions.

- The Grade 12 Mathematics teachers in the Khomas Education Region have insufficient Pedagogical Content Knowledge. Out of 40 participants, only two participants managed to reach half of the total marks of the PCK questions. In addition, the participants scored 28% on average in PCK questions, which is worrisome because the questions of the test were set, based on the basic competencies as stipulated in the NSSCO Mathematics syllabus that they teach to the Grade 12 learners.

- A significant difference existed between the Grade 12 Mathematics teachers' SCK and PCK with a *t*-test calculated value of 11.092 and a degree of freedom of *39* at the significance level, α =0.01 (the $t_{critical}$ = 2.750 value). On average, teachers' SCK scores were 7.5 points higher than PCK scores (95% CI [6.153, 8.897]), which implies that the participants have performed better in SCK questions than PCK questions. The results also revealed that SCK and PCK were close, strongly and positively correlated ($r = 0.681, p < 0.01$) which support Shulman's (1986) theory and other researchers such as Kraus et al. (2008), that SCK is a prerequisite of PCK.

- A strong positive relationship was established between the teachers' SCK and their teaching experiences ($r = 0.76$). However, some teachers that taught Grade 12 Mathematics for as many years as 11 -12 years were outperformed by teachers who only taught for 5-6, and 7-8 years in SCK questions. No clear connection was established between the participants' PCK and their teaching experience of Grade 12 Mathematics (r = 0.52). That is, teachers who taught for 9-10 years were outperformed by those that taught for 1-2, 3-4, and 7-8 years. Further, those who taught for 13-14 years

were also outperformed by those that taught for 3-4, 5-6, and 7-8 in PCK questions.

- The findings also revealed that no clear relationship was established between the participants' types of knowledge (PCK and SCK) and their qualifications. That is some teachers who possessed higher teaching qualifications such B.Ed and B.Ed (Hon) were outperformed by a teacher with lower teaching qualifications such as BETD + MASTEP in both SCK and PCK questions. Similarly, B.Ed (Hon) holders were outperformed by B.Ed holders. Furthermore, qualified teachers (B.Ed and B.Ed (Hon)) were outperformed by unqualified teachers (B.Sc) in PCK questions.

The study also highlights the following:

- The majority of the participants (75%) acknowledged that some topics from the NSSCO Mathematics syllabus are difficult to teach and to explain to the learners. The majority of the participants also acknowledged that they have never attended any sort of Mathematics training, workshops or any professional development training since they started teaching Grade 12 Mathematics. As expected, some participants have confessed that they require training on how to teach some topics in the NSSCO Mathematics syllabus and the topics they required training matched the topics they find difficult to explain to the learners.

- It is evident that more than half of the participants (55%) indicated that they are well prepared to teach the subject content; although, in terms of Pedagogical Content Knowledge, they (33%) felt that were moderately

prepared to teach the NSSC Mathematics syllabus. Nevertheless, they did not perform well in both SCK and PCK questions.

5.3 Conclusion

The findings of this study can only be generalised to the Grade 12 Mathematics teachers in the Khomas Education Region but not to other education regions in Namibia. Hence, based on the findings of the study, it can be concluded that: the Grade 12 Mathematics teachers in the Khomas Education Region have a satisfactory Subject Content Knowledge and insufficient Pedagogical Content Knowledge. This satisfactory SCK and insufficient PCK among the participants might be attributed to non – attendance of professional development training (workshops and conferences) by the teachers. The findings of this study also seem to suggest that there is no clear connection between the teachers' PCK and their teaching experiences. This seems to suggest that experienced teachers do not necessarily have a better PCK of Mathematics than fewer experienced teachers in the Khomas Education Region. If the teachers are not motivated and do not attend professional development courses then their PCK will become weak as the curriculum reforms. That is when a new curriculum is implemented, new subject content and teaching styles are also introduced.

Furthermore, the finding of the study established that there is no clear relationship between the participants' Knowledge (SCK and PCK) and their qualifications. That is, a teacher's qualification does not clearly indicate the degree to which a teacher is trained to teach a particular content or Grade. The results also show that a significant difference exists between the Grade 12 Mathematics teachers' SCK and PCK and

that they (teachers) scored better in SCK questions than PCK questions. However, teachers' SCK and PCK were close, strongly and positively correlated ($r = 0.681, p < 0.01$). It can therefore be concluded that teachers' SCK and PCK are different kinds of knowledge but PCK cannot stand on its own without SCK. Shulman (1986) states that the main idea of PCK lies in the "*teachability",* then it is not surprising that the two knowledge (SCK and PCK) are closely, strongly and positively correlated because the teacher need to understand the content before explaining it to the learners. Thus, it is difficult for a teacher to explain and to make the subject content comprehensive to the learners if he or she does not also understand the content.

5.4 Recommendations

Based on the findings of the study, the following recommendations are proposed in order to strengthen the Grade 12 Mathematics teachers' SCK and PCK and expand on this topic:

Recommendations for improving teachers' SCK and PCK

- The National Institute for Educational Development (NIED), in conjunction with the Ministry of Education, Art and Culture, should organise and implement in-service programs to ensure that Grade 12 Mathematics teachers develop an extensive knowledge base of SCK and PCK in the Khomas Education Region. However, much emphasises should be directed towards PCK since the participants have scored better in SCK questions (average score =60%) as compared to PCK questions (28%).

- Professional development courses in Mathematics as a subject should be conducted on a continuous basis due to curriculum reform to ensure that

teachers are well equipped with new knowledge and modern ways of making the content comprehensive to the learners.

- Advisory teachers for Mathematics should visit schools on a regular basis in order to identify teachers' needs in terms of SCK and PCK so that they can address such needs in workshops and conferences.

- Teacher-training institutions in Namibia should ensure that Pre-service teachers are systematically introduced to both SCK and PCK to enhance the process of teaching and learning.

- Advisory services, School principals and Heads of Departments should encourage teachers to attend workshops, conferences and professional development courses in order to refresh their teaching methods and become abreast with new developments in educational matters.

Recommendations for further studies

- Since the study has indicated that the Grade 12 Mathematics teachers in the Khomas Education Region have a satisfactory SCK and insufficient PCK, a large-scale study that incorporates all the topics as stipulated in the NSSCO Mathematics syllabus should be conducted across all the regions to investigate the status of the Grade 12 Mathematics teachers' SCK and PCK in Namibia. This large-scale study should use both the quantitative and qualitative designs in order to provide more in-depth information about the teachers SCK and PCK.

- According to the information from the Directorate of National Examination and Assessment (DNEA), poor Mathematics results among the Grade 12 learners have been noted in the Khomas Education Region over the past

years. Therefore a correlation study should be done to determine if Grade 12 Mathematics teachers' SCK and PCK have an impact on the Grade 12 learners' academic performance of Mathematics.

- Policy makers and curriculum developers should conduct research to identify teachers' needs in terms of SCK and PCK and develop courses that equip teachers with an SCK and PCK.

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Appendix 1: Permission letter from the University of Namibia Research Ethics Committee

UNAM
UNIVERSITY OF NAMIBIA

ETHICAL CLEARANCE CERTIFICATE

Ethical Clearance Reference Number: FOE /254/2017   **Date: 27 September, 2017**

This Ethical Clearance Certificate is issued by the University of Namibia Research Ethics Committee (UREC) in accordance with the University of Namibia's Research Ethics Policy and Guidelines. Ethical approval is given in respect of undertakings contained in the Research Project outlined below. This Certificate is issued on the recommendations of the ethical evaluation done by the Faculty/Centre/Campus Research & Publications Committee sitting with the Postgraduate Studies Committee.

Title of Project: An Investigation On Grade 12 Teachers' Subject And Pedagogical Content Knowledge Of Mathematics In Some Selected Public Schools In The Khomas Education Region, Namibia

Researcher: JOSEPH JEULUKENI KANDJINGA

Student Number: 200254758

Faculty: Faculty of Education

Supervisor: Dr. H. M. Kapenda

Take note of the following:
(a) Any significant changes in the conditions or undertakings outlined in the approved Proposal must be communicated to the UREC. An application to make amendments may be necessary.
(b) Any breaches of ethical undertakings or practices that have an impact on ethical conduct of the research must be reported to the UREC.
(c) The Principal Researcher must report issues of ethical compliance to the UREC (through the Chairperson of the Faculty/Centre/Campus Research & Publications Committee) at the end of the Project or as may be requested by UREC.
(d) The UREC retains the right to:
(i) Withdraw or amend this Ethical Clearance if any unethical practices (as outlined in the Research Ethics Policy) have been detected or suspected,
(ii) Request for an ethical compliance report at any point during the course of the research.

UREC wishes you the best in your research.

Prof. P. Odonkor: UREC Chairperson   Ms. P. Claassen: UREC Secretary

UNIVERSITY OF NAMIBIA
P / BAG 13301, WINDHOEK, NAMIBIA

2017 -09- 2 5

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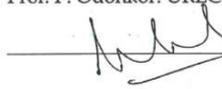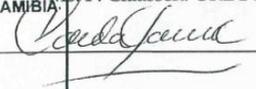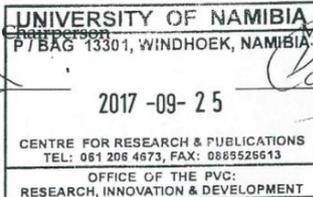
Appendix 2: Letter requesting permission from the researcher

P .0 Box 55284, Rocky Crest, Windhoek
Mobile: **0812857038**
Email : *jkandjinga@yahoo.com*

05 October 2017

The Permanent Secretary
Ministry of Education, Arts and Culture
Private Bag 13391
Windhoek

RE: PERMISSION TO CONDUCT AN EDUCATIONAL REARCH IN THE KHOMAS EDUCATIONAL REGION

I am Joseph J. Kandjinga, a Master of Education (Mathematics Education) student at the University of Namibia. I would like to conduct a research study entitled: *An investigation on Grade 12 Teachers' Subject and Pedagogical Content Knowledge of mathematics of mathematics in some selected public schools in the Khomas Educational Region.*

I am humbly seeking permission from your good office to conduct the above-mentioned study. This research is required for the fulfillment of the degree of Master of Education. This study will only be conducted at public secondary schools and will only focus on the Grade 12 teachers. The study will be conducted at the following schools;

The findings of this study might help policy makers and curriculum developers to formulate policy that will strengthen Subject and Pedagogical content knowledge of Pre-service Mathematics teachers. It might also help Mathematics educational officers to organize workshops specifically meant to address problematic areas of mathematics experienced by Grade 12 Mathematics teachers. Further, the study might provide hints for assessing teachers' Mathematical Knowledge in Namibia. The study will also add to the field of research in the Namibian context, since it is the first of its kind that will examined the Grade 12 Mathematics teachers' Subject and Pedagogical Content knowledge.

Thank you for your consideration in this regard

Yours faithfully

Joseph Kandjinga

Appendix 3: Permission letter from the Permanent Secretary

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION, ARTS AND CULTURE

Tel: +264 61 -2933200/86
Fax: +264 61- 2933922
Enquiries: C. Muchila/G Munene
Email: Cavin.Muchila@moe.gov.na/gm12munene@yahoo.co.uk

Luther Street, Govt. Office Park
Private Bag 13186
Windhoek
Namibia

File no: 11/1/1

Mr. Joseph Kandjinga
Private Bag 55284
Windhoek
Cell: 081 285 7038
Email: jkandjinga@yahoo.com

Dear   Mr. Kandjinga

SUBJECT: PERMISSION TO CONDUCT RESEARCH IN KHOMAS REGION

Kindly be informed that permission to conduct research for your Master's Degree in *"An Investigation on Grade 12 Teachers' Subject and Pedagogical Content Knowledge of Mathematics in some selected Public Schools in Khomas Region"* is herewith granted. You are further requested to present the letter of approval to the Director of Adult Education to ensure that research ethics are adhered to and disruption of curriculum delivery is avoided.

Furthermore, we humbly request you to share your research findings with the ministry. You may contact Mr C. Muchila/ Mr. G. Munene at the Directorate: Programmes and Quality Assurance (PQA) for provision of summary of your research findings.

I wish you the best in conducting your research and I look forward to hearing from you soon.

Sincerely yours
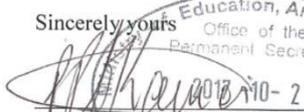
SANET L. STEENKAMP
PERMANENT SECRETARY

26-10-2017
Date

All official correspondences must be addressed to the Permanent Secretary

Appendix 4: Letter requesting permission to the Educational Director of Khomas Region

P .0 Box 55284, Rocky Crest, Windhoek
Mobile: **0812857038**
Email : *jkandjinga@yahoo.com*

06 October 2017

The Director: Khomas Regional Council
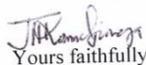Directorate of Education, Arts and Culture
Private Bag 13391
Windhoek

RE: PERMISSION TO CONDUCT AN EDUCATIONAL REARCH IN THE KHOMAS EDUCATIONAL REGION

I am Joseph J. Kandjinga, a Master of Education (Mathematics Education) student at the University of Namibia. I would like to conduct a research study entitled: *An investigation on Grade 12 Teachers' Subject and Pedagogical Content Knowledge of mathematics of mathematics in some selected public schools in the Khomas Educational Region.*

I am humbly seeking permission from your good office to conduct the above-mentioned study. This research is required for the fulfillment of the degree of Master of Education. This study will only be conducted at public secondary schools and will only focus on the Grade 12 teachers. The study will be conducted at the following schools;

The findings of this study might help policy makers and curriculum developers to formulate policy that will strengthen Subject and Pedagogical content knowledge of Pre-service Mathematics teachers. It might also help Mathematics educational officers to organize workshops specifically meant to address problematic areas of mathematics experienced by Grade 12 Mathematics teachers. Further, the study might provide hints for assessing teachers' Mathematical Knowledge in Namibia. The study will also add to the field of research in the Namibian context, since it is the first of its kind that will examined the Grade 12 Mathematics teachers' Subject and Pedagogical Content knowledge.

Thank you for your consideration in this regard

Yours faithfully

Joseph Kandjinga

Appendix 5: Permission letter from the Director of Education of Khomas Region

REPUBLIC OF NAMIBIA

KHOMAS REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS AND CULTURE

Tel: [09 264 61] 293 9411
Fax: [09 264 61] 231 367/248 251

Private Bag 13236
WINDHOEK

9 October 2017

Joseph Kandjinga
P.O. Box 55284
Rocky Crest
Windhoek
Mobile: 081 285 7038

PERMISSION TO CONDUCT AN EDUCATIONAL RESEARCH IN KHOMAS REGION

Your letter dated 5 October 2017 on the above topic refers.

Permission is hereby granted to you to conduct research for your Master of Education Degree (Mathematics Education) titled "An investigation on Grade 12 Teachers' Subject and Pedagogical content knowledge of Mathematics" in the following selected public schools: ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, ▬▬▬▬▬, H ▬, ▬▬▬▬▬S and ▬▬▬▬▬ in Khomas Region.

The following must be adhered to:

- Permission must be granted by the School Principal;
- Teaching and learning in the respective schools should not be disrupted;
- Teachers who will take part in the research should do so voluntarily;
- A copy of your thesis with the findings/recommendations must be provided to the Directorate of Education, Arts and Culture, Khomas Regional Council.

I trust this confirmation will assist.

Yours sincerely

09/10/2017.

Gerard N. Vries
Director of Education, Arts and Culture

MINISTRY OF EDUCATION
ARTS AND CULTURE
PRIVATE BAG 13236 WINDHOEK
09 -10- 2017
DIRECTOR
KHOMAS REGION

Appendix 6: Consent letter for the participants

<div align="right">

PO BOX 55284, Rocky Crest,
Mobile: +264812857038
Email: jkandjinga@yahoo.com
21 July 2017

</div>

Dear participant,

I am Joseph J. Kandjinga, a Master of Education (Mathematics Education) student at the University of Namibia doing research on "*The Grade 12 Mathematics teachers' subject content and pedagogical content knowledge in the Khomas educational region*". You are humbly invited to participate in this research study. There are no risks involved in participating in this research. This study consists of a closed-ended questionnaire and a test based on the NSSCO Mathematics syllabus. Your sincere responses will help in improving the effective teaching of Mathematics in our schools. You are assured that the information you will give in this research study will be treated with maximum confidentiality since your name and that of your school will not be used on the instruments; hence counting numbers will be used. Your participation in this research is entirely voluntary. If you are willing to participate in this research, please fill in your full name and sign in the space provided. Signing the consent form means you declare that you will participate willingly in this research and you have the right to withdraw from the study at any time.

Thank you very much.

Yours truly,

_____

Mr. J. J. Kandjinga

Consent

I_____ agree to participate in the research entitled "*An investigation on the Grade 12 Mathematics teachers' subject content and pedagogical content knowledge in some selected public schools in the Khomas Educational Region.*"

_____   _____
Signature   Date

Appendix 7: A closed-ended questionnaire for the teachers

TEACHERS' PROFILE QUESTIONNAIRE

Grade 12 Mathematics Teachers

Circuit

KHOMAS EDUCATION REGION

<u>**Instructions**</u>:

- ➢ Please answer all the questions on the space provided.
- ➢ **Do not** write your name on this paper.
- ➢ This paper consists of 6 printed pages including the cover paper.
- ➢ Tick or cross the appropriate box where applicable.

SECTION A: TEACHER BIOGRAPHIC INFORMATION

1. Gender

Male	Female

2. Age category

21 – 25	26 - 30	31-35	36-40	41-45	46-50	51-55	56-60	60+

3. Teaching experience of **Grade 12 mathematics**

1 – 2 Year (s)	3 – 4 years	5 – 6 years	7 – 8 years	9 – 10 years	11 - 12 years	13 – 14 years	15-16 years	17+ years

4. What is/are your Mathematics teaching qualification(s)- **for qualified teachers only.** You may tick more than one box

Number	Qualification	Tick	Specify. **E.g. BETD**
1	Certificate (s)		
2	Diploma (s)		
3	Bachelor Degree		
4	Honours Degree		
5	Masters		
6	Doctor of Philosophy (PhD)		

5. What is your area of specialisation? (**for Unqualified teachers**)

Number	Qualification	Majored in mathematics (Tick)	Majored in another subject (s) Specify, e.g. Chemistry
1	Bachelor Degree		
2	Honours Degree		
3	Masters		
4	Doctor of Philosophy (PhD		

SECTION B: GENERAL QUESTIONS REGARDING THE TEACHING OF THE NSSCO MATHEMATICS SYLLABUS

1. The following topics are taken from the NSSCO Mathematics syllabus. Tick the topic (s) that you **find easy to explain** to the learners. You may tick more than one box.

Numbers	
Algebra	
Mensuration	
Geometry	
Trigonometry	
Statistics	
Probability	
Money and finance	
Logarithms	
Locus	
Vectors	
Graphs of functions	
Graphs in Practical Situations	

Linear programming	
Functions	
Others (specify) ………………………………………. ……………………………………….	

2. The following topics are taken from the NSSCO mathematics syllabus. Tick the topic (s) that you **find difficult** to teach and explain to the learners. You may tick more than one box.

Topic	
Vectors	
Probability	
Geometry	
Trigonometry	
Mensuration	
Algebra	
Sequence and series	
Limit of accuracy	
Estimation	
Logarithms	
Others (specify) ………………………………………… ……………………………………….	

3. What kind of support do you get from the Circuit or Regional office regarding the teaching of Grade 12 Mathematics? You may tick more than one box.

Marking examination papers training	
Setting examination papers training	
Moderations of examination training	
Teaching specifics topics training	
Others (specify) ---	
None	

4. Have you ever attended a workshop, conference or a NIED professional development training regarding the teaching and learning of Grade 12 Mathematics? Tick

Yes	
No	

5. If your answer for number 4 is yes, how often? You may tick more than one box

	Workshops	Conferences	Professional development training	Others (specify) ………………..
Once in each term				
Once in two terms				
Once a year				
Once in two years				
Once in three years				
Never attended it since I started teaching Grade 12 Mathematics				

6. Do you need training about any of the topic (s) listed below? If yes, tick the topic (s). **You may tick more than one box**

Topic	
Vectors	
Probability	
Geometry	
Trigonometry	
Mensuration	
Algebra	
Sequence and series	
Limit of accuracy	
Estimation	
Logarithms	
Others (specify) ………………………………………… …………………………………………	

7. To what extent does the tertiary institution (s) prepare you to teach Grade 12 Mathematics ordinary level syllabus? Tick

Extent of preparation	In terms of **the content of the subject**	In terms of **methods of teaching Mathematics**
Sufficiently prepared		
Moderately prepared		
Insufficiently prepared		

Thank you very much for your time!

Appendix 8: A Mathematics test for the teachers

MATHEMATICS PROFICIENCY TEST

Grade 12 Mathematics Teachers
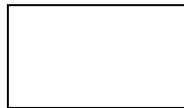
CIRCUIT

KHOMAS EDUCATION REGION

➢ Please fill in the circuit number on the cover page.
➢ **Do not** write your name on this paper.
➢ Answer all the questions and show your work.
➢ This test consists of Section A and Section B.
➢ This paper consists of **7** printed pages including the cover page.

SECTION A (Subject content knowledge)

1. Is it true that $0.9999999\ldots = 1$? Please give detailed reasons for your answer.

2. When John wakes up each morning, the probability that he feels happy is $\frac{9}{10}$. What is the probability that he will feel unhappy when he wakes up, on both Monday and Tuesday?

3. Two lines A ($y = mx + c$) and B ($y = \frac{-x}{2} + 4$) are parallel. Line A pass through the point $(4; 0)$. Find the value of m and c

4. Solve for x in $2^{x+1} = \sqrt{93}$, show all your working

5. Determine which term in the sequence: 2,   6,   18,   54 ... . is equal to 1458.

6. A cylindrical water tank has a volume of $6000\ cm^3$ correct to 1 significant figure. A cup full of water from the tank has a volume of $300cm^3$ correct to the nearest hundred.
   Calculate the maximum number of full cups of water that can be obtained from the tank

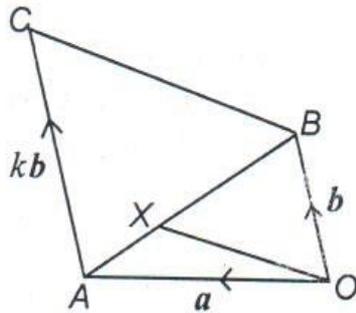7. Express m in terms of t and z in: $t + m = \dfrac{mz}{t}$

8. Calculate the value of angle PQR

NOT TO SCALE
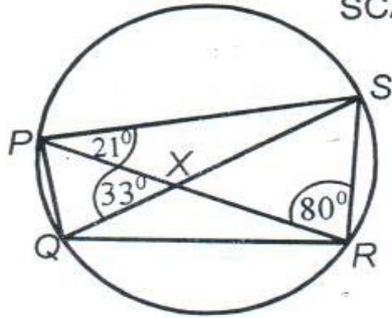
9. $\overrightarrow{AX} = \dfrac{1}{4}AB$, Express $\overrightarrow{AX}$ in terms of a and b

NOT TO SCALE

SECTION B (Pedagogical content knowledge)

1. Suggest any way of helping learners understand that $\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}$

2. In solving the equation; $3x - 5 = 10$, Petrus subtract 5 from both sides. What would you do to help Petrus to overcome this misunderstanding?

3. Selma solved a pair of a simultaneous equation; $3x - y = 7$ and $x + y = 9$ as shown below

$$3x - y = 7 \text{ Equation 1}$$
$$\underline{x + y = 9} \quad \text{Equation 2}$$
$$2x = -2$$
$$x = 1$$
$$\text{Substitution into equation 1: } x + y = 1$$
$$-1 + y = 9$$
$$y = 10$$

    (a) What might she be thinking when she decided to solve the simultaneous equations in that way?

(b) How will you address Selma's difficulty of solving the simultaneous equations?

4. How does the surface area of a square change when the side length is tripled? Show your reasoning

5. The area of a parallelogram can be calculated by multiplying the length of its base by its height. Please sketch an example of a parallelogram to which learners might fail to apply this formula

6. Use different methods to solve $2x + 5 = 11$

7. A teacher asked Selma to round off 0.9875 correct to there (3) significant figures. Selma does this task as follow: 0.98<u>75</u> (7 is bigger than 5, so I add 1 to 7)

    (a) What is the error in her thinking?

    (b) What kind of approach can be used to correct her understanding?

8. A learner says; I don't understand why $(-1) \times (-1) = 1$
    Please outline as many different ways as possible of explaining this mathematical fact to your learner

Thank you very much for your time!

Adapted from Krauss et al (2008); Li (2007); and French (2005).