EXPLORING GRADE 12 LEARNERS' DIFFICULTIES IN SOLVING DIFFERENTIAL CALCULUS PROBLEMS AT A SELECTED SECONDARY SCHOOL IN TSUMEB, NAMIBIA

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ABSTRACT

Differential Calculus is a fundamental topic in Mathematics that deals with the study of how quantities change over time. Grade 12 learners have difficulties in solving Differential Calculus questions as per the examiners' reports year after year. This study aimed to explore Grade 12 learners' difficulties in solving Differential Calculus. One secondary school in Tsumeb was purposely and conveniently selected for this study. A total population sampling method was used to ensure a representation of all 74 Grade 12 Mathematics AS learners and all 3 grade 12 Mathematics AS teachers.

The research study was based on a mixed methods exploratory approach; i.e., data was collected from classrooms' observations, teachers' questionnaires and learners' diagnostic test. Only questions involving the learning objectives from Differential Calculus topic were considered. A pilot study was carried out prior to the main study in order to test for feasibility and effectiveness of the research design. An analysis was done in order to determine learners' common mistakes/misconceptions and errors, challenges and also the great responses given by the learners.

The results revealed that learners had difficulties in 1) applying the product and quotient law, 2) differentiating trigonometric functions, 3) applying conceptual and procedural knowledge to solve Differential Calculus problems (like determining the nature of stationary points), 4) integrating instead of differentiating and 5) just applying differential rules in general. These difficulties may be caused by 1) lack of algebraic knowledge (learners fail to substitute values into functions, solve equations), 2) lack of sufficient time (for both teachers and learners), and 3) teachers' teaching approaches. The study recommends that teachers should expose learners to a variety of algebraic manipulation problems from an early stage, so that learners begin differential calculus with solid foundations of algebra; and that teachers should do diagnostic assessments based on algebraic and functions before teaching Differential Calculus.

KEY WORDS: Differential Calculus; Learners’ Difficulties**;** Misconceptions; Solving Problems

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DEDICATION

I dedicate this paper to my sister Josephine Nangula Matheus. To my mother, Frieda Mutota-Phillipus for her unconditional love and my father, Nghishoono Kafunga for his continuous encouragement and support towards my studies.

DECLARATION

I, Martha Ndakumwa Kafunga, hereby declare that this study is my own work and is a true reflection of my research, and that this work, or any part thereof has not been submitted for a degree at any other institution.

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ACRONYMS AND ABBREVIATIONS

MoEAC: Ministry of Education Arts and Culture

DNEA: Directorate of National Examinations and Assessment

NSSC: Namibia Senior Secondary Certificate

AS: Advanced Subsidiary

CHAPTER ONE: INTRODUCTION

The study sought to explore Grade 12 learners' difficulties in solving Differential Calculus at the selected secondary school in Tsumeb, Namibia. The background, problems statement, research questions, significance of the study, limitations, delimitations and definitions of key terms are in this Chapter. This Chapter therefore, overall gives an overview of this study.

1.1. Background of the study

Mathematics stands as a cornerstone in global education, as an indispensable framework woven into the fabric of everyday life (Brijall, et al., 2017; Saadati & Celis, 2023). Michael (2015) articulates Mathematics as the science governing reasoning and computations, encompassing the study of numbers, quantities, and shapes. Within the realm of science, Mathematics stands as a pivotal branch, propelling significant contributions to the progress of science and technology (Kusmaryono, 2014; Hodanova & Nocar, 2016). Mathematics in science and technology may go beyond just understanding the world.

In the realm of Mathematics, symbols play a crucial role, making it imperative for learners to master and use them accurately (Brijlall, et al., 2017; Fitzmaurice et al., 2021). Proficiency in Mathematics goes hand in hand with actively participating in its practice, allowing learners to build their understanding as they delve into the subject (Kusmaryono, 2014). Consequently, a deficiency in comprehending mathematical language and symbols can lead to inadequacies within the subject matter itself (Fitzmaurice et al., 2017). If learners are not familiar with the terms used in Differential Calculus or Mathematics in general, they may struggle to comprehend problems and communicate effectively.

Performance in Mathematics is not solely reliant on the capabilities of learners. It is imperative for educators to grasp the fundamental essence of mathematical learning before passing on this knowledge to effectively attain learning objectives (Kusmaryono, 2015). Moreover, the demands of contemporary life underscore the importance of robust mathematical proficiency, serving as a critical asset for both instructors and students (Hodanova & Nocar, 2016). Therefore, it is of vital importance to equip learners with technical prowess, as a strong foundation in Mathematics and technical disciplines equips students to secure employment and thrive in the competitive job market (Hodanova & Nocar, 2016).

In Namibia, Mathematics stands as a fundamental subject within the core curriculum (all learners are expected to take, regardless of their educational goals), universally taught across all schools from Grade 0 through Grade 11. Upon reaching Grade 12, students have the option to pursue Mathematics further provided they have attained a grade of C or higher in their Grade 11 Mathematics exams.

In Namibia, Mathematics stands out as one of the most challenging subjects for school learners. Over the years, the Namibia Senior Secondary Certificate (NSSC) has consistently reported Mathematics as one of the subjects with the poorest performance (Bezuidenhout & Ntinta, 2022). The attainment of high marks in Mathematics remains rare among students (Brijlall et al., 2017), resulting in only a small fraction meeting the stringent university entrance requirements for fields reliant on strong mathematical proficiency. Despite the practical significance of Mathematics in our daily lives, a notable number of learners struggle to excel in this subject (Brijlall et al., 2017). Multiple studies corroborate these findings, highlighting that the majority of learners do not find Mathematics enjoyable due to its challenging nature, making mastery difficult (Bezuidenhout & Ntinda, 2022; Brijlall et al., 2017; Dwedi, 2022; Hamukwaya, 2022). Please synthesize the literature.

Differential Calculus, constitutes of a fundamental branch of Mathematics widely taught in secondary schools across various countries (Makgakga & Makwakwa, 2016). This mathematical discipline holds significance in fostering essential skills like mathematical reasoning, computational proficiency, effective mathematical communication, and problem-solving capabilities (Da, 2022). Its core objective revolves around optimizing real-world applications by enhancing practical human activities through the development of these crucial competencies.

Differential Calculus serves as the cornerstone for understanding dynamic systems in various fields, from physics to economics and engineering. The concept of derivatives allows us to explore not only instantaneous rates of change but also tangents, slopes, and optimization. For instance, in physics, it's pivotal for grasping concepts like velocity, acceleration, and the behavior of objects in motion through space. Moreover, Differential Calculus enables economists to model changes in variables like cost, revenue, and profit, aiding in making informed decisions in markets. Understanding derivatives also lays the groundwork for higher-level mathematical tools such as integral calculus, differential equations, and multivariable calculus. Despite its widespread applications, the intricacies of chain rule, product rule, and quotient rule often pose challenges for learners grappling with the nuanced methods of solving Differential Calculus problems (Da, 2022). Additionally, the visual interpretation of derivatives through graphs and functions can sometimes be complex, requiring a solid grasp of concepts for a comprehensive understanding.

Understanding the concept of limits, which is a fundamental idea in Differential Calculus, is one of the main challenges learners' encounters. According to Goldsmith and Mark (2016), pupils often struggle with the concept of limits for the reason that it is abstract and requires a deep understanding of mathematical notation. In addition, learners may not have a solid foundation in topics (for instance Algebra and Trigonometry) which are essential for solving calculus problems (Lopez, 2017).

Learners face the complexity of the mathematical notation used in Differential Calculus as another challenge. According to Lerman (2017), learners may find it difficult to understand the notation and symbols used in calculus textbooks, which also hinders their ability to solve problems effectively. Additionally, students might encounter challenges when attempting to apply calculus concepts to real-world scenarios, often stemming from difficulties in visualizing the problems or comprehending their practical relevance (Goldsmith & Mark, 2016).

Moreover, learners may also struggle with the problem-solving process in Differential Calculus. According to Stacey (2018), learners may face difficulty in applying the correct calculus procedures to solve problems, as they may not be able to recognize the relevant concepts or apply them appropriately. In addition, learners may struggle with the interpretation of the results obtained from solving calculus problems, which can hinder their ability to apply the solutions effectively (Lopez, 2017).

Furthermore, learners may also experience difficulties in understanding the interrelatedness of different calculus concepts. Learners often view calculus concepts as separate and disconnected, which can incumber their ability to see the bigger picture and apply the concepts correctly (Bressoud, 2017),. This could result in a diminished sense of motivation among learners, as they might fail to recognize the practical relevance of calculus concepts in their future academic pursuits or professional endeavors.

Learners may also face difficulties in understanding the application of calculus concepts in different fields of study. For instance, learners in engineering or physics may struggle with the application of calculus concepts in their respective fields due to the complex nature of the problems they encounter (Garnier & Taylor, 2019). Learners in economics or finance may also find it challenging to apply calculus concepts due to the abstract nature of the problems and the need to interpret results in real-world contexts (Mankiw, 2014).

Furthermore, learners may struggle with the abstract nature of Differential Calculus concepts, such as derivatives and integrals. According to Selden and Selden (2017), learners may find it challenging to understand the meaning of derivatives and integrals, as they may not have a clear understanding of the graphical representation of these concepts. This may result in an inadequate grasp of the core concepts in Differential Calculus, hampering their capacity to proficiently tackle problem-solving tasks.

Another common challenge that learners face in solving Differential Calculus problems is the use of mathematical software and technology. Through their study, Hassani and Shad (2019), found that learners may find it difficult to use software to solve calculus problems effectively. This could be because learners may not have a clear understanding of the mathematical algorithms used by the software, which can hinder their ability to identify and correct errors.

Additionally, learners may struggle with the use of technology in calculus instruction, such as online resources and interactive media. According to Alkhateeb and Al-Okaily (2018), learners may find it difficult to engage with online resources and interactive media, as they may not be able to navigate the materials effectively or understand the instructional design. This may result in diminished motivation and engagement, consequently impeding learners' capacity to effectively grasp new knowledge.

Just as other countries, Differential Calculus is part of the Namibian Advanced Subsidiary [AS] level (Grade 12) syllabus and is aimed to prepare learners for tertiary courses. In the Namibian curriculum, Differential Calculus is envisioned to prepare learners to enroll for university courses in Engineering (used in studies of machinery and electric lighting), Physics (optics and thermodynamics), Chemistry, Economics (used in finding maximum profit or minimum cost), and various other courses. The Grade 12 learners' academic achievement in Mathematics has recorded unimpressive results over the years (Ilukena, et.al, 2016; Maile & Makofane, 2019; Mabena et al., 2021). Among the topics which recorded low performance in the Grade 12 syllabus is Differential Calculus (Directorate of National Examinations and Assessment [DNEA], 2019; 2020; 2021; 2022). According to DNEA (2021; 2022), very few learners attempted to answer questions involving Differential Calculus and on those that attempted to answer these questions, only a few managed to score marks. Differential Calculus was also noted to be posing difficulties for South African learners as well as most learners could not solve problems based on first principles and rules of differentiation as well as applications of optimization (Makgakga & Makwakwa, 2016).

Multiple studies have investigated the underlying factors responsible for low academic performance in Mathematics and possible solutions to mitigate such factors (Hamukwaya, 2021; Makondo & Makondo, 2020; Chand et al., 2021; Kafetwa & Mbetwa, 2016). Scholars have looked at difficulties experienced by learners in mathematical topics such as Algebra (Sikukumwa, 2017; Mukelabai 2018) and challenges contributing to learners' low academic achievement (Zoubi & Younes, 2015; Mabena & Mokgosi, 2021). An improvement in Mathematics results can be achieved if the factors that cause low academic performance in certain topics can be identified and proper intervention strategies implemented (Dlamini, 2017). The present study therefore seeks to explore difficulties experienced by Grade 12 learners when solving Differential Calculus problems. This pursuit is justifiable as Differential Calculus is a topic that has been exclusively part of the higher-level Mathematics curriculum in Namibia and is still part of the new AS syllabus.

1.2. Statement of the Problem

In Namibia, a persistent trend of inadequate academic performance in Differential Calculus has been substantiated by recurring comments within examiner's reports spanning multiple years (DNEA, 2020, 2021, 2022). These reports consistently highlight shortcomings in the understanding and application of Differential Calculus concepts, reflected in the students' responses categorized as 'poorly answered,' 'moderately answered,' or 'extremely poorly answered' in the 2022 Differential Calculus examination.

While existing research has extensively investigated challenges faced by learners in mastering subjects such as algebra (Sikukumwa, 2017; Kaufilua, 2019; Sezuni, 2022), geometry (Ugulu, 2019; Kanandjebo and Ngololo, 2017), and Probability (Abisai, 2018), the prevailing issue of consistently low performance in Differential Calculus across consecutive years has spurred the need for a dedicated investigation. This study aims to delve into the specific difficulties encountered by grade 12 learners when tackling Differential Calculus problems at a chosen secondary school in Tsumeb. The study seeks to identify and analyze persistent challenges experienced by learners in order to provide insights and suggest potential interventions that can aid in enhancing the learning experience and performance outcomes in Differential Calculus for learners in similar educational settings.

1.3. Research Questions

The study was guided by the following questions:

1. What difficulties do AS Mathematics learners encounter in solving problems on Differential Calculus?

2. How do the difficulties encountered by Mathematics AS learners affect performance?

3. How do Grade 12 Mathematics learners understand concepts in Differential Calculus?

1.4. Significance of the study

This research could add to the existing body of knowledge concerning the challenges learners face in understanding Differential Calculus. By identifying these challenges, educators can develop alternative teaching methods for Differential Calculus. Implementing novel approaches might expose learners to more effective ways of grasping concepts, potentially aiding them in solving Differential Calculus problems. Consequently, an improved comprehension of these concepts could boost learners' academic performance.

1.5. Limitations of the study

This research study was conducted at a selected secondary school in Tsumeb which offers Mathematics at Grade 12 level; therefore, the study does not generalize the study findings to schools outside Tsumeb. However, other schools outside Tsumeb may learn from this study on difficulties encountered by learners while solving Differential Calculus problems. The researcher's presence in the classrooms during observations may have had an influence on teachers' usual teaching strategies. Due to limited literature on Differential Calculus in the Namibian context, the discussions were mostly aligned to studies on Differential Calculus in different contexts.

1.6. Delimitation of the study

The study focused only on Grade 12 Mathematics learners and teachers at a selected school in Tsumeb.

1.7. Definitions of key terms

Differential Calculus (or differentiation) - elucidated by Dlamini (2017), stands as a specialized realm within Mathematics devoted to exploring derivatives, encompassing their determination, properties, and practical applications.

Difficulties: - can be defined as failure to attain learning objectives, marked by an inability to reach a minimum proficiency level, insufficient performance, unmet developmental tasks, and a lack of the necessary mastery level (Fatimah, 2019). For this study, difficulties are challenges which learners encounter when solving Differential Calculus.

Performance: - refers to the attainment of predetermined objectives and goals within the realm of basic Mathematics in academic pursuits (Isack, 2015). Regarding this study, performance is the extent to which learners can answer well and solve Differential Calculus problems.

CHAPTER TWO: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1. Introduction

This chapter consists of the conceptual framework which guides the study, as well as the review of some related literature. This chapter covers overall sources that have a link to the problem being explored, which helps to give more sight of the connection between the existing knowledge and the research problem being investigated.

2.2. Theoretical Framework

Learners must attain not only mathematical knowledge and skills but also the confidence necessary to effectively apply the mathematics they have acquired (Kilpatrick et al., 2001). This acquisition of knowledge and skills occurs through diverse classroom teaching approaches, enabling learners to achieve proficiency in Mathematics (Makgakga & Makwakwa, 2016). This study was guided by Pragmatic theory and Kilpatrick et al. (2001)'s Mathematical proficiency.

Pragmatic theory suggests that the difficulties learners face in solving Differential Calculus problems can be attributed to their lack of familiarity with the language and symbols used in this domain (Lakoff & Núñez, 2000). Conferring to this theory, learners may struggle to understand the abstract concepts and symbolic notations used in Differential Calculus because they have not yet developed the necessary conceptual framework to make sense of them.

Furthermore, pragmatic theory posits that learners must actively engage with the language and symbols of Mathematics in order to develop the conceptual framework, which means that learners need to practice working with Mathematical (i.e Differential

Calculus problems and become familiar with the language and symbols used in this domain (Lakoff & Núñez, 2000).

The pragmatic theory empowers researchers to prioritize research problems and leverage diverse approaches to comprehend these issues, rather than fixating solely on methodologies (Rossman & Wilson, 1985). Employing the pragmatic theory facilitated the identification and investigation of challenges encountered by learners when tackling Differential Calculus within this study. These challenges encompass difficulties in adhering to procedures. Consequently, the methodologies rooted in pragmatic theory were substantiated by Kilpatrick et al. (2001)'s five pillars of mathematical proficiency, which encompass: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. Although the study is based on two different theoretical perspectives, Kilpatrick's theory underpinned this study more.

Additionally, pragmatic theory suggests that the difficulties learners face in solving Differential Calculus problems may also be related to their prior knowledge and experiences (Lackoff & Núñez, 2000). For example, their study showed that learners who have had limited exposure to mathematical concepts and symbolic notations may struggle more with Differential Calculus than those who have had more extensive mathematical training (Lakoff & Núñez, 2000).

Moreover, pragmatic theory emphasizes the importance of context and relevance in learning (Lakoff & Núñez, 2000). In their study, Lakoff and Núñez claim that learners may be more motivated to learn Differential Calculus if they can see its relevance to real-world problems and applications. By providing meaningful contexts for learning, teachers can help learners develop a deeper understanding of Differential Calculus and its applications (Lakoff & Núñez, 2000).

Although this study originally aimed to employ Kilpatrick et al.'s mathematical proficiency framework, it deliberately chose to emphasize conceptual understanding, procedural fluency, and strategic competence as the foundational pillars. These components serve as the basis for discussing the amalgamation of knowledge, skills, abilities, and beliefs that collectively constitute mathematical proficiency (Makgakga & Makwakwa, 2016). This study is therefore based on learners' conceptual understanding, procedural fluency and strategic competence to weigh learners' knowledge, skills and abilities to solve Differential Calculus problems.

Conceptual understanding denotes an amalgamated and functional grasp of mathematical concepts and principles (Kilpatrick et al., 2001). Procedural fluency encompasses the adaptability, precision, and efficiency in executing appropriate Mathematical procedures (Kilpatrick et al., 2001; Awofala, 2017). Learners' concepts, principles, efficiencies, precisions and adaptabilities to execute Differential Calculus problems are a guidance through Procedural fluency.

Grade 12 Mathematics learners should know the procedures of how to use different rules of differentiation as $\frac{dy}{dx}$, to differentiate functions. The difficulty might not solely lie in the methods employed by learners, but rather in the necessity for a harmonious integration of higher-order thinking abilities, conceptual understanding, and procedural knowledge to effectively complement each other (Makwakwa & Makgakga, 2014). Moreover, this research employs strategic competence as a key factor that reinforces both conceptual comprehension and procedural fluency. As per Kilpatrick et al.' strategic competence refers to the adeptness in formulating, representing, and resolving mathematical problems. Grade 12 Mathematics learners may need to know how to apply rules and formulas of Differential Calculus to solve differential functions. For example, learners may need to know how to differentiate a function using the first principles: $\frac{dy}{dx}$ $or\ f'(x)\ or\ y' =$ $\frac{f(x+h)-f(x)}{h}$ and rules of differential functions.

2.3. Difficulties in solving Differential Calculus

Multiple studies examining the underperformance of learners in Mathematics have pinpointed several critical factors. These include language application in teaching and learning, available educational resources, time constraints, financial limitations, deficient curriculum delivery, and teachers' inadequate grasp of the subject (Makofane & Maile, 2019; Mateya et al., 2016; Naidoo, 2007; Makgkga & Makwakwa, 2016). However, a specific investigation into the causes of poor performance among Grade 12 learners in calculus tasks revealed a distinct factor: their struggles stemmed from shortcomings in other fundamental Mathematics topics, such as algebra, functions, and inequalities. For instance, students would often fail to factorize a derivative or solve equations for 'x' (Dlamini, 2017). Similarly, Dlamini's research corroborate findings by Muzakwa and Shifamba (2012), whose study on undergraduate students identified that difficulties encountered in calculus predominantly arose from deficiencies in basic algebra knowledge. This study is therefore informed through literature that basic algebraic knowledge may be one of the basic tools to solving Differential Calculus problems.

Shikongo and Ndalipondja (2016) delved into exploring the challenges learners face while solving problems in Differential Calculus. Employing a qualitative research methodology, they gathered data via interviews and classroom observations. Their findings revealed that learners struggled notably with grasping concepts related to limits, derivatives, and integration. Consequently, the study suggested that educators incorporate diverse representations and real-world instances into their teaching methods to aid learners' comprehension of these intricate concepts (Shikongo & Ndalipondja, 2016). In accordance with researchers, this could be due to the fact that the learners' performance depends on a combination of factors.

Conferring to their study, Kong et al. (2020) learners often struggle with key concepts in Differential Calculus such as limits, derivatives, and integration, as well as with their application to real-world problems. In addition to the difficulties with key concepts, the study by Kong et al. (2020) also identified other challenges faced by learners in Differential Calculus. This includes the complexity and abstractness of the subject matter, difficulty in understanding and interpreting mathematical notations and symbols, and inadequate preparation in precalculus topics such as algebra and trigonometry. The study further identified some effective strategies for improving learners' understanding and performance in Differential Calculus. These effective strategies encompass elucidating concepts through clear explanations and relatable examples, employing real-life applications to inspire learning, fostering active involvement and engagement in the learning journey, and offering ample opportunities for practice and feedback.

Similarly, learners may face difficulties understanding the relationship between derivatives and graphs. all et al., (1980), identified that earners often struggle to connect the geometric properties of a curve with its algebraic representation. This complexity can pose challenges for learners in understanding the outcomes of their computations or in picturing how a function behaves (Tall et al., 1980). The study revealed that there could be challenges in learners understanding the relationship between Differential Calculus and graphs. Moreover, learners may also have difficulty with the application of differentiation in real-world problems. Yeo and Yeo (2017) found that students often have difficulty identifying the appropriate mathematical model to use in solving real-world problems, as well as difficulty with the interpretation of the results.

In addition, learners may face difficulties with the notation and terminology used in Differential Calculus. For example, students may struggle with understanding the meaning of symbols such as *dx* and *dy*, or the difference between a derivative and a differential. According to a study by Stewart, Morselli, and Alcock (2017), learners may also have difficulty with the use of formal mathematical language, such as the use of "let" statements and the use of symbols to represent variables.

In a study conducted by Ozdemir and Akbulut (2014), the challenges faced by high school students in grasping the intricacies of derivatives in Differential Calculus were examined. The research highlighted significant hurdles encountered by students, notably in comprehending the concept of instantaneous rate of change, a fundamental aspect of Differential Calculus. Additionally, the study revealed students' struggles with manipulating derivative rules algebraically and applying these rules effectively to problem-solving scenarios. Misconceptions among students regarding the derivative concept were identified, such as erroneously equating it to the slope of a tangent line or conflating it with the slope of a secant line.

Furthermore, the research shed light on a pervasive lack of conceptual understanding regarding the relationship between the derivative and the original function, resulting in errors when solving calculus problems. To mitigate these challenges, the study recommended employing visual aids like graphs and diagrams to augment conceptual understanding, integrating real-world examples to stimulate learning, and implementing interactive activities to actively engage students in the learning process.

In Kaur and Syed's (2015) investigation into challenges encountered by students regarding the comprehension of limits in Differential Calculus, several key difficulties emerged. Students notably struggled with grasping the notion of approaching a value, comprehending infinity, understanding continuity, and establishing the connection between limits and continuity. Additionally, applying limits to real-world scenarios posed a hurdle due to a lack of comprehension. To mitigate these challenges, the study recommended employing interactive pedagogical approaches like group discussions and problem-solving activities to actively involve students in the learning process. Moreover, the use of diverse representations, such as graphical depictions and diagrams, was suggested to augment students' comprehension of these intricate concepts.

Grade 12 learners encounter a significant hurdle in mastering Differential Calculus, primarily centered around comprehending the concept of limits. Kiziltepe and Ozdemir (2017) emphasize the pivotal role of understanding limits in grasping the core principles of Differential Calculus. However, the abstract and intricate nature of this concept often poses a challenge for learners, impeding their ability to effectively tackle Differential Calculus problems.

Additionally, Turan and Karatas (2016) identified another obstacle faced by these learners: a lack of adeptness in problem-solving. Their research revealed that the difficulty in solving Differential Calculus problems stems from deficient problem-solving skills among grade 12 students. The study establishes a correlation between problem-solving abilities and the successful application of Differential Calculus concepts and formulas. Consequently, educators should prioritize enhancing students' problem-solving skills within the framework of their instruction in Differential

Calculus. In essence, addressing both the comprehension of limits and the development of problem-solving skills is crucial in enabling Grade 12 learners to overcome challenges and excel in Differential Calculus.

Additionally, AS learners encounter difficulties in grasping the correlation between the graphical portrayal of a function and its derivative. Çevik and Çetin (2019) noted that learners struggle in associating the slope of a tangent line to a curve with the derivative's value at a specific point. This challenge poses a barrier to effectively applying Differential Calculus in real-world scenarios, particularly when analyzing graphically presented data. Furthermore, learners confront obstacles in comprehending the fundamental concept of derivatives, which is pivotal in Differential Calculus. Learners often struggle to understand the meaning of derivatives and the process of finding them (Almukhaizim and Almukhaizim, 2018). This difficulty can be attributed to the abstract nature of the concept, which requires learners to think critically and abstractly.

Another challenge that grades 12 AS learners face when solving Differential Calculus problems is applying the concepts and formulas they have learned to problems in real-world contexts. Learners often struggle with the application of concepts and formulas of Differential Calculus to real-world problems, as they find it challenging to translate the problems into mathematical equations (Ammar et al., 2019). This difficulty highlights the need for educators to provide learners with real-world examples and practice problems that require the application of Differential Calculus concepts. Furthermore, learners also encounter difficulties in understanding the relationship between Differential Calculus and other branches of Mathematics, such as algebra and geometry. Ammar et al., further stated that learners find it challenging to make connections between Differential Calculus and other branches of Mathematics, which can hinder their ability to solve Differential Calculus problems. This challenge emphasizes the need for educators to integrate Differential Calculus instruction with other branches of Mathematics to help learners see the connections between them.

Learners also face difficulties in understanding the concept of continuity, which is another fundamental concept in Differential Calculus. Learners find it difficult to understand the concept of continuity, which makes it challenging for them to solve problems that require the application of this concept (Ezeilo, 2018). This difficulty can be attributed to the abstract nature of the concept and the fact that it requires learners to think critically and analytically.

Furthermore, learners encounter difficulties in understanding the concept of differentiability, which is closely related to the concept of derivatives. Learners often struggle to understand the relationship between differentiability and the existence of derivatives, which can hinder their ability to solve Differential Calculus problems (Ogunleye and Adeniyi, 2017). This difficulty highlights the need for educators to provide learners with clear explanations and examples of the concept of differentiability.

Moreover, learners face challenges in understanding optimization, which is an important application of Differential Calculus in real-world problems (Ogunleye & Adeniyi, 2017). Additionally, learners often find it challenging to apply Differential Calculus concepts to optimization problems, as they struggle to translate the problems into mathematical equations (Ogunleye & Adeniyi, 2017). This difficulty can be addressed by providing learners with real-world examples and practice problems that require the application of optimization concepts (Adesoji & Akinbobola, 2016).

Shilongo and Lubben (2012) reports that learners in Namibia struggle with translating the basic concepts into rigorous mathematical forms of Differential Calculus, such as limits, derivatives, and integrals. Shilongo and Lubbenlso revealed that learners face difficulties in applying these concepts to solve problems. These difficulties are attributed to factors such as a lack of motivation, inadequate teaching materials, and insufficient time devoted to the subject (Shikongo & Lubben, 2012).

A study by Hangula and Lubben (2013) also revealed that learners in Namibia have difficulty in visualizing geometric shapes and graphs, which is an essential skill in solving Differential Calculus problems.

Another study by Tjikuzu and Lubben (2014) found that learners in Namibia faced difficulties in differentiating and integrating various functions, as well as applying the concepts of Differential Calculus to real-world problems.

Tjikuzu and Lubben (2014) concluded that a lack of conceptual understanding and inadequate teacher training were the main reasons for these difficulties, however Hangula and Lubben recommended using digital devices and software in teaching Differential Calculus to enhance learners' visualization skills.

Additionally, studies have also been conducted in other African countries and other parts of the world, which shed light on the difficulties experienced by learners in solving Differential Calculus problems. For example, a study by Oyelami and Adesope (2018) investigated the difficulties that learners encounter in learning calculus in Nigeria. The study revealed that learners encountered challenges comprehending the principles of limits, derivatives, and integration, mirroring previous findings from Namibia. It suggested that educators employ diverse teaching strategies and incorporate real-world examples to facilitate a better grasp of these concepts among students.

In one study, Nkambule and Dlamini (2019) investigated the hurdles learners faced when grappling with calculus in Eswatini. They identified struggles in comprehending fundamental concepts like limits, derivatives, and integration. To address these issues, Nkambule and Dlamini proposed employing diverse teaching methods, incorporating real-world illustrations, and implementing problem-based learning to aid comprehension.

In a separate study conducted by Nyaumwe et al. (2020) focusing on learners in Zimbabwe, challenges emerged in both understanding Differential Calculus concepts and applying them to problem-solving. The study suggested prioritizing the development of a strong conceptual foundation in learners before delving into problem-solving techniques.

Similarly, Singh and Naidu (2018) explored the obstacles encountered by learners in Fiji while tackling Differential Calculus problems. Their findings highlighted difficulties in grasping and applying rules like the chain rule, product rule, and quotient rule. To mitigate these challenges, the study recommended the utilization of various pedagogical strategies, including visual aids and real-life examples, to bolster learners' comprehension of these concepts.

In a study conducted by Chirikure et al. (2019), the focus was on examining the challenges faced by Zimbabwean learners when tackling optimization problems in Differential Calculus. The research revealed notable hurdles encountered by students, particularly in correctly identifying the objective function and constraints.

Additionally, applying differentiation to determine maximum or minimum values posed significant difficulties for these learners. Chirikure et al. (2019) suggested that educators enhance students' proficiency by incorporating increased practice sessions focused on optimization problems, coupled with comprehensive feedback to support their learning process.

Learners experienced difficulties in understanding the fundamental concepts of Differential Calculus, such as limits and derivatives (Adakhovskaya et al., 2017). Learners struggle with the application of Differential Calculus to real-world problems. Adakhoyskaya et al (2017) suggested that teachers should use a variety of teaching methods, including the use of technology, to enhance learners' understanding of Differential Calculus.

Similarly, Al-Rawashdeh and Al-Shdifat (2015) which examined the challenges that learners in Jordan face in solving Differential Calculus problems, found that learners had difficulty understanding the concepts of limits, derivatives, and integrals. The study also revealed that learners struggled with the application of Differential Calculus to real-life situations. The study recommended that teachers should provide learners with ample opportunities to practice solving Differential Calculus problems and offer feedback on their work.

A study by Heidari and Aziminezhad (2013) which investigated the difficulties that learners in Iran face in solving Differential Calculus problems, also found that learners had difficulty understanding the concepts of differentiation and integration, as well as applying these concepts to solve problems. Heidari and Aziminezhad also revealed that learners had difficulty in visualizing geometric shapes and graphs, which is an essential skill in solving Differential Calculus problems. The study recommended that teachers should use appropriate teaching materials, such as computer software, to enhance learners' visualization skills.

In a study conducted in China, Li and Chen (2018) found that learners often have difficulties in understanding the concept of limit and its relationship to differentiation. The study also identified a lack of visualization skills as a major obstacle to solving Differential Calculus problems. Similarly, a study by Alkhateeb and Al-Momani (2019) in Jordan revealed that learners do come to calculus problems with algebraic deficiencies

Another study conducted in Turkey, Kocakülah and Özdaş (2017) identified common misunderstandings concerning derivative, limit, and continuity among Turkish learners. They observed challenges in the application of differentiation rules to problem-solving. Similarly, Ncama and Naidoo (2017) in South Africa found learners struggling with graph interpretation and applying differentiation in real-world scenarios due to inadequate prerequisite knowledge and weak algebraic skills. In India, Sivaramakrishnan and Venkatesan (2018) pinpointed a lack of comprehension in fundamental calculus concepts like limits, derivatives, and integrals as a primary hurdle for learners tackling Differential Calculus problems. From these studies it can be seen that there are various factors which contribute to difficulties of learners solving Differential Calculus.

Several studies have explored learners' difficulties in Differential Calculus in the context of specific instructional approaches or learning environments. In a similar study conducted in Iran, Aghanikar et al. (2019) investigated how an inquiry-based learning method impacted learners' grasp of Differential Calculus. The research revealed that participants engaging in inquiry-based learning activities demonstrated notable advancements in comprehending the subject's concepts and exhibited enhanced problem-solving abilities, especially with more intricate problems.

Another study by Lee and Kim (2016) in Korea focused on learners' difficulties in understanding the usage of graphical representations in solving problems in Differential Calculus. The researcher discovered that many learners faced challenges in understanding the significance of graph shapes and slopes, thereby impeding their capacity to effectively utilize differentiation in problem-solving. While, in the United States, a study by Kwon et al. (2018) investigated the impact of integrating technology, specifically graphing calculators, on learners' understanding and performance in Differential Calculus. The study found that learners who used the graphing calculators exhibited deeper conceptual understanding and were able to solve more complex problems (Kwon et al., 2018). These studies show that learners solve Differential Calculus problems better with integrating technology such as graphing calculators.

In China, Li et al. (2019) delved into the significance of metacognition in the context of learners' problem-solving abilities in Differential Calculus. They found that learners who exhibited higher levels of metacognitive awareness were more successful in solving complex problems and were better able to identify and correct their own errors.

Collectively, these studies indicate a shared set of challenges encountered by learners across different countries when tackling Differential Calculus problems (Shikongo & Ndalipondja, 2016; Kong et al, 2020; Yeo & Yeo, 2017; Ozdemir & Akbulut, 2014; Kaur & Syed’s, 2015; Cevik & Cetin, 2019; Ammar et al., 2019). These challenges notably encompass a fundamental grasp of core concepts, deficiencies in algebraic proficiency, and struggles in visualizing and interpreting graphical representations. These findings underscore the need for targeted instructional approaches and interventions to support learners' development of Differential Calculus skills.

Literature also suggests that instructional approaches, learning environments, and metacognitive awareness can all play important roles in supporting learners' development of Differential Calculus skills. Studies further highlight the importance of designing effective instructional strategies that consider learners' individual needs, backgrounds, and learning preferences.

2.3.1. Teaching approaches

Scholars argue that learners' academic achievements in calculus appear to have been affected by teaching approaches (Ganzela, 2015; Makgakga & Makwakwa, 2016; Dlamini, 2017). Generally, a study on factors influencing inadequate performance among learners in Mathematics proposes key strategies for improvement. The study underscores the critical need for ongoing enhancement of teachers' skills in constructivist teaching methods, along with continuous education for school administrators and parents in supporting Mathematics education (Mabena, et al., 2021). Emphasizing the consistent prioritization and reinforcement of these measures is crucial. Additionally, the study advocates for fostering collaboration among Mathematics researchers on a global scale, highlighting the importance of government funding to advance and enhance the teaching and learning of Mathematics.

Traditional Teaching Methods: Traditional teaching methods, which emphasize rote memorization and repetition of mathematical procedures, have been criticized for not promoting deeper understanding of mathematical concepts (Van de Walle & Lovin, 2011). Thus, this type of approach may not adequately prepare learners to solve complex problems in Differential Calculus.

*Lack of Conceptual Understanding**:*** Many learners struggle with Differential Calculus because they lack a strong conceptual understanding of the underlying mathematical principles. This can be attributed to a lack of emphasis on conceptual understanding in traditional teaching methods (Hiebert & Lefevre, 1986). A lack of conceptual understanding seems to have an impact on learners solving Differential Calculus problems.

Insufficient Preparation: Learners may not have sufficient preparation in prerequisite mathematical concepts that are necessary for understanding Differential Calculus. This can include algebraic concepts, trigonometry, and geometry (National Council of Teachers of Mathematics, 2014).

Overemphasis on Symbol Manipulation: Learners may be taught to focus on symbol manipulation rather than understanding the underlying mathematical concepts. This approach can lead to an apparent understanding of Differential Calculus and exertion in applying it to real-world problems (Sfard, 1991).

Inadequate Use of Technology: The use of technology, such as computer-based simulations, can help learners develop a deeper understanding of mathematical concepts. However, many classrooms may not have access to the usage of technological tools or teachers may not be adequately trained to use these technological tools effectively (Huang & Hsieh, 2018).

Overall, it is significant for educators to adopt teaching approaches that emphasize conceptual understanding, build on prerequisite knowledge, and incorporate technology to support learners' understanding of Differential Calculus.

2.3.2. Errors, misconceptions and incompetence Prerequisite algebraic topics to Differential Calculus

In a study focusing on learners' difficulties in finding the derivative of functions using first principles and rules of differentiation, Makgakga and Makwakwa, (2016) revealed that learners performed more poorly in finding the derivatives using first principles than using the rules of differentiation. Likewise, Fatimah (2019) discussed that students also experience difficulties with creating and analyzing elements in the graph of the function, the graphical relationship functions with limit. Fatimah further conferred that the students who are less able to learn independently still use the memorization method and tend to be less careful when they answer questions. In their study, they also determined that teachers' teaching approaches and learners' lack of procedural and conceptual knowledge could possibly be the cause of Mathematics learners' poor performance in calculus. This is also concurred in Dlamini (2017), who discovered that learners perform poorly in calculus due to incompetence in other Mathematics topics such as algebra, functions and inequalities. Both Dlamini (2017); Makgakga and Makwakwa (2016)'s studies revealed a lack of learners' mathematical proficiency strands.

a) ***Errors:*** Learners may make errors in their calculations, which can lead to incorrect solutions. Common errors include misinterpreting notation, misapplying rules, and making arithmetic errors (Lai & Murray, 2011).

b) ***Misconceptions:*** Misconceptions are beliefs or ideas that are inconsistent with accepted mathematical concepts. Learners may have misconceptions about Differential Calculus, such as believing that derivatives only apply to linear functions or that the derivative of a function is always positive (Tall & Vinner, 1981). Many learners have misconceptions about basic Calculus concepts, such as derivatives, limits, and continuity. For example, learners may confuse the definition of the derivative with the derivative rule and may not understand the concept of a limit as a value that a function approaches but may never reach (Tall & Vinner, 1981; Dubinsky & Harel, 1992)

c) ***Incompetencies***: Learners often face challenges in mastering Differential Calculus due to several factors. One significant hurdle is a deficiency in essential competencies necessary for solving problems in this field. These deficiencies encompass a lack of proficiency in algebraic manipulation, difficulties in employing effective problem-solving strategies, and a lack of confidence in their mathematical abilities (Buchholtz, 2015). Moreover, learners frequently encounter struggles when attempting to apply calculus principles to practical problem-solving scenarios. This struggle can be attributed to either an inadequate grasp of fundamental concepts or insufficient practice in employing effective problem-solving methodologies (Selden & Selden, 2006; Zandieh & Tarmizi, 2011). Furthermore, a notable issue arises from the inability of learners to correctly apply calculus rules, including the Chain rule, product rule, and quotient rule. This struggle often stems from a lack of comprehension of these rules or insufficient practice in their application across various functions (Jones & Lee, 2019; Smith, 2017).

Addressing errors, misconceptions, and incompetence is crucial in helping learners develop a strong foundation in Differential Calculus. Educators can address these issues by providing opportunities for learners to practice problem-solving, identify and correct errors, and engage in meaningful discussions about mathematical concepts (Kilpatricket al., 2001).

2.3.3. Learners' attitude towards Differential Calculus

A study on enhancing students' attitude, conceptual comprehension, and procedural proficiency in Differential Calculus utilizing Microsoft Mathematics revealed notable improvements (Mendezabal & Tindowen, 2018). Mendezabal and Tindowen (2018) concluded that employing Microsoft Mathematics effectively enhances students' conceptual understanding, procedural skills, and overall attitude towards learning

Differential Calculus. Notably, students viewed the use of Microsoft Mathematics as equally beneficial as the traditional instructional methods. Mendezabal and Tindowen referred to a traditional approach as an approach which puts emphasis on computational procedures instead of an understanding of the underlying concepts of Differential Calculus. They further pointed out that Microsoft Mathematics affords learners opportunities to learn calculus concepts and processes by exploration and discovery, allowing them to be more engaged in learning. Certainly, learners' attitude towards learning Differential Calculus is observed through the following:

a) ***Negative Attitudes:*** Negative feelings that learners harbor about Mathematics may result in reduced motivation, a lack of engagement, and increased anxiety (Martin, 2000). These emotions can act as obstacles, impeding their capacity to comprehend and tackle problems in Differential Calculus.

b) ***Low Self-Efficacy:*** Enhancing learners' self-efficacy, denoting their confidence in achieving success in Mathematics, can impact their performance in Differential Calculus. Learners with low self-efficacy may be less likely to persist in problem-solving and may feel a lack of control over their learning (Bandura, 1997).

c) ***Perceived Difficulty***: Learners may perceive Differential Calculus as difficult or too abstract, which can discourage them from engaging in problem-solving and seeking help (Krejcie & Morgan, 2000).

d) ***Lack of Relevance***: Learners may perceive Differential Calculus as irrelevant to their future goals or interests, which can lead to disinterest and a lack of motivation to learn (Hannula & Pehkonen, 2002).

A study suggested that in order to address learners' attitudes towards Differential Calculus, educators can promote a positive learning environment, offer learners with opportunities to succeed and build their self-efficacy, and connect mathematical concepts to real-world applications and learners' interests (Boaler, 2016).

e) ***Insufficient Preparation***

Learners may not have sufficient preparation in prerequisite mathematical concepts that are necessary for understanding Differential Calculus. This can include algebraic concepts, trigonometry, and geometry (National Council of Teachers of Mathematics, 2014).

2.3.4. Lack of motivation

Many learners may lack the motivation to engage with calculus content, which can lead to difficulties in learning and applying calculus concepts. This lack of motivation may be due to a belief that calculus is too difficult or not relevant to their interests or career goals (Hannula & Pehkonen, 2002; Leshem & Markovits, 2013).

As a result, to address learners' attitudes towards Differential Calculus, studies recommended that educators can promote a positive learning environment, create environments that foster learner success and bolster their self-efficacy by integrating real-world applications of mathematical concepts tailored to their individual interests (Boaler, 2016). Researchers found that there was a need for calculus reforms. One of the reforms is to consider reasons why it is important to encourage learners to do calculus and how important it is in real life.

Overall, although items requiring learners to do applications of Differential Calculus are usually high order questions (Dlamini, 2017), they underscore the necessity for an increased emphasis on employing a problem-solving approach in the teaching and learning of Mathematics (Zulu & Nalube, 2020). A study conducted on the analysis of difficulties faced by learning of calculus revealed incompetence in other topics of

Mathematics such as algebra and functions to be contributing to learners’ poor performance in Differential Calculus (Dlamini, 2017; Zulu & Nalube, 2020).

2.4. Chapter summary

The cited literature draws closer to the idea that learners in various African countries, including Namibia, Zimbabwe, and Fiji, face challenges in solving Differential Calculus problems. These challenges include a lack of conceptual understanding, difficulty in applying concepts to solve problems, and struggles with specific techniques such as the chain rule and optimization. Furthermore, learners in Namibia also face several difficulties in solving Differential Calculus problems. These difficulties are attributed to factors such as a lack of motivation, inadequate teaching materials, insufficient time devoted to the subject, a lack of conceptual understanding, inadequate teacher training, and poor visualization skills.

Educators need to be aware of these challenges and develop instructional strategies that address them to improve learners' understanding and performance in Differential Calculus.

Differential Calculus presents a range of difficulties to learners, from conceptual to procedural to interpretive challenges. Addressing these difficulties requires targeted instruction and practice that focuses on building a deep understanding of underlying concepts, as well as developing skills in problem-solving, interpretation, and communication

Although literature reviewed highlighted factors that cause poor performance in Mathematics generally and difficulties in solving Differential Calculus specifically, it can be concluded that there may still be varieties of factors and strategies that need to be discovered in order to help improve and mitigate these factors. Additionally, it is imperative to uncover difficulties experienced by Grade 12 learners while solving Differential Calculus at a secondary school in Tsumeb. The next chapter delves into the methodology employed for this study.

CHAPTER THREE: METHODOLOGY

3.1. Introduction

This Chapter describes the procedures for conducting the research study. Through the research design and research approach, the researcher indicated how this research study was set up and all the methods of data collections that were used, including the research samples and sampling procedures, research instruments, population, data collection procedures, data analysis and ethical issues of the study.

3.2. Research design

This research study is based on a mixed methods exploratory approach with the aim of exploring Grade 12 learners' difficulties in solving Differential Calculus. Both Quantitative and Qualitative methods were considered for this study. The Quantitative aspect is however, limited as compared to Qualitative aspect. Qualitative data was first collected from classroom observations and semi-structured interviews in order to allow the researcher to explore learners' difficulties in solving Differential Calculus as well as observing teachers to share their experiences in teaching Differential Calculus. Thereafter, a follow up through quantitative data with diagnostic tasks for the learners was then conducted, which allowed the researcher to use numerical and descriptive statistics in exploring and analyzing learners' scores in depth from solving Differential Calculus questions.

3.3. Population

The population of this study consisted of all Grade 12 Mathematics learners (74 learners) and three Grade 12 Mathematics teachers at the selected secondary school in Tsumeb. One school was purposely and conveniently selected as it has been offering Mathematics on higher level for the past several years with teachers considered to have the experience needed in providing data for this research study. The researcher's proximity to the school also allowed the study to be conducted utilizing the available limited resources.

3.4. Sample and Sampling procedures

The sample was used to generalize information from the population in the context of the study. Bhardwaj expounded on sampling as a systematic methodology aimed at selecting a subset of individuals from a vast population for specific research objectives.

A total population sampling method at the selected school was used in this study as it included all the 74 Grade 12 Mathematics AS learners and the three Grade 12 Mathematics AS teachers at the selected school. Total population sampling at the selected school was used as the study population was relatively small and leaving out participants with experiences would risk leaving out rich information (Etikan et al., 2016). The researcher believes that considering total sampling technique gives a full prospect to not leave out quality data as the population was relatively small. **Research Instruments**

Data was collected via classroom observations, a diagnostic task on Differential Calculus, and Semi-structured interviews.

3.4.1. *Classroom Observations*

Two lessons per Grade 12 Mathematics teacher were observed. Observations done with the aim to help determine the actual learning and teaching process during the topic of Differential Calculus (Bonnie, 2019). The lesson observation has assisted the researcher to further recognize and determine the level of cognition of grade 12 Mathematics learners to Differential Calculus. An observation sheet was used, and the researcher further jot down incidents as they manifested during the lessons.

3.4.2. Diagnostic Task

Five questions were set for the learners after lesson observations of topics related to Differential Calculus. Questions to the diagnostic task were selected as per the learning objectives from the NSSC(AS) Mathematics syllabus. Learners were expected to complete the diagnostic task in approximately 30 minutes during an agreed suitable time from both the learners and the researcher. Diagnostic tasks were set with the aim of; a) allowing the researcher to determine difficulties that Grade 12 learners may display in their diagnostic task scripts when answering Differential Calculus questions and b) determining how learners understand the concepts in Differential Calculus by answering different questions in the diagnostic task which are calculus related.

3.4.3. Questionnaire (for teachers):

An unstructured questionnaire which consists of open-ended questions was distributed to all AS Mathematics teachers, upon completion of the lesson observations and diagnostic task on Differential Calculus. This questionnaire allows participants to describe their experiences.

3.5. Pilot study

The pilot study was designed as a small-scale preliminary investigation which was conducted prior to the main study to test for feasibility and effectiveness of the research design, procedures and instruments (Pilot & Beck, 2021). Pilot studies are essential to obtain preliminary data, determine sample sizes, and test the feasibility of the study's procedures and instruments (Creswell, 2014).

The pilot study was carried out with 24 grade 12 learners doing AS level Mathematics and one AS Mathematics teacher at another school in Tsumeb which has been offering Mathematics at higher level and still offers AS level Mathematics, just as the school where the main study was carried out. The researcher assumed that the AS level

Mathematics learners and teachers at these two schools had the same characteristics. There was one grade 12 AS level Mathematics class at the school where the pilot study was conducted, and hence only one teacher for the said class. Due to a relatively small population, a total sampling method was used to select all 24 learners and one teacher.

A diagnostic task was administered to the learners after they were taught Differential Calculus as well as an interview was carried out for the teacher. Piloting of the study helped to point out flaws in the instruments, selection of the sample and the data collection procedure.

One of the problems discovered was in the interviews for the teachers, as the researcher feels that the interview limited more details from the teacher due to the limited time and also lack of explanations. Hence the replacement of an interview with questionnaires which would allow unlimited expressions from the teachers. Another problem was observed during the lesson observations as some of the questions were set unnecessarily as open-ended questions in the observation sheet, which made it difficult for the researcher to complete the observation sheet during the observation session and sometimes the researcher even skipped questions during the observation sessions to fill in after the presentation was done. Therefore, some of the questions were then changed to closed ended questions as a selection type of questions during the main data collection.

Although some learners did not answer Question 6 of the diagnostic task, time allocated to the diagnostic task was observed to be enough as most learners completed the task within the stipulated time. No reputation of questions was observed in the learners' diagnostic task; therefore, the diagnostic task was not changed.

3.6. Data collection procedures

The researcher first conducted classroom observations. Each of the three Mathematics teachers was observed twice. After observations, the researcher then administered a diagnostic task for approximately 30 minutes to the Grade 12 learners to determine the difficulties experienced by learners while solving Differential Calculus. The diagnostic tasks were administered in the afternoon to avoid disturbing normal classes. The questionnaires were distributed after the preliminary analysis of the learners' scripts and classroom observations.

3.7. Data analysis

For the qualitative data, the researcher considered an exploratory factor analysis of themes to analyze data. The learners' scripts were analyzed to determine the difficulties learners displayed in solving Differential Calculus. Learners' descriptions of concepts were analyzed to determine how well the learners understand the Differential Calculus concepts. Thereafter, the researcher then coded the transcribed questionnaires and observations data. Codes were organized in categories which generated themes for analysis.

For the quantitative data, scores from the learners' diagnostic task were analyzed using descriptive statistics. The diagnostic task scripts underwent moderation by teachers following the researcher's marking, ensuring the accuracy of the assessment process and memo. The researcher used measures of centrality and dispersion of data to describe the performance of AS level Mathematics learners on Differential Calculus.

3.8. Ethical Considerations

The ethical clearance certificate was issued by the University of Namibia's Decentralized Ethics Committee. The researcher further sought permission from the Ministry of Education, Arts and Culture from the Executive Director.

Information sheet for the participants was prepared in advance. Participants received full information (informed consent) about the purpose and objectives of the study. This has enabled participants to make informed decisions whether to participate in the research or not. Participants were duly notified of their absolute freedom to withdraw from the study at any time they deemed it necessary. The information provided by the participants is treated with the utmost confidentiality and anonymity. Issues of confidentiality and maleficence were well taken care of; therefore, teachers' names did not appear on the interview report sheets or in the final thesis report as well as learners' names did not appear on the diagnostic task sheet.

Consent forms for Mathematics AS learners were provided so that learners take the consent forms to their parents or guardians to sign which allowed learners to participate in the study. As for the teachers, they were provided with an agreement form that they have signed which confirmed their participation in the study. The agreement forms were destroyed as soon as the data collection process was complete in contemplation of protecting participants' identities. The study was not in any way harmful to the participants either physically or psychologically. The researcher promised to remain objective and honest enough throughout the study till the end. The study report was compiled with precision and integrity.

CHAPTER 4: PRESENTATION AND DISCUSSION OF FINDINGS

4.1. Introduction

This chapter unveils the outcomes of the research and engages in a comprehensive discussion thereof. The initial segment delves into the revelations concerning learners' reactions to questions pertaining to Differential Calculus, which constituted a diagnostic task administered to Grade 12 students at a chosen secondary school in Tsumeb, Namibia. Subsequently, the data analysis section dissects learners' responses to all inquiries by employing three of the five strands outlined by Kilpatrick. The subsequent segment expounds upon an in-depth discussion of the study's findings.

4.2. Demographic information

Demographic data on gender for both teachers and learners, as well as the teachers' background in teaching higher-level Mathematics, was collected. A total of 44 AS level Mathematics learners and three AS level Mathematics teachers took part in the study. Among the learners, 18 (41%) were male, and 26 (59%) were female. In terms of teacher demographics, there was 1 (33%) male teacher and 2 (67%) female teachers. Notably, one teacher had over ten years of experience teaching the higher-level Mathematics curriculum, while two teachers lacked prior experience in teaching higher-level Mathematics curriculum.

4.3. Presentation of findings

This study sought to explore difficulties experienced by the Grade 12 learners while solving Differential Calculus problems at a selected secondary school in Tsumeb, Namibia. It administered a diagnostic task to AS level Mathematics learners to determine and identify errors, misconceptions and other difficulties; teachers' questionnaires which helped to explore other difficulties that teachers have experienced from the learners, as well as classroom observations which helped the researcher to observe the teaching-learning approaches used.

4.3.1 Data from classroom observations

The researcher conducted classroom observations during class sessions and several noteworthy practices and dynamics emerged. At the outset of each topic, the objectives were notably emphasized, providing a clear direction and focus for the lessons. Teachers initiated the learning process with concise self-assessment tasks to gauge students' existing knowledge, facilitating a better understanding of their starting point in the subject matter.

A commendable practice observed was the provision of lesson plan books to the researcher before each class, showcasing an organized and structured approach to teaching. However, a mild trend of absenteeism of 7 learners for two consecutive lessons out of 5 lessons was noted, potentially impacting their continuity and engagement in the learning process.

Throughout the sessions, a predominant teacher-centered approach prevailed, where instructors predominantly solved problems on the board instead of fostering more active learner involvement. The researcher observed a lack of Learner-Centered Education (LCE) during lesson observations in all the lessons observed. Nonetheless, learners actively participated in discussions and responded well to teacher-initiated queries, indicating their eagerness to engage despite the instructional style.

Encouragement for learners' inquiries and clarifications was evident as teachers kept encouraging learners to ask questions after every explanation that the teacher gives to the learners. The encouragement created an environment where learners felt comfortable seeking clarification when needed. Homework assignments were consistently provided, with a commendable practice of beginning subsequent lessons with feedback on the previous tasks for both lessons observed, enabling learners to comprehend their errors and improve.

The coverage of all objectives, along with focused emphasis on key elements within lessons, underscored a comprehensive teaching strategy for all the observed teachers. Exposure to various question types from past papers and guidance on tackling them demonstrated a proactive approach to exam preparation.

While group discussions and collaborative tasks were relatively infrequent, instances were noted where teachers facilitated peer learning by allowing students to explain concepts to their classmates, fostering a more interactive learning environment.

Notably, the alignment between the planned lessons and their execution was evident as their lesson presentations observed aligned with the objectives they emphasized from the beginning of their lessons - reflecting the effective implementation of prepared lesson plans. The engagement levels of students when responding to their teachers' questions showcased a positive learning atmosphere, despite the limited use of group-based activities. Overall, while the classes predominantly followed a teacher-centered approach, the efforts to encourage participation, address queries, and prepare students for assessments were evident throughout the observations.

4.3.2 Difficulties encountered by learners on the diagnostic task

The primary focus of the researcher during the analysis of learners' responses on the items of the diagnostic task was to discern and interpret their thought processes and reactions to various questions on Differential Calculus. The researcher documented common errors, misconceptions, and instances where learners exhibited a lack of proficiency in solving specific problems. Subsequently, the researcher delved into understanding the reasons behind these mistakes. The analysis revealed a spectrum of performance among the learners: some demonstrated a thorough understanding of the material, while others exhibited partial comprehension, and a subset appeared to lack understanding altogether. Table 4.1 shows the scores from the diagnostic task, which gives sight on the general performance of the learners on the diagnostic task.

Table 4.1. Results of learners' diagnostic task

Scores (x) (Marks out of 30)	Frequency	Fx	Cumulative Frequency	$(x-\bar{x})$ = (x-10.7)	$(x-\bar{x})^2$
0	1	0	1	-10.7	114.49
1	0	0	1	-9.7	94.09
2	0	0	1	-8.7	75.69
3	2	6	3	-7.7	59.29
4	3	12	6	-6.7	44.89
5	1	5	7	-5.7	32.49
6	3	18	10	-4.7	22.09
7	2	14	12	-3.7	13.69
8	1	8	13	-2.7	7.29
9	4	36	17	-1.7	2.89
10	3	30	20	-0.7	0.49
11	5	55	25	0.3	0.09
12	5	60	30	1.3	1.69
13	5	65	35	2.3	5.29
14	2	28	37	3.3	10.89
15	1	15	38	4.3	18.49
16	0	0	38	5.3	28.09
17	2	34	40	6.3	39.69

18	0	0	40	7.3	53.29
19	1	19	41	8.3	68.89
20	1	20	42	9.3	86.49
21	0	0	42	10.3	106.09
22	0	0	42	11.3	127.69
23	1	23	43	12.3	151.29
24	1	24	44	13.3	176.89
25-30	0	0	44	14.3-19.3	204.49-372.49

Measures of Centrality and dispersion:

a) Mean:

$$\bar{x} = \frac{\sum x}{n} = \frac{(0+0+0+6+12+5+18+14+8+36+30+55+60+65+28+15+0+34+0+19+20+0+0+23+24+0)}{44}$$

≈ 10.7

b) Mode: 11, 12 and 3

c) Median: $= \frac{11+11}{2} = 11$

d) Range $= 24 - 0 = 24$

e) Variance = $\sigma^2 = \frac{\sum f(x-\overline{x})^2}{n-1} = \frac{3053.19}{43} \approx 71.0$

f) Standard Deviation = $\sigma = \sqrt{\frac{\sum f(x-\bar{x})^2}{n-1}} = \sqrt{71.0} \approx 8.43$

As on table 4.1, a total of 24 learners out of 44 (54.5 %) scored below average of 10.7 marks, according to the calculated average mean score. Most learners scored 11, 12 and 13 marks as indicated in the calculation of the mode. The standard deviation shows appears to be less than the average mean mark for the learners' marks which shows that most learners have scored marks around the average marks. This suggests that the learners had difficulties in solving problems in Differential Calculus. The next sub-sections present learners' responses to items on the diagnostic task.

Question 1. Application of the rules of differentiation

This question required learners to apply different rules of differentiation. It was observed from learners' scripts that the majority of learners could not apply the differential rules correctly. Learners' responses to the sub-questions under Question 1 are presented next:

Question 1. a) required learners to *differentiate* $y = 4\sqrt{x} - \sqrt[3]{x}$. First, learners were expected to know when to use differentiation notations such as $\frac{dy}{dx}$, $g'(x)$, $f'(x)$ etc. From the lesson observations carried out, all teachers explained to learners when to use the different differentiation notations. Therefore, learners were expected to know that for this problem, they were supposed to express the differentiation of this function of y with respect to x, i.e $\frac{dy}{dx}$. Learners were also expected to realize (as observed during lesson observations) that any root is the same as a fractional exponent, i.e $y = 4\sqrt{x} - \sqrt[3]{x}$ could be expressed as $y = 4x^{\frac{1}{2}} - x^{\frac{1}{3}}$.

They had to differentiate by multiplying the exponents with the coefficients and reduce the exponent by 1. That is,

$$\frac{dy}{dx} = 4 \times \frac{1}{2} x^{\frac{1}{2}-1} - 1 \times \frac{1}{3} x^{\frac{1}{3}-1}$$

$$= 2x^{-\frac{1}{2}} - \frac{1}{3}x^{-\frac{2}{3}}$$

which is the same as

$$\frac{2}{\sqrt{x}} - \frac{1}{3\sqrt[3]{x^2}}$$

In analyzing learners' scripts, the researcher had to analyze all scripts including those where candidates had earned full marks. Below is an example of a correct response from one of the learners. The learner correctly expressed the given function with fractional exponents and then continued to derive correctly. The learner also denoted the correct derivative notation of the given function.

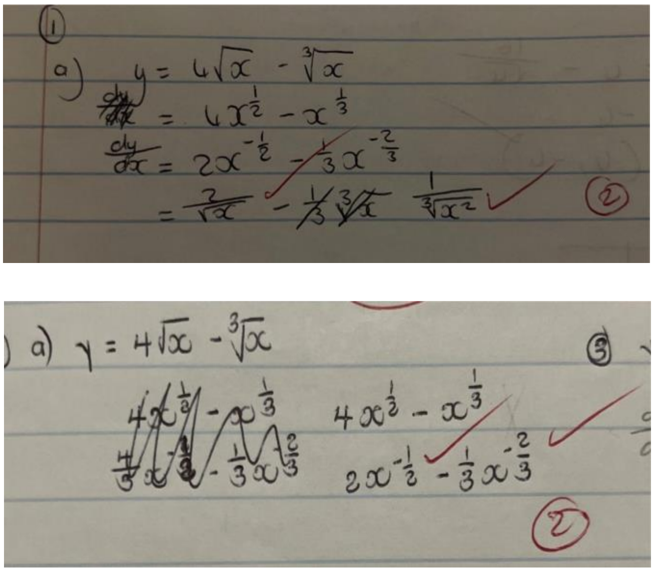


Figure 4.1. Correct responses by Learners

Some learners seemed a bit unsure of what to do and how to differentiate the function, some did not even write the derivative form. They denoted the function as *y'* and also did not multiply the exponent with the coefficient, although the candidate remembered to subtract one from the exponents. This means that these learners somewhat understood how to differentiate, although not fully. Here is an example of the partially incorrectly answered question.

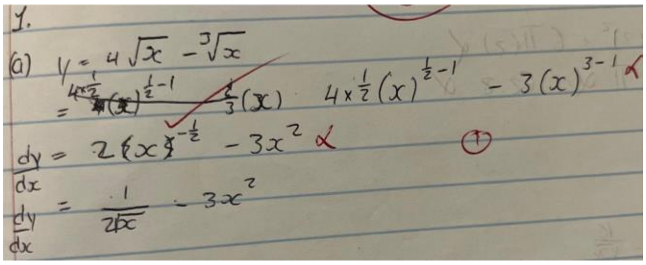


Figure 4.2. Picture of an incorrect response

The learner in Figure 4.3 seemed to have answered correctly but failed to change to a correct final answer with a positive exponent, leading to a learner losing a mark.

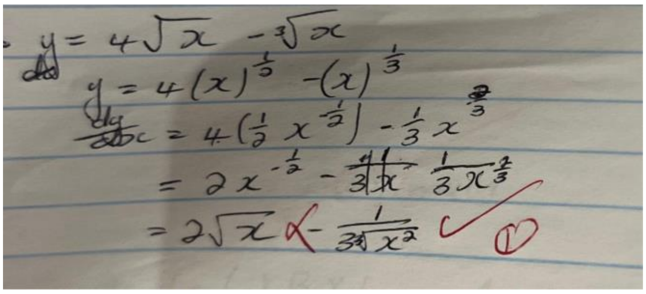


Figure 4.3. Learner who gave incorrect exponent.

Some learners did not have any idea on how to differentiate the function, hence the lack of procedural fluency and conceptual understanding. Figure 4.4 is an example of a wrong response due to an incorrect procedure.

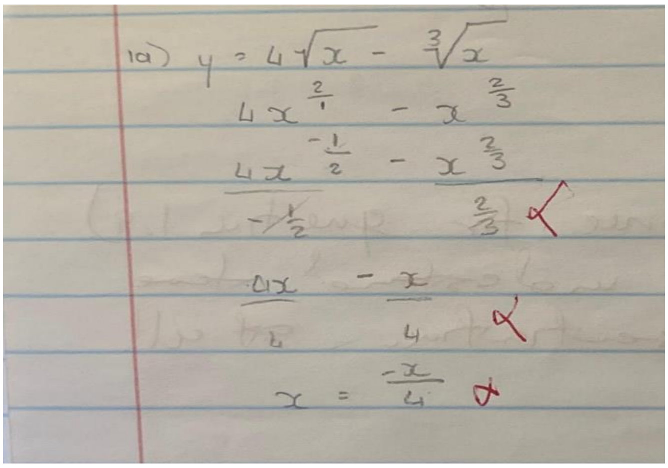


Figure 4.4. Picture of incorrect procedure.

Overall, this question was answered well as the majority of learners obtained full marks as they have answered this question correctly. Thus, 81.8% passed question 1.a), 11.4% obtained 1 mark due to errors and/careless response during their calculations and 6.8% have not obtained any mark for this Question 1. A) as a result of not knowing what to do at all, and turning the function into an equation that they have attempted to answer.

Question 1 b) Differentiate

$$f(x) = 2cos^3x$$

Learners were expected to differentiate the trigonometric function as

$$f'(x) = 2 \times 3cos^2x \times (-sinx)$$
$$= -6cos^2x\ sinx$$

This question was poorly answered as most learners (about 93%) of those that answered this question failed it as most do not seem to understand what the derivative of $cos\ cos\ x$ is. Figure 4.5 shows some answers of learners who failed to get this question correctly.

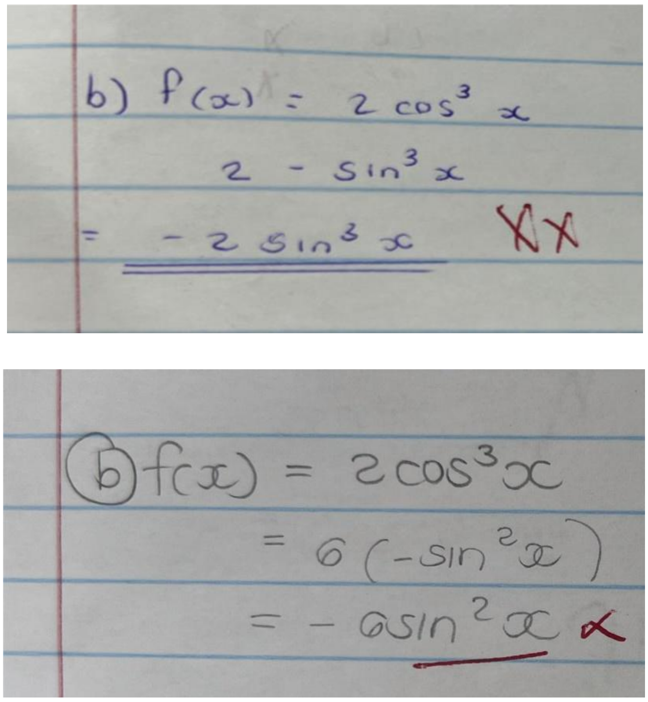


Figure 4.5. Learners who did not differentiate the cosine function correctly.

The learners in Figure 4.5 did not correctly apply the chain rule. Although the learners remembered that

$$(cos\ cos\ x)' = -sinx$$

, they could not remember that they needed to apply the chain rule because of the exponent i.e.

$$2cos^3x$$

.

Some learners answered the question partially correctly as they have made some slip mistakes during their calculations. It also appeared that some learners have no idea on differentiating trigonometric functions.

Question 1. c. Differentiation of a natural log function

$$y = ln\ \frac{1}{\sqrt{4x^2 - 1}}$$

Learners were supposed to remember (as observed from the lessons) that the derivative of a natural logarithm is given by:

$$\frac{d}{dx} ln\, x = \frac{1}{x} . 1$$

$$= \frac{1}{x}$$

Therefore, learners were then expected to derive this function of natural log of y with respect to x as:

$$\frac{dy}{dx} = \frac{1}{(4x^2 - 1)^{\frac{-1}{2}}} . -\frac{1}{2}(4x^2 - 1)^{\frac{-3}{2}} . (4 \times 2x)$$

$$= \frac{-4x}{(4x^2 - 1)}$$

$$or = -4x(4x^2 - 1)^{-1}$$

A chain rule is also applicable for this function as some learners may prefer using a chain rule.

This question was poorly answered by the learners, as 86.4% learners attempted to answer the question but did not score any mark from this question. most learners seem to have forgotten the derivative of a natural log. For example;

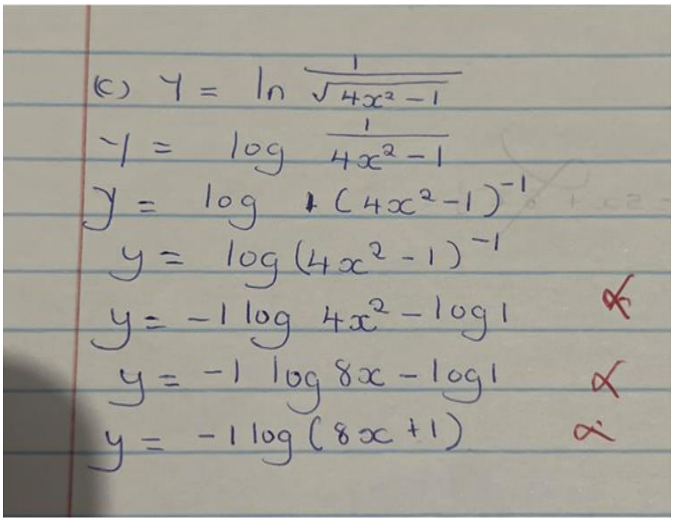


Figure 4.6. An example of one of the learners who did not score any mark in question 1.c)

In Figure 4.6, a learner changed natural logarithm (*ln*) to log in their answers, and carried through to differentiate without indicating that it is a derivative of $y\, with\, respect\, to\, x$. This indicates that the learner either thought of solving the function instead of differentiating. This shows a lack of understanding from the learner as they seemed to have no clue in what was required from them.

13.6% learners who scored full marks for *Question 1 c*, applied correctly either the chain rule or the derivative of the natural logarithm correctly.

The Figure 4.7 shows an example of the learner who used a chain rule correctly.

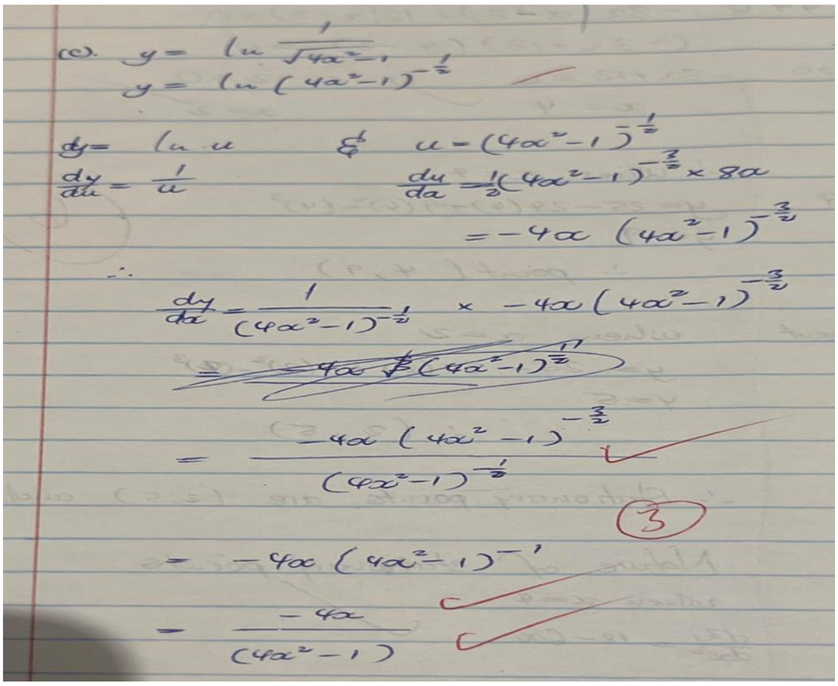


Figure 4.7. An example of a learner who correctly used the chain rule.

This learner shows a full conceptual understanding and applied the procedural fluency in differentiating this function.

Question 1.d) Differentiate

$$g(x) = \frac{(x+2)(2x-3)}{4x^5}$$

Learners were expected to apply both the product and quotient rules to differentiate or multiply the numerator expression first and then apply the quotient rule.

$g(x) = \frac{(x+2)(2x-3)}{4x^5}$,

learners could expand the numerator into $2x^2 + x - 6$,

Then,

$$g'(x) = \frac{d/dx(2x^2 + x - 6) \times 4x^5 - d/dx(4x^5) \times (2x^2 + x - 6)}{(4x^5)^2}$$

$$= \frac{(4x + 1) \times 4x^5 - 20x^4(2x^2 + x - 6)}{16x^{10}}$$

$$= \frac{16x^6 + 4x^5 - 40x^6 - 20x^5 + 120x^4}{16x^{10}}$$

$$= \frac{-24x^6 - 16x^5 + 120x^4}{16x^{10}}$$

$$= -\frac{3}{2x^4} - \frac{1}{x^5} + \frac{15}{2x^6}$$

(Learners may leave their answer in anyway correct e.g., with negative exponents for this question)

9.1% of the learners scored full marks for this question, a further 9.1% scored 1 mark out of 3 for this question while 81.8% did not score any mark for this question. This reveals that majority of the learners lack conceptual and procedural understanding of finding the derivative of a function using the rules of differentiation.

Figure 4.8 is an example of the learner who scored full marks. These learners showed full understanding and procedural fluency of finding the derivative of a product and quotient rule.

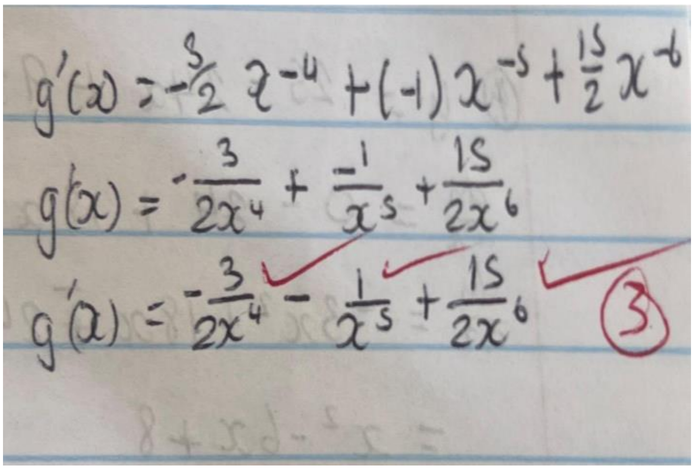


Figure 4.8. an example of the learner who scored full marks in Q.1 d.

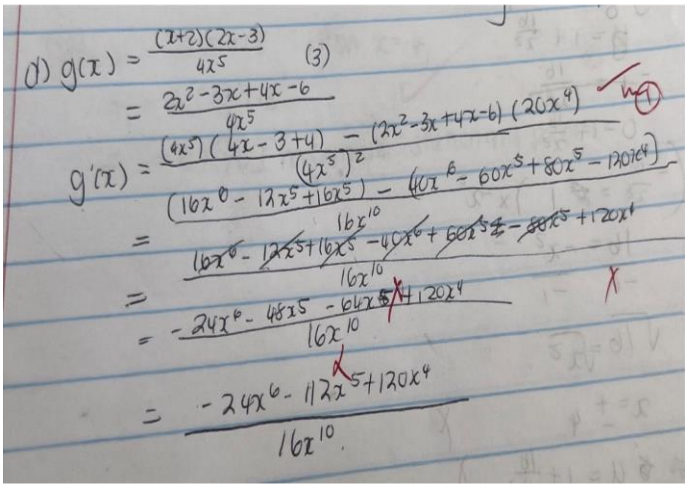


Figure 4.9. Learners mistake on Q.1d

Figure 4.9 of a learner who obtained one mark for this question as there is a slip in her calculations after applying the Product and Quotient rules incorrectly. It is evident from the learner's script in Figure 4.9 that the learner understands how to apply the product and quotient rules but made mistakes in expanding out expressions after correctly applying the differentiation rules.

Figure 4.10 is an example of learners who do not understand how to differentiate this function at all.

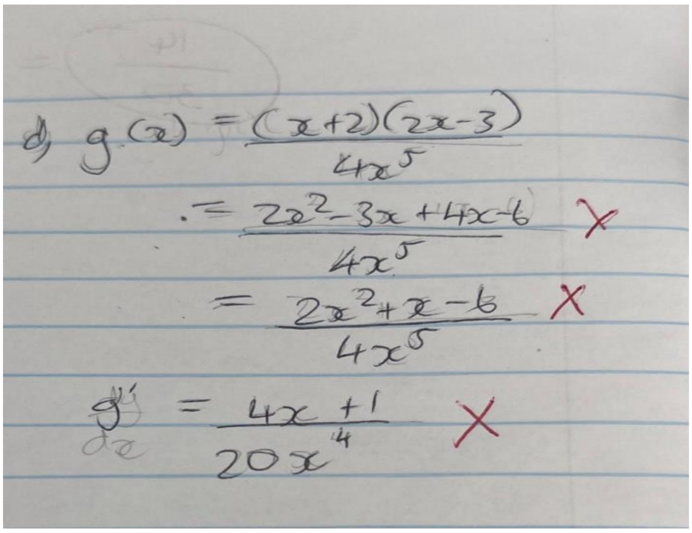


Figure 4.10. Learners who failed Q.1d

Question 2. Application of Differential Calculus

This question required the learners to determine the rate of change of $A = 2\pi r^2 + 6\pi r$ with respect to $r, where\ r = 2$.

Learners were expected to differentiate as:

$$\frac{dA}{dr} = 4\pi r + 6\pi$$

and thereafter they evaluate

$$\frac{d}{dA}(2) = 4\pi(2) + 6\pi$$

$$= 8\pi + 6\pi$$

$$= 14\pi\ or\ 44.0$$

Learners answered this question fairly, whereby 47.8% of the learners scored full marks for this question, 9.9% scored one mark out of (they all differentiated corrected but have a slip in substituting into their differentiated answer correctly, hence the loss of one mark in their calculations). 43.2% could not score any mark for this question as they could not apply the rate of change as a differentiation of a given function.

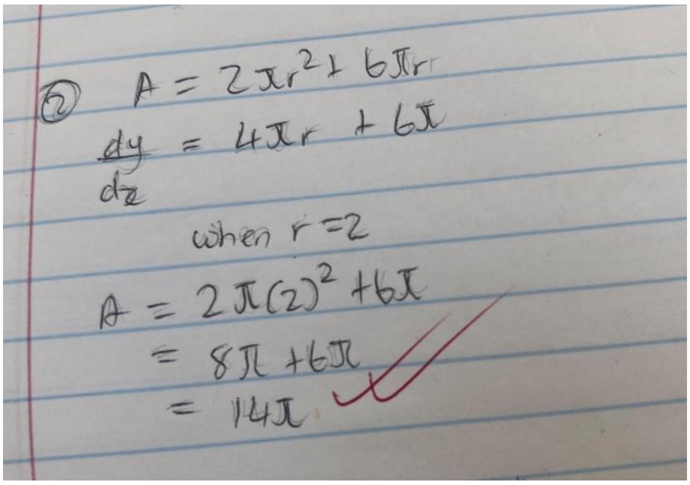


Figure 4.11. A correct response on Q.2.

Figure 4. 11 Shows a response for a learner who showed conceptual knowledge as he/she followed the procedures correctly to the final step of the answer for Question 2.

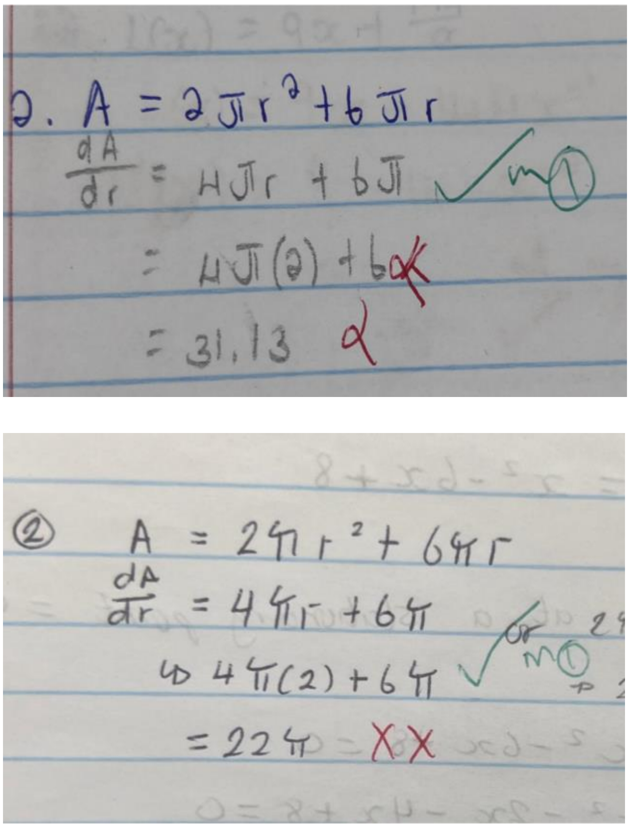


Figure 4.12. Learners' incorrect responses due to procedural flaws.

Figure 4. 12 are responses by learners with a slip in their procedural steps to the final answer after differentiating correctly.

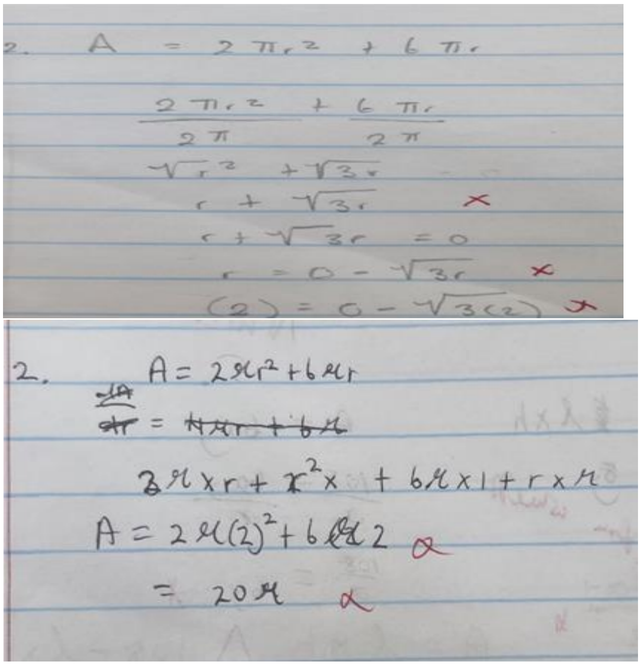


Figure 4.13. An example of a response by a learner lacking conceptual knowledge

Lastly, Figure 4.13 shows the replies of learners that lack conceptual knowledge and portrays no understanding of what to do in attempting the question. A majority (57.7%) of the learners seemed to have no idea.

.

Question 3*. *A curve has an equation

$$y = x - \frac{16}{\sqrt{x^3}}$$

,

$$x > 0$$

.

***Find the equation of the tangent to the curve at the point* $x = 4$.** **[4 marks]**

Learners were expected to first find the value of y when $x = 4$,

$$i.e\ y = 4 - \frac{16}{\sqrt{4}}$$

$$= 4 - 8$$

$$= -4$$

The point is (4, -4) which should be substituted into the general straight-line equation

$$\therefore y - y_0 = m(x - x_0)$$

$$y - (-4) = 2(x - 4)$$

$$y + 4 = 2x - 8$$

$$y = 2x - 8 - 4$$

$$Equation\ is: y = 2x - 12$$

From the learners' scripts 34.1% have scored full marks for this question. These learners portray a good conceptual understanding; procedural fluency as well as an adaptive reasoning (see Figure 4.14). About 47.7% of the learners scored two marks out of the total four, where they either got the y-value correct and/ the first derivative of the given function, which shows a lack of conceptual understanding and procedural fluency in the learners. Only 2.3% scored 3 marks whereby the learner had a slip off in their last step of calculations. Although it was observed that the teachers emphasized that the derivative of a function is the gradient of a function, all 47.7% did not show that in their calculations.

15.9% did not score any mark for this question. The diagrams below show how learners answered question 3.

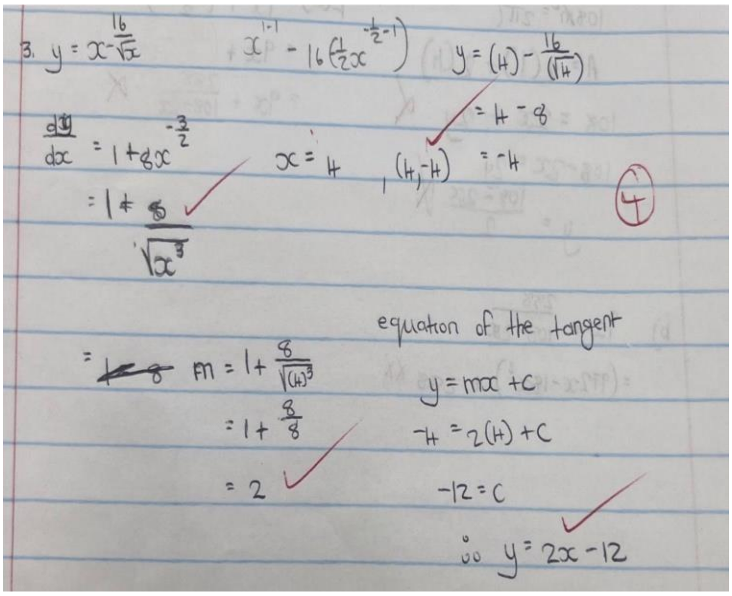


Figure 4.14. learners who scored full marks, showing full conceptual understanding.

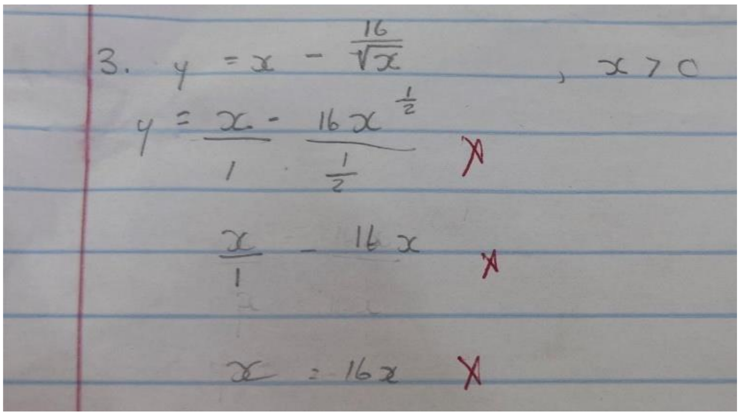


Figure 4.15. Learners who did not score any mark.

The learner did not even indicate the differentiation symbol as

$$\frac{dy}{dx}.$$

The learner further failed to change the fraction

$$\frac{16}{\sqrt{x}}$$

into a negative exponent, which is supposed to be

$$16x^{-\frac{1}{2}}$$

This learner lacked basic algebraic applications of the negative law of index. The learner also ended up equating

$$x = 16x$$

which shows that the learner is not very sure of how to go about differentiating this function.

Question 4. A curve has an equation

$$y = 25 - 24x + 9x^2 - x^3$$

Find the coordinates of the stationary points and determine their nature.

[6 Marks]

In order to find coordinates of stationary points, learners should use the first and second derivative of the given function. Therefore, for function

$$y = 25 - 24x + 9x^2 - x^3$$

;

$$\frac{dy}{dx} = -24 + 18x - 3x^2$$

Since

$$\frac{dy}{dx} = 0$$

at any stationary point then

$$-24 + 18x - 3x^2 = 0$$

; this gives a platform to solve for the variable x in the first derivative of the given function.

Learners may choose any method to calculate the value of x (i.e by factorization, completing the square and/ quadratic formula). Therefore,

$$\frac{-24}{-3}+\frac{18x}{-3}-\frac{3x^2}{-3}=\frac{0}{-3}$$

$$0 = 8 - 6x + x^2$$

$$0 = (-2 + x)(-4 + x)$$

$$x = 2 \; or \; x = 4$$

Thereafter, learners should calculate the second derivative of the given function, which is

$$\frac{d^2y}{dx^2} = 18 - 6x$$

Then when

$$x = 2; at\ point\ (2; 5)$$

$$= 18 - 6(2)$$

$$= 18 - 12$$

$$= 6 > 0, the\ function\ has\ a\ minimum\ value$$

$$while\ when\ x = 4; at\ point\ (4; 9)$$

$$= 18 - 6(4)$$

$$= 18 - 24$$

$$= -6 < 0, the\ function\ has\ a\ maximum\ value$$

It evidently shows that from the learners' scripts, the majority 54.5% scored 4-6 marks thereby displaying full conceptual understanding in calculating coordinates of the stationary points and determining the nature of the stationary points. About 25% of learners scored 2-3 marks while a further 20.5% of the learners scored either 0 or 1 mark out of the total 6.

Figure 4.16 shows the workings of a learner who scored 1 out 6 marks in Question 4.

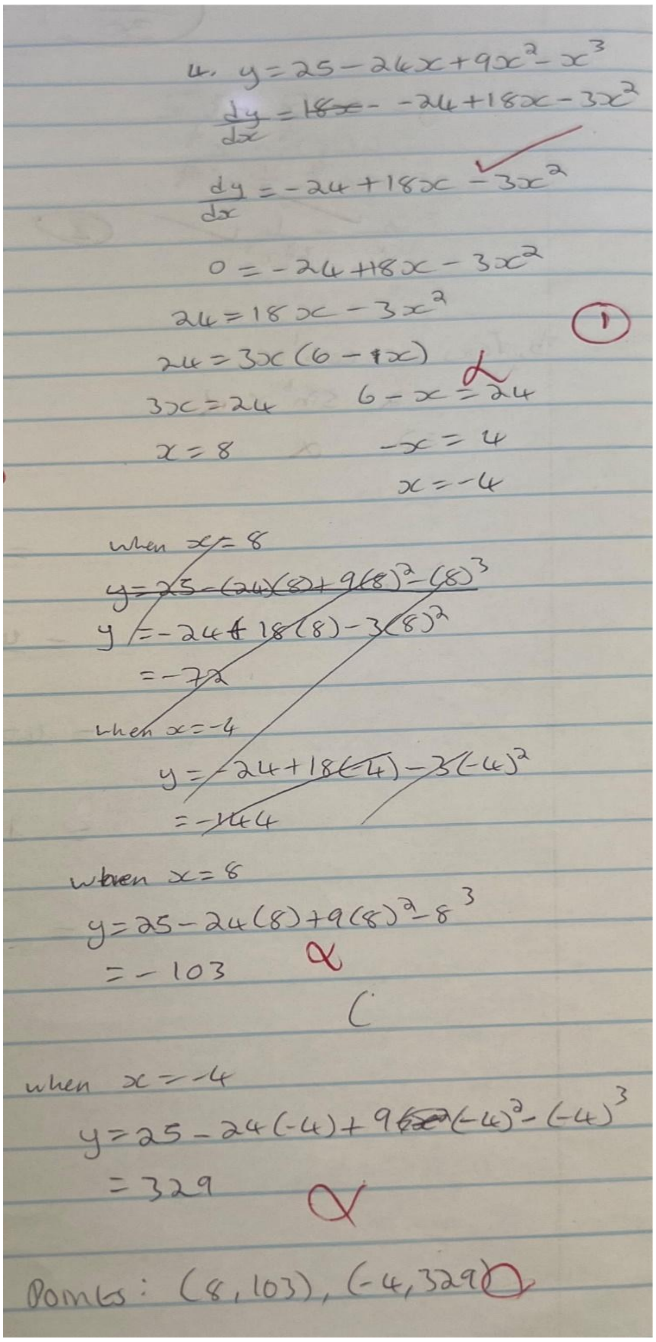


Figure 4.16. A learner who scored 1 out 6 marks in Question 4

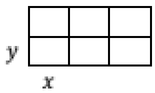
The learner Figure 4.16 displayed a lack of understanding and procedural fluency. Although the learner remembered and indicated clearly that the first derivative of a function is = 0 at its stationary points, the learner further failed to solve the equation

$$0 = -24 + 18x - 3x^2$$

leading to incorrect stationary points and further failing to remember to find the second derivative to give him/her the nature of the stationary points.

Question 5. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food. Each plot will be a rectangle measuring x meters and y meters as shown in the diagram.

a) The area of land being set aside is $108m^2$. Show that the total length of fencing, L meters, is given by



$$L(x) = 9x + \frac{144}{x}$$

b) Find the value of x that minimizes the length of fencing required.

Learners were expected to figure out and use the general properties of a rectangle and connect them together.

$$TotalArea(TA) = 6xy$$

$$108 = 6xy$$

Then they find y in terms of x, using the TA

$$\frac{108}{6x} = \frac{6xy}{6x}$$

$$\frac{18}{x} = y$$

$$If\ L = 9x + 8y$$

Then,

$$L(x) = 9x + 8(\frac{18}{x})$$

$$= 9x + \frac{144}{x}$$

1.Learners were expected to find the value of x that minimizes the length of fencing required.

Learners were then supposed to find the first derivative of L(x), i.e

$$\frac{dL}{dx} = 9 - 144x^{-2}$$

$$= 9 - \frac{144}{x^2}$$

$$\frac{dL}{dx} = 0$$

at a stationary point, thus;

$$0 = 9 - \frac{144}{x^2}$$

$$-9 = \frac{-144}{x^2}$$

$$\frac{-9x^2}{-9} = \frac{-144}{-9}$$

$$\sqrt{x^2} = \sqrt{16}$$

$$x = \pm 4$$

$$\therefore the\ length\ of\ fencing\ is\ minimum\ at\ x = 4,$$

From the learners' scripts 18.1% did not attempt Question 5 at all while, 52.3% attempted the question and scored zero. Learners are taught to continue with solving the equation by the first differentiation. It however seems that these learners did not know that the first differentiation of any function is equal to zero at a stationary point, hence failure to solve the function. Figure 4.17 shows the workings of one learner on *Question 5* who could not score any mark due to lack of understanding.

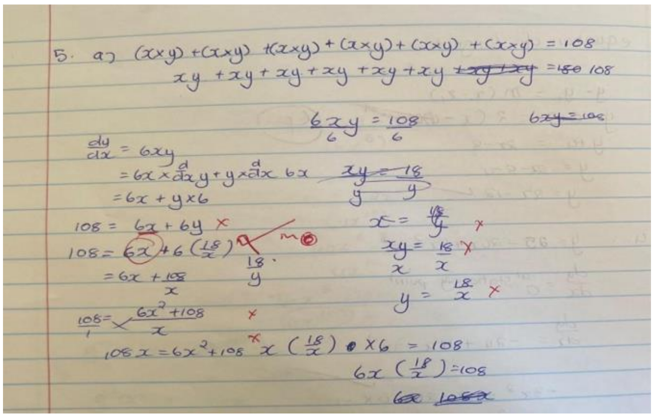


Figure 4.17. Learner's response to Question 5.

Although the learner shown in Figure 4.17 could figure out one of the main keywords as total area, the learner could not connect the area to the total length of the field. Instead, the learner differentiated the area function they formulated in their first step ($6xy$) as an implicit function incorrectly. Since the learner failed to derive the length of the farm in terms of x and y, s/he further failed to get to the length of the field as provided.

4.3.3 Teachers' perspectives on difficulties encountered by learners

A questionnaire was administered to teachers of the AS level. The items of the questionnaire sought to determine errors, misconceptions, challenges and recommendations that teachers have picked up from learners regarding Differential Calculus.

The following are responses from the AS level teachers from the questionnaire:

Question 1. b) Does your experience of teaching Mathematics Higher level perhaps have an effect on teaching Mathematics AS-level?

Response: ***"****Yes, it helps as most, if not all topics that were covered in higher level are now part of AS-curriculum".*

Question 2. How many Mathematics AS level learners do you have in your class and how does this number affect your teaching strategy?

One participant indicated that they had 28 learners whilst another had 23 learners.

Respondents further added that:

"*Although 28 learners seems to be fewer compared to other grades, it is still very difficult to give full assistance and attention to all individual learners. The number may be small but the time allocated for completion of the content is still less. We therefore, instead make efforts of extra classes whereby we continue with the normal lessons - not even to go through the contents the learners do not really understand."*

"The number of learners is fine, but regardless, most of the learners doing AS Mathematics seem to not really have a clear understanding of most of the Mathematics concepts, especially algebra. Most learners achieved the minimum requirement in Mathematics Ordinary level (Grade 11) which is mostly why the learners struggling with AS Mathematics generally."

"If the requirement could change to A and B symbols specifically for AS Mathematics requirement, it would really make AS Mathematics much easier to deal with for both the teachers and learners themselves. Now we mostly give loads of homework to the learners but most of them will still lie and say they do not understand how to go about answering the questions when they just do not want to work on the homework, which makes it very difficult to truly understand their area of struggle in the topic."

Question: 3.a) Is there any prior knowledge that learners are expected to have before they do the Differential Calculus?

Responses: According to the teachers' responses, learners already have an idea of the following.

- "*Learners should know how to solve equations as well as substitution*".
- "*They already know how to calculate a gradient of a curve at a point*"
- "*They know how to apply different laws of indices*'

b) How does prior knowledge help learners understand Differential Calculus concepts?

Responses: *"Applying Differential Calculus requires basic algebraic knowledge and skills, especially with the solving of equations (linear or quadratic). Learners are mostly solving equations when calculating stationary points, or determining the nature of the stationary points.*

"Learners need to know how to apply laws of indices as mostly, with differentiations of fractional or radical terms, they will be required to change from fractional terms to negative exponents or change from radical roots to fractional exponents. It makes it easier for the learners to differentiate such functions"

"Most learners struggle to find equations that are normal to the curve or perpendicular to the curves. But if learners understood the geometry topic from grade 10-11, they can easily relate it to such types of questions in Differential Calculus."

"Sometimes, learners will be required to substitute through their answers, if they know how to substitute correctly then it will be easier for them to know when and how to substitute."

Question 4. a) Do you provide assessment activities to your learners (on Differential Calculus).

Responses: all teachers indicated that they indeed give assessment activities to their learners,

"*I give tests and homework mainly*"

Another responded by saying; "*I usually give tests and homeworks of many TRY-NOWS from the textbook that learners use"*

Question 5. Do learners show any misconceptions and/errors in answering Differential Calculus questions in their answers to their assessments?

Responses: "*Learners integrate sometimes instead of differentiating"*

"They do not know how to differentiate implicit functions"

"*Learners fail to apply product and quotient rule*"

Question 6. Do you have any recommendations in helping learners to deviate from misconceptions and these common errors observed?

Responses:

"I advise learners to practice more and do more revision so that they get to master and understand better the concepts of differentiation, it may help them to differentiate between integration and differentiation as well as to master how to use a product and quotient rule"

"Regarding the product rule and quotient rule, train learners to always indicate a general formula before they answer, to help minimize mistakes in the calculations"

"*There is really less one can do from the teachers' side regarding common errors and misconceptions, teachers should just encourage their learners to practice more and revise more so that they get to master Differential Calculus."*

"Teachers should emphasize on learners to master algebra, geometry and trigonometry topics from the lower grades so that learners will not struggle with solving equations, substitution and or solving geometrical questions"

4.4. Discussion of findings

The following section delves into a comprehensive discussion of the research outcomes, aiming to address the key inquiries that steered this study. The findings are dissected across several subsections, primarily focusing on: the challenges experienced by AS level Mathematics learners when tackling Differential Calculus problems, the impact of these difficulties on learners' overall performance, and an evaluation of AS level Mathematics learners' grasp of concepts within the realm of Differential Calculus.

4.4.1 Difficulties encountered by AS level Mathematics learners in solving Differential Calculus

The results from the learners' diagnostic task displayed that many learners still struggle with Differential Calculus. Although teachers used different approaches and had made emphasis on concepts in Differential Calculus during the teaching of Differential Calculus, as observed by the researcher, learners still showed that they did not acquire mathematical knowledge, skills and confidence they required to tackle mathematical problems. This is evident from the learners' diagnostic task results and teachers' responses to the questionnaires. Learners acquire mathematical knowledge, skills and confidence they need in order to use the Mathematics they have learnt during different approaches used in classroom teaching, which also enable them to be proficient in Mathematics (Makgakga & Makwakwa, 2016). This means that learners acquire knowledge and skills from the classroom, and failure to do so will contribute to a lack of acquiring the knowledge and skills. A lack of conceptual knowledge in solving

Differential Calculus questions was also supported by both the teachers in their responses to the questionnaires and learners' results from the diagnostic test.

Some learners failed to use symbolic notations in Differential Calculus, for example, in *Question 1.c*, some learners could not denote differentiation symbolic notations. This could be due to lack of familiarity with the language and the symbols used, This finding is of the same opinion with Lackoff and Nunez (2000)'s study, which concurred that learners may struggle to understand the abstract concepts and symbolic notations used in Differential Calculus because they have not yet developed the necessary conceptual framework to make sense of them. Furthermore, learners who have had limited exposure to mathematical concepts and symbolic notations may struggle more with Differential Calculus than those who have had more extensive mathematical training (Lakoff & Núñez, 2000).

Although the majority of learners answered the differentiation of natural log function well as seen from the diagnostic task results, more difficulties were observed as learners showed a lack of procedural knowledge in using other differentiation rules. Regarding the teachers' responses on common errors observed, learners lack understanding and procedural knowledge in using the product and quotient rules of differentiation. Learners' procedural knowledge seemed better than their conceptual knowledge, as they were able to differentiate and evaluate simple functions better than complex ones. This is observed from diagnostic test question 2 whereby majority of learners could determine the rate of change of $A = 2\pi r^2 + 6\pi r \ when \ r = 2,$ correctly although some learners still evaluated the function without determining the rate of change of the given function.

As observed from the learners' diagnostic task scripts, some learners lack algebraic knowledge, which led to some of the incorrect calculations. Dlamini (2017)'s research findings concurred with this study findings as it indicated that difficulties in calculus are caused by the knowledge gaps of basic algebra. This is observed from the script when a learner ends up solving an expression when they are supposed to simplify (see Figure 4.4) whereby, a learner ends up solving for *x* when it is not an equation to solve.

Furthermore, learners' limited knowledge in basic algebra was revealed when learners failed to substitute the terms in the formula, or find the derivative of functions that included fractions (see Figure 4.15).

Learners lacked knowledge of converting the cube root into a fractional exponent before they could find its derivative using rules of differentiation. An example is at Figure 4.2 where

$$f(x) = 4\sqrt{x} - \sqrt[3]{x}$$

in which learners were supposed to write variables with fractional exponents before differentiating. It seems that some learners did not comprehend calculus concepts such as differentiating and integration, which led to them using wrong procedures. Some learners seemed to have also confused differentiation with integration steps as in Figure 4.15.

Some learners differentiated correctly but failed to express their answer with a negative fractional exponent into a fraction (see Figure 4.3).

A study by Chirikure et al. (2019) which sought to investigate difficulties that learners in Zimbabwe encounter when solving optimization problems in Differential Calculus, found that learners had difficulty in identifying the objective function and constraints, as well as in applying differentiation to find the maximum or minimum values. This is evident from the learners' diagnostic task *Question 4* whereby learners were expected to find the coordinates of the stationary points and to determine their nature. Most learners who determined the stationary points could not proceed to determine the nature of these points and this could be due to a lack of application of differentiation to finding the maximum and minimum value. Out of the 33 learners who correctly found the stationary points, 12 could not proceed to determine the natures of the function at the stationary points.

Nyaumwe et al. (2020) examined the challenges that learners in Zimbabwe face in solving Differential Calculus problems and found that learners struggled with both the understanding of the concepts and the application of those concepts to solve problems. This is in agreement with Nyaumwe's study as in diagnostic task *Question 5*, whereby learners were expected to show that the total length of fencing, L meters, is given by

$$L(x) = 9x + \frac{144}{x}$$

when the area of land is given,- A total of 25 out of 44 learners who wrote the diagnostic task did not attempt *Question 5 a)*, although some proceeded to *Question 5.b)* and managed to score some marks.

4.4.2 Effects of learners' difficulties in solving algebraic problems on performance to Differential Calculus

Since the majority of learners portrayed a lack of conceptual understanding and procedural fluency in their diagnostic task responses, most of the errors seen from the data analysis are linked to a lack of prior knowledge, such as: lack of algebraic knowledge, trigonometry, mensuration and geometry. This is evident that, if learners struggled to apply the prior knowledge in solving questions prerequisite topics (i.e in algebra, geometry, mensuration and trigonometry) then there could also be poor performance in Differential Calculus questions. Other research studies conceded this study as they revealed in their findings that courses such as algebra, geometry and trigonometry serve as gatekeepers to more advanced Mathematics and can affect Mathematics achievements at large (Adelman, 2006; Ma & Wikins, 2007).

Furthermore, since some learners have shown a slip of calculations in answering Differential Calculus questions, there could be a possibility that learners may do exactly the same in other topics of Mathematics which would also be a contributing factor to poor performance in Mathematics at large.

4.4.3 AS level Mathematics learners' understanding of Concepts in Differential Calculus concepts

Lack of sufficient time may be a contribution to learners' difficulties in solving mathematical problems, learners may not have sufficient preparation in prerequisite mathematical concepts that are necessary for understanding Differential Calculus which include algebraic concepts, trigonometry, and geometry (National Council of Teachers of Mathematics, 2014). This is evident from the observation carried out, whereby teachers mostly work out the Differential Calculus problems with the aim of covering much content in a short period. One of the teachers in this study reflected on their own approach and that of their colleagues to teaching Differential Calculus at a faster pace in order to complete the curriculum, conceding that it could have played a part in the learners' misunderstanding of calculus. Furthermore, according to the teachers' responses from one of the questions in the questionnaire about "*how the number of AS Mathematics learners in a class have an effect on the teaching strategy,* the respondent revealed that:

“*Most of the learners doing AS Mathematics seem to not really have a clear understanding of most of the mathematical concepts from their previous grades, especially algebra”.*

which is one of the reasons why most learners are failing to understand the Differential Calculus concepts. Teachers stated further that most AS learners act as if they have never even done Algebra before, thus in most cases they end up solving where it is not even equations and vice versa.

CHAPTER 5: SUMMARY, CONCLUSION AND RECOMMENDATIONS

This Chapter presents a summary of the main findings of the study, the conclusion as well as recommendations. The study sought to explore grade 12 learners' difficulties in solving Differential Calculus at a selected secondary school in Tsumeb, Namibia. Towards the end of the chapter, the study makes suggestions for further research avenues.

5.1. Summary

The study focused on learners' difficulties in solving Differential Calculus. The findings revealed that although most learners applied very well as they were taught how to differentiate natural log functions, most of them still performed more poorly in applying differential rules to trigonometric functions as well as functions which require product and quotient rule, and solving Differential Calculus with applications. Learners' diagnostic task revealed that the majority of learners still struggle with Differential Calculus. Learners acquire mathematical knowledge, skills and confidence they need in order to use the mathematics they have learnt during different approaches used in classroom teaching, which also enable them to be proficient in Mathematics (Makgakga and Makwakwa, 2016). Although teachers used different approaches and had made emphasis on concepts in Differential Calculus during the teaching of Differential Calculus, as observed by the researcher, learners still showed that they did not acquire mathematical knowledge, skills and confidence they require to tackle mathematical problems. This is evident from the learners' diagnostic task results and teachers' responses to the questionnaires. A lack of conceptual knowledge in solving Differential Calculus questions was also supported by both the teachers in their responses to the questionnaires and learners' results from the diagnostic task.

For example: Some learners failed to use symbolic notations in Differential Calculus, for example in *Question 1.c,* some learners could not denote differentiation symbolic notations. This could be due to lack of familiarity with the language and the symbols used. Lakoff and Nunez (2000) concurred that learners may struggle to understand the abstract concepts and symbolic notations used in Differential Calculus because they have not yet developed the necessary conceptual framework to make sense of them. Furthermore, learners who have had limited exposure to mathematical concepts and symbolic notations may struggle more with Differential Calculus than those who have had more extensive mathematical training (Lakoff & Núñez, 2000).

Although the majority of learners answered the differentiation of natural log function well as seen from the diagnostic test results, more difficulties were observed as learners showed a lack of procedural knowledge in using other differentiation rules. Teachers' responses on common errors observed revealed that learners lack understanding and procedural knowledge in using the product and quotient rules of differentiation. Learners' procedural knowledge seemed better than their conceptual knowledge, as they were able to differentiate and evaluate simple functions better than complex ones. This is observed from diagnostic test question 2 whereby majority of learners could determine the rate of change of $A = 2\pi r^2 + 6\pi r \ when \ r = 2 \ correctly,$ although some learners still evaluated the function without determining the rate of change of the given function.

As observed from the learners' diagnostic task scripts, some learners lack algebraic knowledge, which led to some of the incorrect calculations. Dlamini (2017)'s research findings concurred with this study findings as it indicated that difficulties in calculus are caused by the knowledge gaps of basic algebra. This is observed when a learner ends up solving an expression when they are supposed to simplify (see Figure 4.4).

Furthermore, learners' poor background in basic algebra was revealed when learners failed to substitute the terms in the formula, or find the derivative of functions that included fractions as well as where learners failed to apply indices laws to either change from radical expressions to fractional indices.

Even after the testing of learners' prior knowledge and teachers revising the prerequisite topics as required prior to Differential Calculus, learners still seemed to have difficulties converting the radical root expressions into fractional exponential expressions before they could find its derivative using rules of differentiation. An example is at Figure 4.2 whereby learners were asked to differentiate $(x) = 4\sqrt{x} - \sqrt[3]{x}$, in which learners were supposed to write variables with fractional exponents before differentiating. Although teachers emphasized on how, when and where to apply different rules of differentiation, it is still evident that some learners did not comprehend calculus concepts, which led to them using wrong procedures. The learners seemed to have also confused differentiation with integration steps (see Figure 4.4).

Some learners differentiated correctly but failed to express correctly their answer with a negative fractional exponent into a fraction.

A study by Chirikure et al. (2019) which sought to investigate difficulties that learners in Zimbabwe encounter when solving optimization problems in Differential Calculus, found that learners had difficulty in identifying the objective function and constraints, as well as in applying differentiation to find the maximum or minimum values. This is seen in diagnostic task question 4 whereby learners were expected to find the coordinates of the stationary point and to determine their nature. Most learners who determined the stationary points could not proceed to determine the nature and this could be due to a lack of application of differentiation to finding the maximum and minimum value. For example: out of 33 learners who correctly found the stationary points, 12 could not proceed to determine the natures of the function at the stationary points.

Nyaumwe et al. (2020) examined the challenges that learners in Zimbabwe face in solving Differential Calculus problems and found that learners struggled with both the understanding of the concepts and the application of those concepts to solve problems. This is seen in diagnostic task question 5, whereby learners were expected to show that the total length of fencing, L meters, is given by $L(x) = 9x + \frac{144}{x}$ when the area of land is given. About 25 out of 44 learners who wrote the diagnostic task did not attempt *Question 5.a* of the diagnostic task, although some proceeded to *Question 5.b* and managed to score some marks.

Insufficient time could have contributed to learners' difficulties in solving mathematical problems, learners may not have sufficient preparation in prerequisite mathematical concepts that are necessary for understanding Differential Calculus which include algebraic concepts, trigonometry, and geometry (National Council of Teachers of Mathematics, 2014). One of the teachers in the present study reflected on their own approach and that of their colleagues to teaching Differential Calculus at a faster pace in order to complete the curriculum, conceding that it could have played a part in the learners' misunderstanding of calculus.

5.2. Conclusion

The study delved into the challenges encountered by learners when tackling Differential Calculus problems. Employing methods such as script analysis, questionnaires, and observations, the research gleaned valuable insights. The study findings shed light on various fronts, pinpointing the root causes of learners' struggles, uncovering misconceptions, and highlighting common errors made when navigating Differential Calculus questions. Notably, it surfaced that most learners exhibited deficiencies in both grasping the concepts and executing the procedures proficiently.

Despite the teachers’ commendable application of certain differentiation rules, such as those pertaining to natural log functions, learners displayed significant shortcomings when applying rules to other areas, like trigonometric functions, product and quotient rules, and practical applications of Differential Calculus. The study also hinted at the potential adverse impact of inadequate time allocated to teaching and learning Differential Calculus, affecting learners' problem-solving abilities in this domain.

Furthermore, the research underscored that poor performance in calculus stemmed not only from the intrinsic complexity of the subject matter but also from gaps in foundational knowledge across various mathematical domains (ranging from algebra and inequalities to functions, trigonometry, mensuration, and Differential Calculus) which contributes to learners' difficulties in diagnostic tasks. Moreover, the findings suggested that errors, barring instances of carelessness, were primarily rooted in a lack of competence in the tested Differential Calculus concepts.

Additionally, the study revealed that teachers' instructional approaches and learners' deficiencies in both procedural and conceptual knowledge, coupled with carelessness and inattentiveness during calculations, could be attributing factors to poor performance. Consequently, it highlighted the necessity for a heightened focus on problem-solving methodologies in the teaching and learning of Differential Calculus and Mathematics as a whole.

5.3. Recommendations

This study explored the AS level Mathematics learners' difficulties in solving Differential Calculus. The analysis of the data pointed to certain factors that contribute to the difficulties that learners experience in Differential Calculus. Therefore, the following recommendations are made based on the findings of the research study:

● The study recommends that for more practice, learners must be taught functions that consist of exponents which are fractions and also coefficients with radical symbol $\sqrt[n]{x}.$

These practices should already start from Grade 10-11 so that they do not continue to make mistakes when changing from radical roots to a fractional exponent.

● The study further recommends that before learners are taught calculus, teachers should carry out a diagnostic assessment to determine learners' prior knowledge in topics such as algebra, functions and mensuration, trigonometry and inequalities and apply remedial measures. As observed from the findings, some learners could not answer questions that had involved algebraic solving of quadratic equations.

● Furthermore, concepts such as algebraic manipulation and factorization should be highly emphasized throughout the teaching of Mathematics, not only in algebra. This will help learners to avoid some of the misconceptions like where learners ended up solving even when the given problem was not an equation.

● The study also recommends that learners should be taught how to differentiate functions using different rules of differentiation especially with trigonometric functions, functions which require application of product and quotient rules. This may help learners to master differentiation rules. Furthermore, teachers should practice more of a Learner-Centered approach than Teacher-Centered approach in classrooms.

- The study recommends a continued working with a variety of algebraic manipulation from an early stage, so that learners come to calculus with really solid foundations, and that teachers should make an initial assessment of current algebraic and functional thinking before embarking on calculus teaching.
- The study further recommends that teachers should make an emphasis as to when to use differentiation and integration, to avoid mixing up the two concepts.

Further research avenues: Since this study tested for conceptual understanding and procedural fluency, further research may consider exploring mathematical proficiency, which includes adaptive reasoning, strategic competency and productive disposition on a larger scale.

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APPENDIX 1: ETHICAL CLEARANCE CERTIFICATE

UNAM
UNIVERSITY OF NAMIBIA

ETHICAL CLEARANCE CERTIFICATE

Ethical Clearance Reference Number: RUC0009 **Date: 30 November 2022**

This Ethical Clearance Certificate is issued by the University of Namibia Ethics Committee (REC) in accordance with the University of Namibia's Research Ethics Policy and Guidelines. Ethical approval is given in respect of undertakings contained in the Research Project outlined below. This Certificate is issued on the recommendations of the ethical evaluation done by the ethics committee.

Title of Project: Exploring grade 12 learners' difficulties in solving differential calculus at a selected secondary school in Tsumeb

Student: Martha Kufunga

Student Number: 201021056

Supervisor(s): *Dr Frans N. Haimbodi*

Centre for Research Services

Take note of the following:

1. Any significant changes in the conditions or undertakings outlined in the approved Proposal must be communicated to the ethics committee. An application to make amendments may be necessary.
2. Any breaches of ethical undertakings or practices that have an impact on ethical conduct of the research must be reported to the ethics committee
3. The Principal Researcher must report issues of ethical compliance to the ethics committee (through the Chairperson) at the end of the Project or as may be requested by the ethics committee
4. The ethics committee retains the right to:

i) Withdraw or amend this Ethical Clearance if any unethical practices (as outlined in the Research Ethics Policy) have been detected or suspected,

ii) Request for an ethical compliance report at any point during the course of the research.

The ethics committee wishes you the best in your research.

Dr Emilia N Mbongo (Chairperson Ethics Committee)

Prof. Davis Mumbengegwi (Head, Multidisciplinary Research

APPENDIX 2: PERMISSION LETTER - REGIONAL DIRECTOR OF EDUCATION

REPUBLIC OF NAMIBIA

OSHIKOTO REGIONAL COUNCIL

Tel: (065) 242500 **DIRECTORATE: EDUCATION, ARTS & CULTURE** Private Bag 2028
Fax: (065) 241660 ONDANGWA
Enquiries: Ms Tende 28 February 2023

Ref: 13/2/9/1

Ms Martha Kufunga
mrthndkmw65@gmail.com
Etosha Secondary School

RE: PERMISSION TO CONDUCT RESEARCH AT A SCHOOL IN OSHIKOTO REGION

The Office of the Director acknowledges receipt of your letter seeking for permission to conduct research studies focusing on *"Exploring grade 12 learners' difficulties in solving differential calculus"* a case study of a selected school in Oshikoto Region.

Kindly be informed that permission has been granted to carry out the research in Oshikoto Region, please be guided by the following:

- You have to consult the school principal well in advance to ensure a proper co-ordination of other school activities
- The research should not interfere with the normal teaching and learning process at the school.
- Participation in the research should be on a voluntary basis.
- The information to be collected should be treated as confidential and only for research purposes.

Thank you for showing interest to do the research in the Oshikoto Region. It is our sincere hope that the information you would gather will be useful towards the completion of your qualification.

Sincerely yours

28/02/2023

MS ALETTA A. EISES
DIRECTOR OF EDUCATION, ARTS AND CULTURE
OSHIKOTO REGION

OSHIKOTO REGIONAL COUNCIL
Director
2023 -02- 28
PRIVATE BAG 2028
ONDANGWA
EDUCATION, ARTS & CULTURE

APPENDIX 3: LETTER TO THE DIRECTOR OF MOEAC - OSHIKOTO

P.O. Box 523, Tsumeb

6th Deceember 2022

The Regional Director

Ministry of Education, Arts and Culture

Oshikoto Region

Private Bag 2028, Ondangwa

Dear Mrs A. Eises

RE: REQUEST FOR PERMISSION TO CONDUCT A RESEARCH STUDY AT A SELECTED SCHOOL IN TSUMEB ABOUT "EXPLORING GRADE 12 LEARNERS' DIFFICULTIES IN SOLVING DIFFERENTIAL CALCULUS"

I am a registered student for a Master's degree in Mathematics Education at the University of Namibia. In partial fulfilment to qualify for a Master's degree, I am required to conduct a research study on the topic stated above. The purpose of my research is to explore grade 12 learners' difficulties in solving Differential Calculus, at the selected school in Tsumeb.

Studies revealed that there has been deficient performance of grade 12 learners in Differential Calculus over the years (Makgakga & Makwakwa, 2016; Dlamini, 2017). Moreover, in Namibia, poor academic performance in Differential Calculus is confirmed through the comments indicated in the examiner's reports over the years (DNEA, 2020, 2021 & 2022).

This study therefore seeks to explore grade 12 learners' difficulties in solving Differential Calculus at the selected school in Tsumeb. Results from this study might help with recommendations on approaches to help with enhancing teachers' and learners' performance in the topic.

I hereby kindly request your good office to grant me permission to use the selected school in Tsumeb as my research site for the research project. I would like to conduct the research study with all Mathematics AS level teachers and leaners at the selected school. Data will be collected via observations, learners' diagnostic tasks as well as teachers' one-on-one interviews. The diagnostic tasks will consist of Differential Calculus questions testing different learning objectives in Differential Calculus. Although normal class teaching time slots will be used for observations, learners' diagnostic tasks and teachers' interview sessions will be set for afternoons suitable for the participants in order to avoid lesson interruptions.

I hope to complete this study before end of April 2023. The school and participants will be assured of confidentiality and anonymity both during experiment and in the final research report.

For any clarifications, please contact me at +264 81 496 3427 (mrthndkmw65@gmail.com) or my research study Supervisor Doctor Haimbodi at +264 66 268 6230 (fhaimbodi@unam.na)

I look forward to hear from your good office.

Yours Sincerely,

Martha Ndakumwa Kafunga

APPENDIX 4: LETTER TO SCHOOL PRINCIPALS

P.O.Box 523, Tsumeb

10 January 2023

The Principal

…………………… School

P/Box …, Tsumeb

Dear Sir/Madam

RE: RESEARCH TO BE CONDUCTED AT …………………………….. SCHOOL

I am a registered student for a Master's degree in Mathematics Education at the University of Namibia. In partial fulfilment to qualify for my Master's degree, I am required to conduct a research report on the topic: "Exploring grade 12 learners' difficulties in solving Differential Calculus".

I therefore kindly, request your good office to permit me to carry out my research at your school. Attached please find the proof of permission to conduct the research at the selected school in Tsumeb, granted by both the office of the Permanent Secretary as well as the office of the Oshana Regional Director of Education, respectively. I will conduct the study with all classes doing Advanced Subsidiary Mathematics. Data will be collected through learners' diagnostic test, classroom observation and teachers' questionnaire. The classroom observations will be done on lessons based on

Differential Calculus only, a topic in the Advanced Subsidiary Mathematics syllabus. I hope to complete this study before the end of April 2023.The participants will be assured of confidentiality and anonymity in the final report. Classroom observations will be done during Grade 12 Mathematics Differential Calculus lessons and will not disturb the normal teaching lessons.

For any clarifications, please contact me at 081 496 342 7 (mrthndkmw65@gmail.com) or my Supervisor Dr Haimbodi at +264 66 268 6230 (fhaimbodi@unam.na) .

Yours Sincerely,

__________

Martha Ndakumwa Kafunga

APPENDIX 5: CONSENT FORM: TEACHERS

Dear Participant

Re: INFORMATION LETTER AND CONSENT FORM FOR RESEARCH STUDY TO BE CONDUCTED

Title of Study: *Exploring grade 12 learners' difficulties in solving Differential Calculus at a selected secondary school in Tsumeb.*

Principal Investigator: Kafunga N. Martha

University of Namibia

Master of Education

(Mathematics Education)

mrthndkmw65@gmail.com

0814963427

Background

You, ………………………………………………., are cordially invited to take part in a research study. Before you decide to participate in this study, it is crucial that you understand why the research is being conducted and what it will entail. Please take some time to read the following information carefully. For more information you may contact me on the contact details provided.

Purpose and Benefit

The purpose of my research is to explore grade 12 learners' difficulties in solving Differential Calculus. The study will be conducted towards the attaining of the Masters of Education (Mathematics Education). Furthermore, the study will make appropriate recommendations for Mathematics classroom practices. Moreover, your active participation in this study will be highly appreciated.

Study Procedure

Observations will be done during lesson presentations on Differential Calculus. Thereafter, an interview session of about 20 minutes will be organized (depending on the time you may be free).

Risks

There will be no risks or harm in participating in this study. If you find anything upsetting in the study, do not hesitate to bring it to my notice. In case of unforeseeable risks, efforts will be made to minimize such risks.

Confidentiality and anonymity

Confidentiality will be maintained throughout the study, since the study will only be used for academic, professional presentations as well as publications purposes. Data collected by this study will be anonymous through the use of pseudonyms. Your name will not be reported, neither on the data collection instrument, research reports nor any final publication.

Costs to Subject and Compensation

There are no costs and no monetary compensation to you for your participation in this study.

Consent

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving a reason and at no harm or cost. I understand that I will be given a copy of this consent form. I voluntarily agree to take part in this study.

Signature________________________ Date ________________________________

APPENDIX 6: CONSENT FORM: PARENTS/GUARDIANS

Dear Parent/Guardian

Re: INFORMATION LETTER AND CONSENT FORM FOR RESEARCH STUDY TO BE CONDUCTED

Title of Study: EXPLORING GRADE 12 LEARNERS' DIFFICULTIES IN SOLVING DIFFERENTIAL CALCULUS AT A SELECTED SECONDARY SCHOOL IN TSUMEB

Principal Investigator: Martha N. Kafunga

University of Namibia

Master of Education

(Mathematics Education)

mrthndkmw65@gmail.com

0814963427

Background

Your child, ___________________________________ is kindly invited to take part in a research study. Before you allow the child to participate in this study, it is important to understand the purpose of conducting the research and what will be involved in the research study.

Please read the following information carefully. For more information and/further clarifications, do not hesitate to contact the researcher on the contact details provided.

Purpose and Benefit

The purpose of this research is to explore grade 12 AS learners' difficulties in solving Differential Calculus at a selected school in Tsumeb. The study will be conducted towards the attaining of the Masters of Education (Mathematics Education). Furthermore, the study will make suitable recommendations for Mathematics classroom practices. Moreover, the learner's active participation in this study will be highly appreciated.

Study Procedure

Learners will be required to write diagnostic tasks on Differential Calculus.

Risks

There will be no risks or harm in participating in this study. If you find anything upsetting in

the study, do not hesitate to bring it to the researcher's attention. In case of unforeseeable risks, efforts will be made to minimize such risks.

Confidentiality and anonymity

Confidentiality will be maintained throughout the study, since the study will only be used for

academic, professional presentations as well as publications purposes. Data collected by this

study will remain anonymous through the use of pseudonyms. The child'name will not be reported, neither on the data collection instrument, research reports nor any final publication.

Participants involved in this study will not be identified and their anonymity will be maintained.

Costs to Subject and Compensation

There are no costs and no monetary compensation for child participation in this

study.

Consent

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions. I understand that my child's participation is voluntary and that s/he is free to withdraw at any time, without giving a reason and at no harm or cost. I understand that I will be given a copy of this consent form. I voluntarily agree on behalf of my child (a minor) to take part in this study.

Signature ___________________ Date __________

APPENDIX 7: LEARNERS' DIAGNOSTIC TEST

Diagnostic Task	Differential calculus	AS Mathematics
09 August 2023	Marks: 30	Time: 60-minutes

Instructions to candidates

- Do not write your names on this question paper or on the answer script,
- Answer all the questions on the answer script attached to this question paper,
- Please show your calculations (where applicable),
- Use only a blue or black ink pen,
- Write neatly,
- Number your questions correctly.

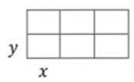
1. Differentiate the following
   (a) $y = 4\sqrt{x} - \sqrt[3]{x}$ [2]
   (b) $f(x) = 2cos^3x$ [2]
   (c) $y = \ln\frac{1}{\sqrt{4x^2-1}}$ [3]
   (d) $g(x) = \frac{(x+2)(2x-1)}{4x^5}$ [3]

2. Given that $A = 2\pi r^2 + 6\pi r$
   Determine the rate of change of A with respect to r, where $\boldsymbol{r = 2}$. [2]

3. A curve has an equation $y = x - \frac{16}{\sqrt{x}}, > 0,$
   Find the equation of the tangent to the curve at the point where $x = 4$ [4]

4. A curve has an equation $y = 25 - 24x + 9x^2 - x^3$
   Find the coordinates of the stationary points and determine their nature. [6]

5. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.
   Each plot will be a rectangle measuring x meters and y meters as shown in the diagram.



   (a) The area of land being set aside is $108m^2$
   Show that the total length fencing, L meters, is given by
   $L(x) = 9x + \frac{144}{x}$ [3]
   (b) Find the value of x that minimizes the length of fencing required. [5]

APPENDIX 8: TEACHERS' QUESTIONNAIRE

QUESTIONAIRE

1. a) Have you taught Mathematics higher-level before?

…………………………………………………………………………………………

……………………………………………………………………

b) How long have you taught Mathematics Higher level-for the previous curriculum?

…………………………………………………………………………………………

………………

c) Does your experience of teaching Mathematics Higher level perhaps have an effect on teaching Mathematics AS-level?

…………………………………………………………………………………………

……………….

2. How many Mathematics AS level learners do you have in your class and how does this number affect your teaching strategy?

…………………………………………………………………………………………

……………….

3. Differential Calculus is one of the topics in Mathematics AS level curriculum content, which has been also part of Mathematics higher level for the previous curriculum. According to DNEA examination reports in Mathematics, one of the highlights is that "Very few learners attempted to answer questions involving Differential Calculus and on those that attempted to answer these questions, only a few managed to score marks".

a) Is there any prior knowledge that learners are expected to know before they do any Differential Calculus topic?

…………………………………………………………………………………………

………………………………………………………………………

4. A) Do you provide assessment activities to your learners (on Differential Calculus).

If yes, state all the assessment activities you provide.

…………………………………………………………………………………………

………………………………………………………………………

5. Do learners show any misconceptions in answering Differential Calculus questions in their answers to their assessments?

……………………………………………………

…………………………………………………………………………………………

………………

6. What are your observed common errors that learners show in answering differential questions?

…………………………………………………………………………………………

………………………………………………………………………

7. A) Do you have any recommendations in helping learners or teachers to help their learners to deviate from misconceptions or these common errors observed?

……………………………………………………………………………………………

……………………………………………………………………………………………

………………………………

APPENDIX 9. OBSERVATION CHECKLIST

1. **Introduction to the lesson**

a) How did the teacher introduce the topic to the learners?

b) Is there any lesson preparation from the teacher? Tick in the appropriate box

Yes	
No	

c) Are the objectives well highlighted in the lesson plan and during the introduction of the lesson to the learners?

d) Is there any prior knowledge expected from the learners?

e) How did learners react to the prior knowledge highlighted-objectives? Did they seem to remember?

f) What instructional sources of information did the teacher and learners use for the lesson?

2. **Presentation of the lesson**

a) Which parts of Differential Calculus do leaners seem to struggle to understand during the lesson?

b) Are learners able to answer questions posed to them by teacher during the lesson? How were the explanations regarding their level of understanding?

c) Did learners ask questions during the lesson?

d) How was the response of the teacher to the learners' questions?

e) Does the teacher encourage learners to ask questions during the lesson?

f) Does the teacher answer learners' questions in depth?

g) i) Did the teacher encourage pair/group work or peer explanations or activities at all during the lesson?

ii) What languages did learners use in discussing answers to the questions asked in the class (if there were pair/group work)?

iii) Did learners seem to understand explanations from their peer much better
than the teacher's?

h) Overall, what teaching mechanism did the teacher use in presenting the lesson to the learners?

i) Does the teacher seem to have sufficient time to assist all learners in his/her class?

3. **Conclusion of the lesson**

a) How did the teacher conclude the lesson to the learners?

b) Was there any specific emphasize on the lesson presented?

c) Is there any homework or sort of assessment given to the leaners at the end of the lesson? What questions exactly?

4. Lesson plan

How did the teacher align the lesson presentation to the lesson plan?