AN ANALYSIS OF TEACHERS’ EXEMPLARY INSTRUCTIONAL STRATEGIES IN RELATION TO NSSCO LEARNERS’ LEARNING DIFFICULTIES IN SOLVING ALGEBRAIC EQUATIONS IN OMARURU CIRCUIT

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ABSTRACT

This study analysed teachers' exemplary instructional strategies in relation to NSSCO learners' learning difficulties in solving Algebraic equations in Omaruru Circuit, Erongo Educational Region, Namibia. This study used Cognitive Constructivism Theory as its theoretical underpinnings. The study applied a convergent mixed method, QUAN and QUAL, approach, using a quasi-experimental design and an existential phenomenology design to identify learners' learning difficulties in Algebra as well as the instructional strategies employed by teachers to assist those learners. One public school was purposively sampled, and one private school was automatically selected for study. Data collection instruments include pre-tests and posts. The participants were two National Senior Secondary Certificate Ordinary (NSSCO) Mathematics teachers and 36 grade 10 learners. The data collected from pre-tests and post-tests were analysed quantitatively and the two proposed hypotheses were tested using the t-test at $\alpha = 0.05$ method of analysis while the data from observations were descriptively analysed. The study's findings revealed the following: (1) Learners have more difficulties in word problems, factorizing, LCM, binomial expansion, reciprocal and inequality direction among all the other difficulties identified. (2) Watching videos assisted learners with learning difficulties in Algebra than peer teaching and group discussion. (3) The t (stat) = 2.56 is more than t (critical) = 2.04 and this reject the null hypothesis. This indicates that there is a significant difference in the mean scores of learners taught using exemplary instructional strategies and learners taught using traditional methods. Learners taught using exemplary instructional strategies performed better than those taught using the traditional methods. The study recommends that teachers must use instructional strategies that allow learners to actively participate in their learning such as peer teaching, group discussions and allow them watch educational videos since these strategies enhance learners' interest in learning. Hence, Educational regions should organize numerous Mathematics workshops, so that teachers meet, discuss and share exemplary instructional strategies which are ideal for teaching Algebra.

Key words: Mathematics, Algebra, teachers' instructional strategies, learners learning difficulties, solving Algebraic equations, instructions, traditional methods

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LIST OF ABBREVIATIONS AND/OR ACRONYMS

NSSC-AS: Namibia Senior Secondary Certificate Advanced Subsidiary

VVEIS: Video Viewing Exemplary Instructional Strategy

NSSCO: Namibia Senior Secondary Certificate Ordinary

NIED: National Institute for Educational Development

CEIS: Cooperative Exemplary Instructional Strategy

MTOS: Mathematic Teacher Observation Schedule

NSSC: Namibia Senior Secondary Certificate

PEIS: Peer Exemplary Instructional Strategy

LTIS: Lecture Tradition Instructional Strategy

LCM: Lowest Common Multiple

UNAM: University of Namibia

HCF: Highest Common Factor

PBL: Problem- Based Learning

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DECLARATIONS

I, Beath Avehe Lumeta, hereby declare that this study is my own work and is a true reflection of my research, and that this work, or any part thereof has not been submitted for a degree at any other institution.

I, Beath Avehe Lumeta, grant the University of Namibia the right to reproduce this thesis in whole or part, in any manner or format, which The University of Namibia deem fit.

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CHAPTER ONE

INTRODUCTION

1.1. Introduction

The aim of this research is to analyse instruction strategies used by teachers to teach algebra to NSSCO learners experiencing learning difficulties when solving algebraic equations. The background context of the study, the statement of the problem, and the questions of the study together with the hypotheses, the significance of the study, the limitation and delimitation of the study, as well as the definition of terms and the form of the structure of the report, are described in this chapter.

1.2 Orientation of the study

Prior to Namibia's independence in 1990, several high schools meant for Black learners did not offer mathematics in compulsory mode as per the educational curriculum for Bantu education (Andima, 2001), and the teaching and learning of mathematics was optional (Angula, 2015). Even then, the content of mathematics was only taught with a sturdy potency of memorisation of facts with slight acquaintances into factual of real-life experiences. Moreover, learners were disheartened from majoring in mathematics as it was dignified to be tough (Mwetulundila, 2000, as cited in Andima, 2001).

On the contrary, currently, the Namibian education system considers mathematics to be an essential subject of the science discipline at the senior secondary school level, and it is one of the fundamental constituents of the science discipline (Algani, 2019 & Tashtoush et al., 2022). Worldwide, mathematics is recognised as one of the most important subjects in most areas of human endeavours (Kiplagat et al., 2012). It is also considered to be a science that is closely related to everyday life (Sugiarti & Retnawati, 2019). Additionally, many people use it in their daily activities, for instance, during transactions, purchasing in the market, budgeting, following rules for medication prescriptions, in architecture. Yasar (2016) defines mathematics as a sequence of guidelines to be remembered, or as arithmetic calculations, secretive and algebraic equations, and geometrical substantiations. As it is very crucial in everyday life, the Namibian education system decided to include it in its curriculum for all to enhance their numerical skills and number values. Furthermore, Mathematics is crucial for the "*development of science, technology, and commerce, and mathematical skills, knowledge, concepts, and processes enable individuals to investigate, model, and interpret numerical and spatial relationships and patterns that exist in the world*" (NIED, Ministry of Education, Art and Culture, 2010, p. 18).

The Namibian Government attaches great significance to the teaching of mathematics in Namibian schools. Thus, now mathematics is taught in a compulsory mode to all learners from pre-primary phase to senior secondary phase in the revised school curriculum of 1992, 1998, as well as in the last revised curriculum of 2016. Even though mathematics becomes compulsory in the Namibian education system, a larger number of learners have already built a view that mathematics is a challenging subject (Sibuku & Mukwambo, 2019).

Additionally, mathematics plays a major role in developing learners' thinking abilities to evaluate situations, to prove a point, and thereby providing solutions to real lives problems. Furthermore, mathematics knowledge can contribute to the scientific and technical progress of nations if the significance and the application of mathematics in life is well understood. The knowledge in mathematics may allow learners to apply it in every situation of their lives and thereby, develop an understanding of how mathematics plays a role in the world around them, and develop learners' ability to apply mathematics in other field of studies (NIED, Ministry of Education, Arts and Culture, 2019). Nevertheless, Algani (2019), highlighted that there is a need for educators to emphasise and to utilise all resources as well as strategies available to them to support all learners to have a better understanding of various mathematical concepts and thereby, motivate learners to consider mathematics as a normal subject rather than seeing it as a hard subject.

In knowledge-based societies, numeracy skills are some of the core skills that everyone needs to acquire (NIED, Ministry of Education, Arts and Culture, 2016). In addition, the goal of the Namibian National Curriculum for Basic Education (2016) is to highlight that "learners need to be fully counted for and must be able to use mathematical language confidently and effectively" (p. 9). Moreover, mathematics is a pre-requisite subject to admission to numerous occupations in Namibia and elsewhere, such as the natural sciences, engineering, medicine, architecture, and many more areas (Fumador & Agyei, 2018; Hamukwaya, 2020).

In addition, the Mathematics Namibia Senior Secondary Certificate Advanced Subsidiary Level (NSSC-AS) Level syllabus describes some of the objectives of Mathematics as:

(i) *"to develop learners' Mathematical knowledge and skills in a way which encourages confidence and provides satisfactions and enjoyment as well as*

(ii) *to develop learners' ability to analyse problems logically, recognise when and how a situation may be represented Mathematically,*

(iii) *identify and interpret relevant factors and, where necessary select an appropriate Mathematical method to solve problems" (NIED, Ministry of Education Arts and Culture, 2020, p. 2).*

Although these objectives are relevant, the focus of this study is the NSSCO Mathematics learners (NSSCO) syllabus. Algebra is one of the topics in the NSSCO curriculum in the Namibian Education system. Additionally, Fumador and Agyei (2018) consider algebra as an obligatory learning content that is offered to learners who major in science and technical programs, some programs in arts, as well as in business. Algebra requires learners to adopt logical thinking, which, in turn, allows them to focus on arithmetic operations while focusing on the use of symbols in representing equations as well as to establish the relationships in mathematical operations (Ying et al., 2020).

Algebra's, as one of the topics in mathematics' teaching and learning, contribution to a collective understanding of society and the world at large, is vital to the development of science, technology, and engineering (Ojaleye et al., 2018). To many learners, algebra has been a stumbling block and challenging (Francis, 2011), and many learners in Namibian schools still seem not to find their ways in solving simple algebraic equations as specified in examiner reports (2019, 2020, and 2021) for the Namibia Senior Secondary Certificate Ordinary (NSSCO) level.

In 2019, Mathematic Paper 1 (core), the examiners reported that "a significant number of candidates seem to have entered examination without solid algebra bases." Question 14 (b) of paper 1, 2019: " $simplify\ 8f - 2e + 10f - 12e$" the examiner reported that "this question was poorly answered as some learners added unlike terms and a common wrong answer of $18f - 10e\ was\ abtained$" (Ministry of Education Arts and Culture, 2019, p. 400). This notion was repeated in 2020, where they indicated that the majority of learners who wrote examinations for the NSSC Ordinary level in that year experienced difficulties in solving algebraic questions, and algebraic questions ''were very poorly solved''. Question 15 of paper 1, 2020: "simplify $\frac{2x^2+x}{2x^2-7x-4}$", the examiner reported that "this question was poorly answered as learners failed to factorise the quadratic expressions" while question 17(write $2x^2 + 4x - 5\ in\ the\ form\ a(x+p)^2 + 9$) the examiner reported that majority of learners proved that they lack the knowledge of expressing quadratic trinomials expressions in the given form (Ministry of Education Arts and Culture, 2020, p. 396).

In 2021, examiners also pointed out that an "emphasis must be put on understanding algebraic manipulation, application of fractions, and direct numbers." Question 5 (b) and (c) of paper 2, 2021: "$simplify\ \frac{x^2-x-12}{x^2+6x+9}\ and\ \frac{y}{(y-1)^2} - \frac{y}{y-1}$"the examiner reported that candidates failed to factorize trinomial and lack of cancelation rules was displayed. While for question (c) most candidates could not find the LCD of $(y-1)^2\ and\ (y-1)$. They further added that multiplying of the signs proved to be a challenge" (Ministry of Education Arts and Culture, 2020, p. 416). Most learners in Namibia have problems identifying like and unlike terms, and thus, learners end up adding or subtracting even terms that are unlike as found by Iilonga and Ogbonnaya (2023). They further stated that learners could not apply addition and subtraction law to solve algebraic fractions despite being taught how to add and subtract fractions since the primary school phase.

Studies conducted by Kaufilua (2019) on ‘The Effects of Problem-Based Learning on Grade 11 Learners’ Retention of Algebraic Knowledge’ and by Kleopas (2020) on ‘Challenges of Teaching Mathematical Problem-Solving Skills' have revealed that learners failed to relate algebra to everyday life situations. Furthermore, Kaufilua (2019) assumed that learners lack basic knowledge in algebra, and that many learners seem to have developed negative attitudes towards algebra as well as to have developed fears in learning algebra. Such concerns clearly show the need for interventions to improve on learners’ understanding of and applicability of algebra in everyday life. It becomes important that instructional strategies used in teaching algebra should be structured in such a way that learning problems and difficulties as faced by learners can be identified early and assistance is provided. It is of outmost importance to develop learners’ problem-solving skills to solve algebraic equations that would assist them in arriving at the correct answer.

Moreover, Ojaleye et al. (2018) revealed that learners frequently commit procedural and conceptual errors while solving algebraic-related problems. Procedural errors occur when learners incorrectly apply rules or algorithms, such as the formula or step-by-step procedure for solving a mathematics problem (Brown et al., 2016). Additionally, Ncube (2016) conducted a study that focused on the “analysis of errors made by learners in simplifying algebraic expressions." This study found that learners made procedural errors such as “misapplication of rules, misuse of distributive property, and substituting letters by numbers” (Ncube, 2016, pp. 24–26). During procedural errors, learners tend to apply rules inappropriately to situations or vice versa, that is, incorrectly apply known rules to appropriate situations.

On the other hand, conceptual errors occur when learners hold misconceptions of lack of understanding of the underlying principles and ideas related to given algebraic problems. These are errors, such as “lack of understanding of like and unlike terms and misinterpretation of symbolic notation, where learners are expected to expand algebraic fractions and the coefficient is invisible” (Ncube, 2016, p. 24 - 25). Learners tend to assume that the invisible coefficient is zero instead of one, deducing that the whole term becomes zero. However, Ying et al. (2020) pointed out that if the learning difficulties are detected early, the teachers could easily assist learners by designing learning activities that will minimise algebra learning difficulties. Therefore, early detection of the learning difficulties that learners experience is of essential importance.

In addition, Odumosu and Olisama (2018) alluded that imperative variables, such as the pedagogy, have an influence on learners’ performance in algebra. In general, teaching requires well-constructed instructional strategies to deliver the content in a well-understandable and crafted way to learners. Berta and Hoffmann (2020) argued that the teacher-centred instructional strategies, such as drilling and teaching for memorising, used in schools are not in compliance with the present expectations of society. Those teacher-centred instructional strategies do not allow learners to be critical thinkers, as they make learners dependent and less self-esteemed by only believing what the teacher taught is correct, and that is the only method Berta and Hoffmann (2020). Today’s world expects learners to think outside the box, especially in technical areas.

Berta and Hoffmann (2020) further explained that currently young people, who are adaptive to the rapidly changing world and innovative in the labour markets, are indeed in demand in societies to teach others basic problem-solving skills. Some of these skills could be learnt in schools through various constructivists' instructional strategies, such as "to solve problems, to look for sources needed for the task, to cooperate with each other in groups, to express their opinion, as well as to focus on a given problem" (Berta & Hoffmann, 2020, p. 270). Additionally, Iyambo (2010) proposed that it is crucial that teachers use instructional strategies that allow learners to participate actively in their own learning and create social settings in which learners learn problem-solving skills through interactions with their fellows.

Nambira (2015) is of the assumption that teachers' knowledge of selecting appropriate instructional strategies (pedagogical content knowledge, PCK) has an influence on how teachers plan their instructions. Competency in lesson preparation requires in-depth cognition, that is, essential in translating the concepts of the algebra content and breaking them into minute components that are cooperated into the lesson preparation. The National Curriculum for Basic Education (2016) defined competence as a "combination of knowledge with understanding, specific objectives and skills, and the will to use them appropriately" (p. 36). Luis et al. (2023) indicated that "throughout the history of mathematics teaching, many pedagogues have tried to decipher the best forms of teaching that allow human beings to acquire the skills that come from mathematics" (p. 519). Some teachers insist that some concepts are too abstract and could not necessarily be aligned to learners' everyday experiences and hence impeded their use of everyday contexts such as irrational numbers and algebra (Kapenda et al., 2015).

The main purpose of the study was to identify learning difficulties as experienced by the NSSCO learners in solving algebraic equations and the exemplary instructional strategies used by teachers in assisting learners to have a better understanding of algebraic equations

1.3 Statement of the problem

Erongo Regional Council (2020; 2021) statistics for NSSCO examination indicated that the average of candidates who wrote examination in Erongo region was below 40% as compared to other educational regions. The Omaruru Circuit ranked the lowest, with 15% in 2020 and 20% in 2021, respectively, of other circuits in the region. Department of National Examinations and Assessment (DNEA, 2022 and 202) of NSSCO examinations highlighted difficulties learners have in algebra. For example, Mathematic Paper 1 (2022), the examiner reported indicated that “the algebraic manipulation to make the subject of the formula was poorly executed" and that “many learners failed to find the correct solution due to algebraic mistakes” (DNEA, Ministry of Education, Arts and Culture, 2022, p. 485). In addition, it was also reported that many candidates struggled to “correctly move variable terms by division while many candidates rather could subtract variables from both sides of the equation” in paper 2 as well as failing to “understand division of algebraic fractions” (DNEA, Ministry of Education, Arts and Culture, 2022, p. 580). It was also emphasized that learners did not know when to move the term and when to divide with which term (DNEA, Ministry of Education, Arts and Culture, 2022, p. 580).

Analysis of the Mathematics Examination Paper 1, examiner’s report indicated that many candidates “confused simplifying with factorising” (2023, p. 576) and they further observed that many learners lack basic algebraic knowledge, more specifically, with like and unlike terms, as well as on how and when to use ‘like’ and ‘unlike’ terms in solving algebraic equations (DNEA, Ministry of Education, Arts and Culture, 2023). In analysing examination paper 2, the examiners reported that the majority of candidates “wrote wrong quadratic formula, and incorrect substitution into the formula." They further stated that the candidate failed to realise that after factoring out the common factor, the bracket contains a difference between two squares that needs to be factorised further” (p. 580). These are some but clear evidence showing learning difficulties that many learners in Namibia have in solving algebraic equations.

Moreover, Rompas et al. (2023) stated that adding or subtracting unlike terms, algebraic expansion errors, not adding or subtracting exponents during multiplying of variables, being unable to pick a common factor during factorization, and being unable to translate words into mathematical problems are some of the common mistakes learners make when solving algebraic equations, which were evident in solving questions about arithmetic operation in algebraic form.

Iilonga and Ogbonnaya (2023) and Jupri (2016) indicate that learners seem to have problems in translating words problems into algebraic equations, as some learners used different variables to represent the same scenarios as ages, thus making the question more complicated, while some learners could not identify the unknown and the known variables in the question. Furthermore, Pramesti and Retnawati (2019) highlighted that learners experience difficulties such as understanding the problem/concept, understanding the meaning of variables, and operating the algebraic form. Understanding the problem is one of the difficulties many learners have, which is caused by a lack of reading and understanding information (Pramesti and Retnawati, 2019).

Difficulties experienced by learners in understanding the concept or problem is the difficulty in determining the initial idea in solving the algebraic problem so they can get the answer (Rompas et al., 2023). Terms used to ask algebraic questions such as factorise, expand, simplify, and solve are literally what learners' do not understand (Rompas et al., 2023). These difficulties are indications that learners need help in understanding the properties of applying a formula.

In addition, Pramesti and Retnawati (2019) further stated that learners' conceptual misconception is on the concept of variable meaning, which is the basic conceptual error that learners make in algebra. Learners tend to make mistakes when working with variables, and they operate them the same even if they are not like terms. The misconceptions on certain algebraic questions are influenced by previous concepts learnt and influence other related concepts. Difficulties in operating algebraic forms are related to procedural capabilities. Procedural errors are operating rules in algebraic form that also influence learners into failing to solve algebraic equations (Pramesti & Retnawati, 2019).

Most learners seem to make mistakes during equation problem solving and therefore, fail to achieve algebraic basic competences such as, solving simple linear equations, simultaneous linear equations, quadratic equations given in factorised form, multiplying a monomial by a polynomial, using brackets and extracting common factors, as well as expanding algebraic expressions in mathematics. All of these seem to indicate that learners have algebraic learning difficulties.

1.4 Research Questions

The main research question is stated below as:

What exemplary instructional strategies do teachers need to employ during instructions to assist the NSSC Ordinary level learners with learning difficulties in solving Algebraic equations? The following are the sub-questions:

1.4.1 What learning difficulties do NSSCO learners have in solving Algebraic equations?

1.4.2 What exemplary instructional strategies do teachers use when assisting NSSCO learners with learning difficulties in solving Algebraic equations?

1.4.3 Is there a significant difference between teachers' exemplary instructional strategies in assisting NSSCO learners with learning difficulties in solving Algebraic equations?

1.4.4 Hypothesis

1.4.4.1 Null Hypothesis: There is no significant difference between teachers' exemplary instructional strategies in assisting NSSCO learners with learning difficulties in solving Algebraic equations.

1.4.4.2 Alternative Hypothesis: There is significant difference between teachers' exemplary instructional strategies in assisting NSSCO learners with learning difficulties in solving Algebraic equations.

1.5. Significance of the study

The study provides teachers with the identified instructional strategies that proofs to be exemplary to best assist learners with specific learning difficulties, such as, procedural and/or conceptual problems in solving algebraic equations. The study also provides NSSCO learners with possible learning difficulties in solving algebraic equations and thereby hoping to avoid these hiccups when learning algebra.

1. 6 Limitations of the study

The limitations of the study are constraints that influenced the findings of the study (Sezuni, 2022). Naturally, learners hold a virulent hatred against mathematics (Marchis, 2011) and due to that, it was observed that some NSSCO learners dissociated themselves from taking part in the study. The researcher came to realise that out of 66 learners from both classes that were selected for the study at both schools, 36 learners did not turn up for both pre-tests and post-tests. Possible reasons that can be given is that, generally, most learners do not like learning mathematics, and some decided not to write the tests although the researcher explained the purpose of the study and what benefits learners they might gain by taking part in the study. On the contrary, those who took part in the study showed little interest and they too did not answer most of the questions, and others left a lot of questions unanswered. Therefore, this leads to fewer numbers of learners being tested in both pre-tests and post-test than planned.

1.7 Delimitation of the study

This study was strictly conducted in Omaruru circuit of the Erongo region since NSSCO learners performed drastically low for the past three years within the region. Therefore, the findings are not generalizable to other senior secondary schools in Namibia or other educational regions, nor to other content in the NSSCO level mathematics syllabus, as it is a case study of specific schools and learners.

1.8 Definitions of operational terms

In this study, there are certain terms that might prevent the researcher and the readers from comprehending the research. These terms are explained below, and they are defined in the context of this study.

Algebra is referred to as the branch of Mathematics which deals with variables (letters) and numbers connected by arithmetical operations and in addition, this is the study that brings alphabetic letters in the world of Mathematics and connects them with numbers (D'Emiljo, 2012).

Algebraic equations is an algebraic statement that two expressions are equal, and it consists of a left-hand side expression, rights-hand-side expression and an equal sign between them (D'Emiljo, 2012).

Instructional Strategy is referred to as the technique used by teachers to help learners become independent, strategic learners as well as assisting them in order to comprehend the content that the teacher is presenting. (Baker et al., 2015).

Learning problems is referred to learning conditions that can cause an individual to experience problems in a classroom such as inability to learn Mathematics (Mahmud et al., 2020).

Learning difficulty is referred to those difficulties in Mathematics that learners have because of one or a combination of several reasons such as biological differences, limited opportunities due to socioeconomic status, limited access to proper support, Mathematics content, lack of adequate prior knowledge, teaching practices and ineffective instructions and materials (Hamukwaya & Haser, 2021).

Procedural error referred to mistakes that occurs or errors that learners commit after failing to apply the correct rules or algorithms when solving Algebraic questions due to misunderstanding of an Algebraic step, rules or procedures such as failing to change the sign when crossing over to the other side of an equal sign or inequality, failing to add or subtract like terms (Delastri & Lolang, 2023).

Conceptual error is referred to error caused by misconceptions or wrong understanding of principles and ideas underlying an Algebraic concept or when a learner used the wrong logic to answer an Algebraic question (Delastri & Lolang, 2023).

Misunderstanding is referred to when learners do not understand or interpret an Algebraic question appropriately and thus resulting in them answering such mathematics questions wrongly (Ojose, 2015).

Misconceptions are misunderstanding and misinterpretations based on incorrect meaning of Algebraic rules, such as when learners base their work on wrong rules for adding fractions with like and unlike denominator (Ojose, 2015).

Traditional method refers to teaching methods that often follow linear or modular learning, which is highly directed, controlled, and program-centred approach as directed by the teacher, wherein learners complete the given activities without developing the critical reasoning skills (Luke, 2021).

1.9 Summary

This chapter presented the background of the study, the statement of the problem, research questions, and the significance of the study. Furthermore, the limitations, delimitations, and definitions of chapters were also presented. All those chapter one concepts were presented based on the exemplary instructional strategies that were used by teachers to teach learners with learning difficulties in algebra in Omaruru Circuit, Erongo Educational Region. On the contrary, the following chapter presents the theoretical framework and the literature review based on the research questions.

CHAPTER TWO

LITERATURE REVIEW AND THEORETICAL FRAMEWORK

2.1 Introduction

In this chapter, the researcher describes the following information: (i) the theoretical framework that is used as a backdrop for the current study, the extant literature about (ii) the learners' common misconceptions and understanding, and (iii) exemplary instructional strategies used by teachers in assisting learners with learning problems/difficulties in mathematics.

2.2 Theoretical Framework

This study is anchored on the Cognitive Constructivism Theory that was established by Jean Piaget in 1953. Cognitive constructivism theory is an approach that emphasises that 'knowledge is not passively received but actively built up spirally by the cognisant subject' (Powell & Kalina, 2009, p. 242). They further stated that knowing is active, that it is individual and personal, and that it is based on previously constructed knowledge, as it will be illustrated in the interests and experiences of the learners (Powell & Kalina, 2009). Similarly, Syarifuddin & Atweh (2022) also highlighted that Cognitive Constructivism Theory suggests that understandings of individuals are constructed from their unique web of previous concepts and their current subjective experiences. This means that during learning, learners have to apply their past knowledge in order to tackle the difficulties that are faced and understand new information. Each learner interprets experiences and information in the light of their existing knowledge, their stages of cognitive development, their cultural background, and their personal history. According to Vintere (2018) and Powell & Kalina (2009), it is not possible to provide new information to a learner and expect such information to be immediately understood or applied, but rather learners themselves build new knowledge by assimilating and accommodating the acquired knowledge (Syarifuddin & Atweh, 2022).

Assimilation is a process conducted by individuals in inserting new stimulus into existing schema which occurs on one's mental activity (Faizah et al., 2022). when new content is being assimilated into the existing schema (Rahmat et al., 2019). Individuals may sometimes be prompted to understand new content which may further be fostered for individuals to relate the new content to their schema While accommodation is considered as a process of adjusting the schema to build a new scheme based on the existing schema or structures, and such adjustment may be influenced by the object being transformed (Faizah et al., 2022; Syarifuddin & Atweh, 2022). For example, learners may acquire new ideas and try to fit these to their old or existing ways of thinking. Assimilation and accommodation processes allow learners to learn algebra by relating new algebra content to previous learnt algebra content as well as adjusting old algebra content acquired to accommodate the new content when connections are made. For example, learners who already have developed appropriate cognitive structure necessary to solve fractions numbers will have some of the structures necessary to solve algebraic fractions problems. However, such learners may need to modify their existing schema/structures to accommodate the newly information to solve the new type of problem.

Despite knowledge assimilation and accommodation, the most important single factor influencing learning is the learners' existing or prior knowledge (Niss, 2018).

Thus, the reason as to why most learners perform poorly in algebraic equations could be the lack of previously acquired knowledge in algebra.

Moreover, cognitive constructivist theory also deals with personal expression of knowledge and creativity as well as inquiry, exploration, and self-sufficiency information (Syarifuddin & Atweh, 2022), and it focuses on the mental processes of learners during instruction rather than observable behaviour. Leonard (2002) added that cognitive psychologists such as Piaget, Vvgotsky, Brunner, Bransford, and Marzano have mentioned the importance of activities whereby learners, with the teacher's support throughout in the process of exploration and discovery, analysis, and integration, build their own knowledge.

The key concept of cognitive constructivist theory is that learning is not about gaining or transferring knowledge but rather an active process of creating knowledge (Powell & Kalina, 2009). An active approach, especially in the teaching of mathematics, grounded in the learners' own discovery and building of knowledge through various activities and through processes and procedures that the activities stimulate, is the one that enables the internalisation of concepts such as variables, expressions, terms, constants, and coefficients, as well as principles and laws such as multiplication law, division law, addition law, subtraction laws, and power law. Thus, the sustainability and transfer value of knowledge result in a more passive uptake of ready-made knowledge, which helps learners reduce their learning difficulties (Terwel et al., 2009).

The following constructivist principles provide a general framework of Cognitive Constructivism Theory and its relevance to teaching and learning:

i. *"Learners bring distinctive prior knowledge and beliefs to learning situation. Every learner has different ability to learn,*

ii. *Learning is both an active and reflective process.*

iii. *Knowledge is constructed uniquely and individually, in multiple ways, through a variety of tools, resources and context.*

iv. *Learning is developmental based and individuals make sense of the world through assimilation, accommodation, or rejection of new information.*

v. *Learning is internally controlled or mediated by learners themselves''* (Syarifuddin & Atweh, 2022, p. 8).

Cognitive constructivism believes that each learner brings different beliefs and prior knowledge, and they have different learning abilities (Syarifuddin & Atweh, 2022). Hence, instructions need to be employed to accommodate all learners to assimilate and accommodate new knowledge as well as to challenge learners' prior knowledge that might restrict learners to adjust to new knowledge. Teachers should pose thought-provoking questions that prompt learners to share their existing knowledge for the teachers to be able to assist them to make connections with the new information and enable them to comprehend the new content. In doing so, the teachers might reduce learners' learning difficulties challenging their prior or existing knowledge. Posing questions before and during instructions allows learners to be actively involved in what they are learning as an active process of discovery. In addition, the role of teachers in facilitating learning, is to provide the necessary resources and guiding learners as they attempt to assimilate new knowledge to the old and modify the old to accommodate the new (Ganapathy et al., 2017).

It is important for teachers to employ a variety of teaching methods and aids during instruction (Syarifuddin & Atweh, 2022). For instance, teachers may utilize various teaching materials to explain algebra in the real world's context as well as to allow learners to challenge learners' understanding of certain algebraic concepts. Cognitive constructivism instructional strategies allow learners to face up their limitations in their existing knowledge and accept the need to modify or abandon existing schema/structures or beliefs. It is important for learners to develop a kind of internal drive in their mental processes and not just depend on external rewards and punishment which could be unlikely to be insufficient (Syarifuddin & Atweh, 2022).

Furthermore, it is important for teachers to be aware of the different aspects, types, and levels of knowledge of skills. Kozel et al. (2023) identified conceptual skills and procedural skills in learning mathematics. In Mathematics, conceptual skills that focus on the understanding of concepts and facts while procedural skills focus on the effective mastery of procedures, rules, laws, and problem skills (Kozel et al., 2023; Al-Mutawah, 2019). It is believed that conceptual skills assist learners to avoid making errors when solving algebraic problems as these provide learners with appropriate skills on certain concepts, facts, and rules while, procedural skills involve the application of various rules and concepts in addressing novel situations to integrate both conceptual and procedural skills l (Al-Mutawah, 2019; Kozel et al., 2023). It is, therefore, the teachers' responsibilities to decide which conceptual and procedural skills need to be prioritised in different situations and know how to address different mathematics concept during instructions (Cotic, 2012, as cited in Kozel et al., 2023).

Additionally, Faizah et al, (2022) ground the notion of cognitive constructivism on the knowledge that the human brain does not directly replicate the external world. However, the human brain constructs its experience and life through an intellectual and passionate process within a social context, forming individual ideas and concepts

(Vintere, 2018). Providing primary learning in a manner that simplifies secondary learning necessitates creating a comprehensive understanding of the connections between earlier and later concepts. We begin teaching algebra to learners in the primary phase. As they move to the secondary phase, the content just gets advanced that is, previous algebra basics competencies are a build-up of new algebra basics competencies as a learner proceeds to the upper grades. Therefore, primary mathematics teachers should make sure that learners have mastered algebra content before they progress to the secondary phase. By doing so, this might reduce the build-up of algebraic learning difficulties, which might also lead to learners having attitudes towards mathematics.

To enhance the learning of algebraic concepts, teachers must understand the learners' prior knowledge to enable them to tailor instructions accordingly. Activating learners' prior knowledge is a very essential aspect that can assists learners to see the connections between previous learning, and builds on what they already know, and in such doing, providing a framework for learners to better understand new information (Vintere, 2018). As already said in the constructivist principles which provide a framework of Cognitive Constructivism Theory, the existing conceptual knowledge that learners have in algebra could significantly influence their conceptual learning. It is of most important for teachers to take into account learners' prior knowledge and by so doing, such notion assist teachers in selecting the most effective instructional strategies that aid learners in understanding the new content. Teachers should teach algebra in a way that empowers learners to think and construct their knowledge, ignites their interest in learning, and helps them understand the subject content they study.

2.3 Misconceptions and Misunderstandings

Misconceptions are common in the teaching and learning environment just as any other phenomena despite of how well each subject content is taught. A misconception could be the misapplication of a rule, an over- or under-generalisation or an alternative conception of the situation (McDonald, 2010). The next section describes learners' misconception in mathematics.

2.3.1 Learner's Common Misconceptions

Misconceptions in mathematics refer to individuals' internal mathematics ideas that are not in agreement with what iss accepted in mathematics (Ubi & Odiong, 2018). According to Elisha (2014) in algebra learners' thinking about variable has indicated that learner' conceptions are inadequate, particularly with respect to the use of literal symbols. He further stated that learners' misconceptions mostly include viewing variable as abbreviations or label. Generally, learners bring along their perceptions, beliefs, and thoughts about the world around them into the classroom. Learners learn new algebraic content as per the constructivism learning theory by building new knowledge as based on their previous algebraic perceptions, beliefs, and thoughts. For example, for learners to carry out factorisation of quadratic expressions, they first need to add this new knowledge to already learnt knowledge of factorisation and finding common factors in algebraic expressions. These two aspects are related, and learners need to understand that factorising by finding a common factor comes first before they learn factorising quadratic expressions. Failure to master such a procedure might lead to misconceptions in factorising algebraic expressions.

Misconception exists in different ways depending on how learners make sense of the kind of instruction they receive (Ojose, 2015). For instance, the rules of adding fractions with like and unlike denominators are quite different. When learners transition from adding fractions with numbers alone to adding algebraic fractions that include both numbers and variables, they must comprehend the various scenarios and make necessary adjustments accordingly. Failure to clarify the difference between those scenarios might lead to algebraic misconceptions as expressed by Ojose (2015).

Many learners seem to experience a lack of mathematics' technical vocabulary, and this may lead to the language to contribute to mis conceptions (Nasser, 2015. Fr example, learners may be unable to differentiate between various action verbs such as to "expand," "factorize," and "solve". Language problems as mentioned above, may lead failure to interpret algebraic questions correctly and end up applying the wrong rule or formula to solve a given problem due to misunderstanding the meaning of the given action verb (Nasser, 2015). When questions prompt learners to factorize, they might end up in either simplifying the problem or attempt to solve it in a different way. Similarly, Arnawa et al. (2019) noted that reading or interpreting mathematical questions can be challenging due to language barriers.

'Signs' and 'operations' are also aspects that give learners learning problems. Pournara et al. (2016) conducted a study on the use of 'signs' and 'operations in mathematics and found out that some learners struggle with simplifying algebraic expressions, primarily due to their lack of attention to signs and operations. Such a problem is found to be common with questions that include " subtraction and negatives" (p. 6). They further found out that learners are primarily focusing on letters and numbers, and ignore the negative signs.

Similarly, Booth et al. (2017) highlighted that learners who possess an incorrect or incomplete understanding of the negative signs often have misconceptions when dealing with negative signs and numbers in algebraic equations and expressions. Learners tent to use incorrect strategies when solving such algebraic equations due to the abstract nature of negativity concept especially when learners' concept understanding is moving from arithmetic to algebraic thinking. Furthermore, they mentioned that learners tend to link the negative sign with the binary operations of subtraction. For instance, found that "majorities of learners may easily interpret the meaning of negative nine in the expression n-9 but not -9 as presented alone (Vlassis, 2002), as cited in Booth et al., 2017, p. 3).

According to Booth et al. (2017), learners often mistakenly assume that a letter in a number sentence represents an actual object or a label, instead of understanding algebraic letters as varying quantities or specific values. For instance, they further said that when learners are asked to write a mathematical expression to represent the following phrase: "six times as many as teachers, the most common error is 6s = t. Learners with misconceptions believe that's' is a label for learners rather than a variable representing the number of learners" (p. 4). Alternatively, some learners tent to ignore the variable in an expression (Booth et al., 2017). For example, when asked to solve (n+5) + 4, some learners incorrectly respond with 9, ignoring the 'n' in the given equation above (p. 4).

Another misconception is that learners treat 'unlike terms' as if these are 'like terms'. Some learners are found to simplify, for example "b + 2 as being equals 2b, which shows that learners apply their prior knowledge of adding numbers and fail to identify x as a variable term" (Pournara et al., 2016, p. 5

Egodawatte (2011) identified some learners' misconceptions in algebra. He found out that learners' lack of understanding the unitary concept when multiplying a variable with a constant in algebra, such as: "the price of shirts is 's' dollars, find out the price of 3 shirts". Learners considered 's' as the label for shirts rather than the unit price of a shirt, and at the same time considered 's' as the item price" (p. 88). Furthermore, incorrect cross multiplication was found to be another misconception. When learners multiply an algebraic fraction with a letter, for example, " $x(\frac{a}{b})$, learners multiplied both the denominator and numerator of the fraction by the letter $(\frac{ax}{bx})$" (p. 90). Many a time, learners may have assumed that there is no denominator to the letter. Often, this seems to happen to learners when they find no visible denominator. Learners seem to have difficulties in realizing that a single letter can be represented by an algebraic fraction by making the denominator as 1. Because of this lack of understanding, learners tend to assume that both the denominator and the numerator of the fraction should be multiplied by the letter.

Egodawatte (2011), also found out an 'invalid distribution' to be another misconception in algebra (. Invalid distribution is considered to be a kind of misuse of the distributive property in algebra which states that, for example, $a (b + c) = ab +ac$. This property implies that learners can either do the addition first, and then multiplication, or multiplication first and then addition. However, when unlike terms are inside the brackets, it becomes impossible to add the variables. Learners must multiply the brackets by the letter outside of the parenthesis. The distributive property helps learners to simplify algebraic quantities by allowing them to replace terms containing parenthesis with equivalent terms without the parenthesis anymore. When learners have misconceptions, they "mistakenly distribute exponentiation over addition, such as $(A + B)^2 = A^2 + B^2$ where learners proceed further to oversimplify the answer to $A^2 B^2 or\ (AB)^2$ " (p. 91).

Another misconception is when learners do not make a connection between algebra to other mathematical concepts. Many learners fail to perceive algebra as having a connection to other previously learned mathematics subject content. However, McDonald (2010) assumed that true algebra integrates the use of variables into tasks and activities in order to make mathematics versatile. Therefore, making learners aware of the similarities and connecting the new information to previously learnt mathematical concepts might help reduce some of the misconceptions and thus eliminate their learning difficulties in algebra.

In summary, the following misconceptions were identified from reviewed literature:

- Learners' misconceptions in interpreting Algebraic questions correctly,
- Misconceptions of learners not paying attention to signs and operations,
- Misconceptions in dealing with the variables,
- misconceptions in understanding the unitary concept,
- Learners' treating unlike terms as if they are like terms,
- Incorrect cross multiplication and
- Invalid distribution.

2.3.2 Learner's common learning difficulties

Learning difficulties are referred to be those difficulties faced by learners during instructions and/or those conditions that impact on learners' ability to gain knowledge and skills at the same rate as his or her counterparts (Awasthi, 2022). to Cooney, as cited in Sugiarti and Retnawat (2019), learners' learning difficulties arise from two types of knowledge, the knowledge of concepts and knowledge of principles. They further considered the knowledge of concepts refers to the knowledge that learners need in order to tackle each mathematics question by using techniques and procedures that are appropriate while the knowledge of principles is considered as the knowledge of rules, formulas, and operations that learners' need to follow correctly to solve given problems.

Generally, learners' knowledge of the algebraic conceptions is based on their knowledge of variables, constants, expressions, equations, linear equations, and quadratic equations, as well as algebraic principles, such as the multiplication rule, addition rule, and power rule Sugiarti and Retnawat (2019) consider the use of interrelated principles (expanding, simplifying, and solving algebraic expressions) involve applying the multiplication rule and the collection of like terms (either the addition or subtraction rule) are capital to to enable learners to solve algebraic equations appropriately.

Moreover, Samuel, et al. (2016), Ying et al., (2020), Kojii et al. (2016), and Widodo (2017) identified the following difficulties:

- Translating words expressions to algebraic expressions or vice versa
- Misinterpreting the meaning of algebraic expressions
- Misunderstanding and incorrectly substituting signs
- Using wrong algebraic concepts to solve common mathematics problems
- Incorrect grouping of terms
- Incorrect manipulation of positive and negative signs
- Unable to formulate algebraic equations from given situations
- Unable to master mathematical language

- Unable to translate and interpret word statements into symbolic algebraic conjectures reading and
- Unable to interpret symbolic algebraic statements and
- Unable to follow steps to solve algebraic equations.

Rompas et al. (2023) conducted a study on types of difficulties experienced by learners and focused on the mastery of three elements in learning Mathematics:

i. *difficulty in conceptual understanding such as variables, coefficient, constant,*

ii. *development of skill difficulty, and*

iii. *problem solving skills difficulty (p. 2698).*

They further, assume that learners have difficulties in understanding concept to determine preliminary clues in understanding the properties or conditions of applying a formula when answering questions. For example, learners might be asked to expand or simplify algebraic expressions, but they seem to have no clues on what to do and/or fail to apply the correct steps or rules in solving the equation and/or expression. Additionally, Darmayanti et al. (2023) also identified the following learning difficulties as:

(i) *"learner's failure to understand the concept of the "equal to" sign/symbol in solving algebraic equations. Many learners tend to get confused when solving algebraic equations which involves variable, and*

(ii) *learners tend to use arithmetic procedure and neglect the variables that are not like terms thus cannot be added together. For example,* *in the following expression* ($2x^2 + 3x$), *learners only focus on the operation which is addition* (p. 2639)".

For example, most learners seem to have the tendency of thinking that variables cannot be added or subtracted like numbers for example, $3x - x$, cannot be subtracted. However, some learners could subtract these variables and add the power despite that the two terms were subtracted but not multiplied. Pramesti and Retnawati (2019) and Awasthi (2022) noted that some of the learning difficulties are because of a lack of understanding the problem in general, difficulties in making generalisations, a lack of understanding arithmetic operations, a lack of knowledge about the process of solving problems, a lack of applying various steps in solving problems, a lack of conceptual understanding of the meaning of variables, as well as a lack of knowledge on the operation of algebraic forms. They revealed that many learners seemed to have difficulties understanding the meaning of algebraic questions, which then made it difficult for them to interpret the questions, while other learners seemed to find it difficult to solve algebraic problems due to a lack of procedural ability. Further, they stated that the lack of creative thinking, reasoning problems, and inability to understand the meaning of variables in algebra are some of the problems experienced in developing appropriate competencies in algebra (Pramesti and Retnawati, 2019; Awasthi, 2022).

2.4 Teachers' Exemplary Instructional Practices

Instructional strategies are the "fundamental principles of teaching that allow for significant learning and indirect teaching that strengthens inner motivation to be involved in active learning, the building up of knowledge, and the creation of understanding'' (Algani & Alhaija, 2021, p. 506). Instructional strategies are methods used by experienced teachers in order to achieve the basic competencies. It is the duty of the teacher to select the most appropriate teaching strategy in scaffolding learners to acquire both mathematical concepts and procedural skills and help learners not only to remember algebraic concepts but to apply acquired conceptual and procedural knowledge to solving algebraic equations and other practical problems in their daily life. Such knowledge is needed for learners to be productive citizens of the 21st century.

Instructional strategy is one of the vibrant mechanisms in teacher-learner collaboration. Instructional method is "the process by which the teacher meets the learner at their level, starting with their interest and problems, and then establishing conditions that will enable them to proceed with their set goals in the most possibly effective manner" (Zhang 2020, p. 3). Teachers apply different instructional strategies in the classroom to ensure that learners understand what they are taught. The selection of instructional strategies depends on the content to be presented.

Garzon and Bautista (2018) indicated that there is a huge struggle amongst mathematics teachers in identifying and employing the best teaching strategy on algebraic equations as well as on how to learn algebra, whereas Niss (2018) added that many learners around the world, at any education level, experience severe problems in learning mathematics, and learners in the Erongo educational region are no exception. Additionally, Darmayanti et al. (2023) mentioned that "many instructions fail to make connections between abstract algebraic concept expressions and the learners' daily experiences of algebraic concepts in ways that are meaningful to them culturally" (p. 2637).

Therefore, teachers' exemplary instructional strategies could also be a contributing factor towards learners learning difficulties. Khan et al. (2016) revealed that what people learn depends not only on what they are taught but also on how they are taught, their development level, and their interests and experiences. Jameel and Ali (2016) highlighted that teachers' classroom practices are one of the most popular factors contributing to learners' poor performances in mathematics. The aspect of teaching approaches being the leading factors is again emphasised by Fan (2012), as cited in Jameel and Ali (2016), whose study specified that much that teachers' practices are determined by many aspects like class size, learning environment, and learners' needs; teaching approaches remain a profound determinant. Moreover, Tashtoush et al. (2022) added that instructional strategies employed by teachers during instructions are closely related to learners' attitudes towards mathematics, which can influence learners' performances in mathematics.

Likewise, during instructions, teachers could employ different strategies of teaching while learners might lack basic skills in understanding what they are taught, have no prior experiences, simply not be ready at their level of development, or even lack interest in what they are taught. Another study also found that teaching strategies could positively affect the performance of learners while others did not (Mosimege & Winnaar, 2021). Thus, this study attempts to identify learners' learning difficulties in solving algebraic equations in relation to how teachers employ exemplary instructional strategies to assist these learners to solve algebraic equations.

Learners are required to have both practical and cognitive experiences throughout the field of mathematics education (Abramovich et al., 2019). Practical experiences equip learners with the ability to make connections and solve more complex mathematics problems, giving them a strong sense of numbers as well as a stronger mathematical foundation which is needed to deeply understand and apply such knowledge to new situations. Whereas cognitive experiences provide learners with the ability to manipulate and retain numbers, which is a key to solving mathematical equations, and provide insights into how mathematics works inside the brain as well as how it is connected with other subjects as well.

Teachers have indispensable responsibilities to diversify alongside modern instructional strategies (Tashtoush et al., 2022). During instruction, the teacher, as the facilitator and mentor, needs to untangle the subject content to the learners in a clear and understandable way. In doing so, teachers are required to plan their instructions thoroughly by including constructivism instruction strategies so that they can assist learners who experience learning difficulties during instructions. However, teachers seem not to involve learners in creative thinking and participation in creative parts of activities but rather deal with quick coverage of content and rote memorisation (Dagar & Yadav, 2016). Further, they stated that during the teaching and learning process, most of the instructional strategies used by teachers in algebra seem to remain unilateral in such a way that teachers' only use teacher-centered teaching methods, which are only ideal for gifted learners.

There are several exemplary constructivist teaching strategies where successful learning has occurred and/or has been evaluated to be excellent to assist learners in actively learning some science concepts (Ableser, 2012). He further added that during such instructions, learners tend to achieve learning outcomes and have excellent achievements and successfully complete the syllabus. Teachers tend to employ such exemplary teaching strategies to guide learners with learning difficulties and by so doing, challenge learners' prior conceptions in their mind so that they realize their mistakes.

Exemplary instructional strategies involve a high level of the teachers' subject content knowledge (SCK), pedagogical content knowledge (PCK), skills and professional dispositions. Strong SCK and PCK, although necessary for teaching excellence, these are not sufficient to assist learners with learning difficulties. It is expected from teachers to have a high-level commitment to their duties as mediator of knowledge to assist their learners to acquire knowledge (Ableser, 2012). Therefore, exemplary instructional strategies involve more than just content and pedagogical expertise and require a commitment to support successful learning for all learners.

Ableser (2011, 2012) identified ten principles which are used in exemplary instructional strategies that include:

i. *Focusing on educational values, beliefs and philosophies that support teaching all to learners.*
ii. *Focusing on learning that is relevant, purposeful, meaningful and meaning making.*
iii. *Focusing on learning and learning outcomes.*
iv. *Focusing on facilitation of learning.*
v. *Focusing on learner active engagement by providing a range of techniques and authentic learning opportunities to meet the needs, interests and styles of all learners.*
vi. *Focusing on assessing and supporting learners' strengths, interests, needs and learning styles to ensure their success.*
vii. *Demonstrating respect, fairness and care of learners' development and learning to ensure success.*
viii. *Focusing on creating a community of learners.*
ix. *Focusing on intentional and purposeful curriculum planning.*
x. *Focusing on engaging in reflective practices.*

In some instances, it becomes a necessity to lay down the basic content knowledge in solving algebraic equations. Since mathematics is experimental and inductive in nature (Baig, 2015), teachers should therefore make use of constructivists' instructional strategies that promote learners' activeness in knowledge development and challenging learners' prior knowledge during instructions. Various exemplary instructional constructivist strategies are described in the paragraphs that follow.

2.4.1 Discovery method

Discovery method refers to an instruction strategy that involves the creation of learning situations to involve learners in acquiring knowledge or skills actively and independently in finding a concept or theory (Supriadi et al., 2019). The discovery method is based on learners' psychology, who want to discover something independently (Baig, 2015) under the guidance of the teacher to master the intended subject content or skills through several practices. The method allows teachers to gain knowledge about their learners, in finding out what their learners know, and how learners think during instruction so that teachers can become more effective mediator of meaning making (Rahmayanti, 2021).

Discovery method also encourages learners to work in groups to discuss given activities. The instructional strategy provides an opportunity for learners to discuss various concepts actively and have a better understanding of rules and steps that need to be applied during algebraic equations. Furthermore, Rahmayanti (2021) states that the strategy allows room for learners to understand and care about what they are learning as they are challenged to think about what they are doing as well as discovering how things are connected by doing, experimenting, and participating in their own learning. The method could be enhanced by creating opportunities where learners can work in groups or individually discover solutions by themselves and/or under the supervision of the teacher through discussions. Here, teachers act as facilitators and mentors, to promote the principles of the constructivist theory during instructions. According to Supriadi et al. (2019), the amount of assistance provided by the teacher does not affect learner's ability to make their own discoveries as based on teachers' instructions, in the form of a guiding statement.

However, it should not be forgotten that the discovery instructional method is time-consuming and it is not ideal for lengthy syllabi (Baig, 2015). If proper guidance is not provided, learners may be confused and discouraged to work on activities that seem to be meaningless. Thus, it is of outmost important for teachers to provide appropriate and clear instructions to avoid confusion during instructions.

2.4.2 Problem solving method

Behlol et al. (2018) considers the problem-solving method to be a process through which solutions are provided to problems in applying higher-order cognitive operations to manipulate and comprehend concepts. The method may have various features that allows itself to be a constructivist method, by provides room to learners (i) to carry out class activities such as to solve algebraic word problems; (ii) to apply acquired skills under the guidance of the teacher before proceeding to work individually at home; (iii) to apply their prior knowledge when working on tasks individually; (iv) to allow learners to have discussions in assisting each other; and (v) to provide room to promote collaboration in class; (vi) to promote an active role in learning; and (vii) to give homework and assignment activities to encourage learners to discover and learn independently while taking ownership of their learning.

In addition, the method involves an act of analyzing problems, first by breaking up the problem into small segments, and then moving to solve the given problem by providing the solution while it is considered to be time-consuming and requires learners to have access to a lot of teaching resources, such as the internet, since textbooks do not give enough help (Baig, 2015).To use this method in the Namibian context, could be problematic as many of the schools are not connected to Internet to fill up gaps in textbooks.

2.4.3 Classroom discussions method

Classroom discussion teaching method refers to a process where learners either work in groups or where the whole class is involved in a discussion under the guidance of the teacher to perform an activity based on the content presented (Syarifuddin & Atweh, 2022). During such classroom discussions, teachers may act as a facilitator to lead inter-group discussions or whole class discussions to reflect on the tasks they are working on as espoused in the principles of constructivist theory. Such opportunities could allow learners to utilize measurement, manipulation, or construct knowledge to find relationships amongst various concepts that they might use symbolically later to be represented as a law or a rule).

Consequently, classroom discussions involve mutual responses amongst learners (Alsaqqaf & Jawad, 2018) and assist learners to utilise the structure of algebraic representations and formulas, to help them not only to memorize the formulas but understand these fully. Furthermore, the use of either whole-class discussions or small-group work allows the teacher to interact with learners so that learners could understand the presented concepts better (Aydisheh and Gharibi, 2015).

Additionally, in small groups, learners can challenge others' ideas and may work together to reach agreement on their differences, which later may be shared with the entire class. Through shared discussions, learners may come to the realization of their shortcomings ang strengths and change or correct their existing misconceptions they have (Shah, 2019). It is through such platforms that learners are also given opportunities to provide solutions to their questions, and to understand their mistakes and remember the steps involved in providing solutions in the future (Star, Foegan et al., 2015). Similarly, Alsaqqaf and Jawad (2018) and Kaufilua (2019) additionally claimed that classroom discussions promote long-term retention of algebraic concepts and develop further learners' thinking skills.

2.4.4 Drill teaching method

The drill teaching method refers to a process where an action is performed repeatedly with an aim of perfecting a skill permanently (Laleye & Ogunboyede, 2023). Generally, teachers use this method to drill and practice a problem-training action where learners are accustomed to solving various problems (Jannati et al., 2023, p. 14). In learning mathematics itself, drill and practice strategy is important to memorize basic content which is chosen from various alternatives for long-time memory storage (Jannati et al., 2023; Star et al., 2015).

Laleye and Ogunboyede (2023) claimed that drill and practice strategy is often used during instructions to develop certain habits in learners through memorization. However, due to lack of time, teachers tend to avoid developing such habits and this may lead to learning difficulties. Learning mathematics, more specifically, algebraic expressions and equations, drill and practice strategy seems to be ideal to assist learners to master difficult algebraic concepts and procedural skills. For example, many times, teachers show multiple procedural steps in solving a given problem and they tend not to provide opportunities to drill and practice these procedural skills. The purpose is to allow learners to learn a habit of choosing steps/skills that should be completed first and which one last, such as multiplying or dividing or adding or subtracting or grouping. Although the strategy seems to be a memorization process, it allows learners to be actively engaged in developing a habit of analyzing a problem before choosing steps to follow to provide a solution to the problem as espoused by the principles of the constructivist theory. Laleye and Ogunboyede (2023), suggest the use of the ‘drill and practice instructional software packages’ to practice developing repetitive actions and thereby assist learners with learning problems in algebra. Such software package offers determined opportunities to practice content and procedural skills and to provide immediate diagnostic feedback.

2.4.5 Repetition method

Repetition method refers to a process of teaching where the teacher or educator repeats certain concepts for information taught previously over and over to make learners acquire and master concepts better without wasting time (Cardinoh & Cruz, 2020) and is similar to the ‘drill and practice method. Although considered to be a teacher-centred method, and to promote ‘rote memorization’ of concepts, it opens up room for learners to hear the content or to perform the skills many times to activate and put the said content in the long-term memory. Such strategies are traditionally applied with the purpose of acquiring basic information passively by simply listening and modeling.

2.4.6 Activity-based method

Activity-based method refers to a process where the subject content is taught indirectly by performing various activities to inspire learners to apply their innovative thoughts, information and their minds to solve problems (Noreen & Rana, 2019; Abramovich et al., 2019). For example, a teacher may give various algebraic equations and expressions activities to learners to solve during guided instructions, provide scaffolding if necessary, and provide feedback immediately. Such activity-based strategies are considered to be constructivist in nature and are believed to yield high learners' academic performance as compared to many traditional methods (Kaufilua, 2019; Magen-Nagar, 2016; Dagar & Yadav, 2016). Through the activity-based teaching method, teachers may provide learners with activities using geometric figures or algebraic tiles to allow learners to think critically and actively as espoused by the principles of constructivist theory (Garzon & Bautista, 2018).

2.4.7 Problem-based method

A problem-based method (PBM) refers to a process of instruction that integrates teaching and learning over an ill-structured real-life problem which could be researched by learners to provide possible solutions to such a problem under the guidance of the teacher (Ojaleye et al., 2018; Kaufilua, 2019; Ojaleye, et al., 2018; Behlol et al., 2018). Although it is time-consuming, it is a process of teaching and involves active participation from the side of the learners to provide solutions to the said ill-structured problem (Kaufilua, 2019), and that is what constructivist theory believes learning should be. Furthermore, PBM develops learners' and keeps learners busy with the existing activity/project thereby enjoying learning, overcoming their doubts, fostering the spirit of learning and improving their enthusiasm, intensity and skills to improve the overall understanding of the subject matter (Supriadi et al., 2019; Kaufilua, 2019). Teachers used this method by giving learners activities to work on and allowing them to present their findings verbally in class. Through presentations, the teacher can highlight the common mistakes made by learners and address them, and this might reduce learners' learning difficulties.

2.4.8 Cooperative method

Berta and Hoffman (2020) define cooperative method as a process that involves the management of teaching activities in which the attainment of information, (intellectual knowledge, societal motives and skills, and learning motives) is agreed in time and alike in the learners' standing as they are working to solve a problem with a common aim and learners stand a chance of being challenged by others on their ideas. Cooperative method provides opportunities for learners with different backgrounds and different circumstances to work together and depend on each other to solve common prearranged tasks (Algani & Alhaija, 2021; Berta & Hoffman, 2020) and this is believed to allow learners to uncover patterns or compare and connect strategies and steps that help them to strengthen and deepen their algebraic solving skills, thereby, reducing their learning difficulties.

Cooperative method is believed to instill a sense of collaboration among learners as each member of the group is seen as being an expert, and all learners are responsible for learning every part of the content through discussions, and this helps them boost their interest in mathematics, which might reduce their learning difficulties as well (Cardinoh & Ortega-Dela, 2020).

2.4.9 Peer teaching

Peer teaching is a method of instruction that involves learners teaching other learners, a system of instruction in which learners help each other and learn by teaching (Abraham & Ikushagba, 2025). Paul (2016) as cited in Abraham and Ikushagba (2025) defines Peer teaching as a teaching method that partners with students to help one another learn materials, reinforce skills or practice a learned task. The pairing of higher and lower-achieving learners is intended so that learners gain knowledge from each other through practice and reinforcement. In peer teaching, the teacher serves as a facilitator (Abraham & Ikushagba, 2025).

In the study conducted by Wang et al (2025) and, Abraham and Ikushagba (2025) revealed that peer teaching method is more effective when compared with lecture method in the improvement of learners' performance. That is to say, those learners in teaching method achieved better than those taught with Lecture method. This finding is true because, peer teaching leverages technology and focuses on the student-teacher relationship to enhance independence, engagement, and performance. It provides a big convenience for the course to achieve its target by combing the face-to-face interaction in traditional learning and time; place and material richness provided (Wang et al, 2025).

In summary, constructivist approaches to teaching and learning have great potential to improve learners' academic performance and to ameliorate learning difficulties. It, however, be remembered that different teaching approaches have a huge effect on learners' performance (Iyamuremye et al., 2021; Star et al., 2015). In addition, the way the subject content is presented and explained to learners may have a huge effect on learners' understanding and teachers are urged to use different instructional strategies such as classroom presentations, group discussions, drills, and practice methods since these proved to be efficient in teaching mathematics as well as to connect algebraic content with abstract thought in developing conceptual understandings as well as to real, concrete things in their everyday life (Algani, 2019). However, Baig (2015) is warning against time-consuming teaching strategies and might negatively affect slow teachers and therefore, most of these strategies require teachers with higher understanding in algebra content as well as computer literacy.

It is not possible to include all these teaching strategies in the current study and therefore, only peer teaching, group discussions and algebraic videos are considered as exemplary instructional strategies which will be used to assist learners with algebraic learning difficulties. Peer teaching was chosen to provide opportunities to learners with different backgrounds and different circumstances and allow them to work together and assist each other in solving algebraic problems and allow room for learners to assist each other rather than the teacher. Some learners are shy and only seem to open up comfortably to their peers (Algani & Alhaija). Group discussion was chosen to allow learners to work in groups to perform an activity (Syarifuddin & Atweh, 2022) and lastly, videos were used to allow learners to listen to someone else explaining algebraic steps, rules, formula and laws apart from the teacher.

2.5 Traditional method

Traditional teaching method is referred to be as a process that takes place in a linear or modular way and is highly directed, controlled and program-centred approach to teaching and learning as it mostly directed and controlled by the teacher wherein learners mostly are passive observers with little or no focus on developing critical reasoning skills (Luke, 2021). For example, the teacher prepares content to be delivered through speeches to learners through lectures and other teacher-centred instructions. (Baig, 2015). This method allows teachers to give verbally clear explanations and understanding of algebraic concepts such as variables, constants, coefficients, terms, expressions, and equations, which gives learners a better understanding when solving algebraic equations.

Furthermore, it is helpful for giving training of complex skills and procedures such as completing squares steps and solving algebraic fractions, which seems to be challenging to many learners (NIED, Ministry of Education, 2022). In the traditional method/conventional/lecture method, the teacher stands before the learners and deliver their lecture while learners listen, take notes and remained passive throughout the teaching and learning (Abraham & Ikushagba, 2025).

Despite the fact that traditional methods allow teachers to explain the concepts and principles during instruction and remain a part of all other instructional strategies it leads to learners memorizing the steps and methods without understanding and it does not promote active engagement of learners thereby, promoting low thinking skills (Baig, 2015; Luke, 2021).

2.6 Summary

The researcher, firstly, described the theoretical framework that underpins this study. Secondly, literature focusing on learners' misconceptions, learning difficulties and exemplary instructions strategies used by teachers to assist learners with learning difficulties were reviewed. Lastly, different constructivist instructional strategies were reviewed and described. The next chapter deals with the research methodology.

CHAPTER THREE

RESEARCH METHODOLOGY

3.1 Introduction

The researcher outlines the research approaches, designs, methods, and data analysis techniques in this chapter under the following headings: Research design, Research population, Research samples and their sampling techniques. Research instruments, the procedure for collecting data, the analysis of data, and the ethical considerations were also identified and described.

3.2 Research Approach

Research approach refers to the “use of evidence-based procedures, protocols, and guidelines that provide the tools and framework for conducting a research study” (Majid, 2018, p. 1). This study employed convergent mixed research approach, QUAN and QUAL approach, using a quasi-experimental design and an existential phenomenology design as the study intends to involve the collection of different but complementary phenomena to provide a comprehensive analysis of the research problem by merges quantitative and qualitative data (Razali et al., 2019. In this study the researcher collected both quantitative and qualitative data at the same time, analysed the data separately and try to look for differences from two sources of data.

A quasi-experimental design was used to find out whether there was a significant difference between the learning difficulties and teaching strategies employed to assist learners with learning difficulties in algebra. The design allows the researcher to draw more clear-cut conclusions as to the differences between variables under study

(Marsden & Torgerson, 2012). According to Shadish et al. (2002), as cited by Ojaleye et al. (2018), "one inherent advantage of a quasi-experimental design is that it is easier to set up than true experimental designs, and it lacks randomisation of subject treatment conditions" (p. 489) and involves pre-testing and post-testing of non-randomised and intact experimental and control groups in real classroom settings (Creswell, 2013). Learners are already assigned to class groups, and there was no need to construct artificial situations as learners were studied in a natural school setting. This was done to make sure that the systematic arrangement and normal running of schools were not interfered with or disturbed.

An existential phenomenology design was also used to examine the presence of a specific phenomenon, the personal experiences of participants to explain a particular phenomenon, and the conscious experiences of participants, their daily lives, and social actions (Turhan, 2019). It is ideally used to provide meanings attributed to phenomena and, thus, used to identify exemplary instructional strategies used by teachers to teach learners with learning difficulties in algebra.

3.3 Population of the study

Population of the study refers to the targeted group that the study intends to focus on (Majid, 2018). There are three circuits in the Erongo educational region, with a total of 29 senior secondary schools that offer mathematics at the Elementary to junior secondary level, which are grades Grade 1 to 11. Omaruru Circuits is the largest, with seven senior secondary schools. The total population for the study was 29 senior secondary school in Erongo Region, seven senior secondary schools in Omaruru circuit, as well as 68 mathematics teachers, and an approximation of 14 000 senior secondary school mathematics learners in seven senior secondary schools in

Omaruru circuit in Erongo Educational region. Out of seven senior secondary in Omaruru Circuit, only two were sampled for the study. The purpose of involving teachers currently teaching the NSSCO learners was to find out from the mathematics teachers which instructional strategies they use when assisting learners with algebraic learning difficulties. The NSSCO learners were involved to find out the specific learning difficulties they experience in algebra.

3.4 Sample and Sampling procedures

The sample consisted of two senior secondary schools in the Erongo education region out of seven schools using a purposive sampling strategy. The study focused only on learners currently enrolled for the NSSCO syllabus that is Grade 10 -11 and their teachers at the selected school sites. The researcher chooses NSSCO learners since Algebra is taught in the NSSCO syllabus and examined at the end of their Grade 11 academic year.

Sampling is the "process of selecting a statistically representative sample of individuals from the population of interest, and it is an important tool for research because the population of interest usually consists of too many individuals for any research project to include as all the participants" (Majid, 2018, p. 3). The schools were selected purposefully from the seven public schools based on criteria used in identifying prosperous the mathematics teachers at identified schools of interest if they have met the following criteria:

i. The mathematic teachers must at least have three or more years of teaching experience.
ii. The teachers must at least have been trained, teacher qualification degree.

iii. The teachers should have taught at the same school for at least three or more years.

The two selected school sites, School SA and School SB, were used in the study, School SA was used as a control group and School SB was used as an experimental group. At School SB interventions were used through the employment of different exemplary instructional strategies to assist learners with learning difficulties while at School SA, learners were assisted using the traditional teaching strategies.

Furthermore, at School SA, one class group was randomly selected out of three class groups while at SB one class group was randomly selected out of two class groups. At School SA, the control class group consists of 35 learners while at School SB the experimental class group B consists of 31 learners. Altogether, there were 66 learners. However, only 21 learners from School SA, and 15 from School SB turned up to write both pre-tests and post-tests, which made up a total of 36 participants.

3.5 Research Instruments

The study employed Pre-tests and Post-tests which were designed to identify learning difficulties experienced by learners in solving Algebraic equations and expressions for 40 minutes each as well as the exemplary instructional strategies used by teachers to assist learners with algebraic learning difficulties. The researcher used traditional method such as lecture and exemplary instructional strategies such as peer teaching, classroom discussions and algebraic videos at the Controlled group school site (SA). to gather information from the participants.

3.6 Pilot Study

A pilot study preceded the main research study to establish the instruments' content validity and reliability, and the researcher use a small sample of the population of interest, having similar characteristics to the described sample that is to be involved in the actual study (In, 2017). The researcher carried out a pilot study in order to find out whether the research instruments used were reliable and truthful and to provide appropriate information to the research questions. The Mathematic Teacher Observation Schedule and pre and post tests were checked by the two knowledgeable people before the pilot study. The pre and post-test was piloted with one grade 10 class group at another senior secondary school, School site C (SC) in Omaruru circuit. The selected grade 10 class group was divided into two groups, class group C1 and class group C2.

The researcher used class group C1 as the control group and class group C2 as an experimental group. Class group C1 was taught using the traditional teaching method while class group C2 was taught using exemplary instructional strategies to improve the designed instruments. The results from the two selected class groups were recorded to see if they provided enough information as required by the research questions and subsequently, to identify mistakes in the content of the research instruments. During the piloting period, the Mathematic Teacher Observation Schedule (MTOS) was developed by the researcher and administered identified teachers, Teacher 1C (T1C) and Teacher 2C (T2C) at the pilot school site. According to Gleason et al. (2015) Mathematics Classroom Observation Schedule is an instrument designed to measure the degree of alignment of the mathematics classroom and the mathematics teacher with various standards set out by the researcher. The MTOS was used to provide information based on how the teachers presented their lessons and how learners engaged in instructions (Gleason et al., 2015).

The MTOS involves algebraic items that proofed to challenge learners in different ways as identified in literature reviewed. The researcher recorded instructional strategies used by two observed teachers who assisted learners with different algebraic learning difficulties. The researcher observed the selected teachers for seven days. The two teachers observed at School site C used the following exemplary instructional strategies in the table below:

Algebraic learners'' learning difficulties	**Exemplary instructional strategies used**	
	T1C	**T2C**
3.6.1 Difficulty with identifying like and unlike terms, adding or subtracting of unlike terms	Lecture session	Lecture session
3.6.2 Difficulties in removing the brackets by using the distributive law	Group discussion session	Peer teaching session
3.6.3 Difficulties to apply the division law of indices	One-on-one assistance session	Group discussion session
3.6.4 Difficulties in finding the highest common factor (HCF)	Drill and practice	Lecture session
3.6.6 Difficulties in using addition and subtraction signs when grouping terms	Peer teaching session	Repetition session
3.6.7 Difficulties in making the denominator the same	Group discussion session	Peer teaching session
3.6.8 Difficulties to reciprocate the second term	Watch video session	One-on-One discussion session
3.6.9 Difficulties to change the direction the inequalities after division	Repetition session	Peer teaching session
3.6.10 Difficulties to translate verbal phrases into Algebraic/Mathematical expressions	Lecture session	Lecture session
3.6.11 Difficulties with meaning of key words	Lecture session	Group discussion session

Table 3.6: Exemplary Instructional strategies used by T1C and T2C

The pilot study was also conducted to ensure that the pre and post-test and the MTOS were appropriate, relevant and provide correct information to the research questions. Two content experts checked the pre and post-tests items for content validity as well as the MTOS checklist prior to their use in the research study. The Mathematic Teacher Observation Schedule (MTOS) was also improved and revised during the pilot study. The MTOS is composed of the type of learning difficulties experienced by learners under each Algebraic basic competency as well as the instructional strategies used by the teachers to assist the learners with those Algebraic learning difficulties.

During the pilot study the pre and post tests were found to be too long, and several items were removed to shorten the tests. Eventually, the pre-test consisted of nine questions with a total of 40 marks and post-test consisted of eight questions with a total of 40 marks. Furthermore, the pre and post-test questions were revised on the basis of the responses obtained from those who participated in the pilot study. It was found out that the experimental group requires more time as compared to the control group because they needed more time for group discussions and presentations, and the researcher scheduled three weeks at the experimental school (School B) to provide enough time for learners to finish their work. The school site SC used for a pilot study was not used in the actual study to avoid biasness.

3.7 Data Collection Procedures

After permission was granted, the researcher visited the two selected schools, and interacted with the school's principals in order to make appointments with the prospective participants and invite them to take part in the study. The reason was to establish a good relationship between the researcher and the participants. The researcher provided the participants with the study information and explains the importance of why they have to take part in the study. Learners were also given consent letters to take to their parents (Appendix H). The researcher also informed the participants of what was expected of them and ensure them of confidentiality as no participants or school identities will be used instead the researcher used pseudonyms such as school SA or LA1 and School SB or LB1. Before the study commence, the participants were provided with consent letters to complete and sign in order to obtain their written consents. The main study covered a period of five weeks, two weeks at control School SA, and three weeks at experimental School SB.

3.7.1 Pre-tests and post-tests

Firstly, the two selected class group CA at school SA and class group CB at school SB, wrote a pre-test (Appendix I) which was administered in the afternoon. This was found to be the appropriate time to avoid intervening with the normal running of the school. After the learners wrote the pre-tests at the control School SA as well as at the experimental School SB their scripts were marked, and the marks were recorded and analysed.

Secondly, the intervention phase followed after the pre-tests and lasted for two to three weeks respectively; sixteen (16) lessons of 40 minutes each were presented at School SA while at School SB twenty-four (24) lessons were presented. School SB was accorded more time (24 lessons) compared to School SA (16 lessons) since the exemplary instructional strategies used at School SB which were peer teaching, group discussions and watching algebraic videos require more time as opposed to traditional teaching drill& practice and lecture method used at School SA. The researcher taught the topic of Algebra to the selected class group CA and class group CB for both to NSSCO learners at school SA and school SB, respectively. School SA was taught using the traditional teaching lecture method using the drill & practice and lecture methods at school B using selected exemplary instructional strategies (peer teaching, group discussions as well as watching Algebra videos). All the lessons for class group CA at School SA and class group CB at School SB were based on the basic Algebraic competencies as described in the Mathematics NSSCO level syllabus. The basic competencies that were taught at both schools included:

- *"use letters to express generalised numbers and express basic arithmetic processes algebraically*
- *substitute numbers for words and letters in formulae*
- *construct linear equations and inequalities and simple expressions from given situations*
- *transform simple formulae (linear formulae) and complex formulae (those involving roots, powers, fractions and factorisation) multiply a monomial by a polynomial*
- *use brackets and extract common factors*
- *expand products of algebraic expressions*
- *factorise where possible expressions of the form*
- *solve simple linear equations with one unknown*
- *solve linear inequalities in one unknown"(NIED, Ministry of Education, Arts and Culture, 2018. pp. 17-19)*

Furthermore, at the control School SA the researcher explained each content verbally and worked examples on the chalkboard. Learners were asked to do class activities individually to apply what they learned from the examples. The teacher ensures that all the steps written on the chalkboard are exactly shown in learner's works during class activities. For all the sixteen lessons covered at School SA, the teacher entered the class and start writing notes on the chalkboard. There were no questions posed to find out learners' prior knowledge. Learners were expected to take notes which was meant to provide reinforcement learning as it often takes place in pure lectures. Class activities followed immediately after the teacher's explanations and no homework assignments were given. The teacher was not even walking around to assist learners or have discussions with individual leaners since lectures and drill& practice methods only allow for a one-way communication. In short, teaching in the control school SA was largely teacher-dominated lectures and verbal drill & practice method.

Whereas, at the experimental School SB, the researcher started off by restructuring learners sitting arrangement which was done in semi-circular groups with some spaces between the groups for the teacher to walk around groups in order to engage with each individual members of the groups. This was done to allow learners work in groups and be able to assist each other during class activities. The groups were arranged in such a way that each learner faced the chalkboard. The classroom arrangement was done to support group discussion as one of the exemplary instructional strategies used at School SB.

At the experimental School SB the teacher introduced lessons by posing questions to check learners' prior knowledge, engage learners in the lessons and to set the stage for effective activity-based learning. Based on learner's answers, the teacher linked the basic competencies with learner's prior knowledge to boost learners' interest in the lesson. Learners were invited to explain or demonstrate their solution to the entire class after given time to provide a solution individually or as a group. At the end of each lesson the researcher gave learners worksheets with different items to work on as part of their homework. Learners were also required to present their answers to class on the next lesson/day. Learners were given opportunities to presents certain Algebraic questions on the chalkboard and discussed these with the whole class. This was done to enhance learner's retention of information and thereby enhancing their interest in learning as well as to boost their presentation skills. Class activities were given is such a way that learners could either work on these in pairs or as a group or individually. The teacher paired struggling learners with academically gifted learners to assist those with difficulties during instructions. This allowed learners to engage through debating various questions, group discussions and therefore helping each other.

Additionally, the teacher ensured that teaching was done in such a way that at areas were learners were struggling the teacher explained the content using visual aids such as Algebra tiles or show them Algebraic videos. Lessons were also presented using a projector and drill & practice for learners to master the contents presented to them. Furthermore, learners were given complicated tasks as homework daily that demanded them to visit the library or Internet websites in preparation. They were also allowed to approach the NSSC-AS learners for more information before their presentation in the next lesson.

During the first week at the control School SA, the researcher taught the basic competencies under the following topics: (i) Algebraic Representation and formula (figure 3.7.1.1), and (ii) Algebraic manipulation (Figure 3.7.1.2) while at the experimental School SB, the two units were taught for a period of one week and three days. The Figure 3.7.1.1 and Figure 3.7.1.2 below provide more information about the Algebraic content taught during the intervention period.

TOPIC 6: ALGEBRA	
(a) Algebraic representation and formulae	
• gain further understanding of algebraic representation and use algebraic formulae	• use letters to express generalised numbers and express basic arithmetic processes algebraically • substitute numbers for words and letters in formulae • construct linear equations and inequalities and simple expressions from given situations • transform simple formulae (linear formulae) and complex formulae (those involving roots, powers, fractions and factorisation)

Figure 3.7.1.1: Algebraic representation and formula

(b) Algebraic manipulation	
• know how to manipulate algebraic expressions	• multiply a monomial by a polynomial • use brackets and extract common factors • expand products of algebraic expressions • factorise where possible expressions of the form:

Figure 3.7.1.2: Algebraic manipulation

Furthermore, during the last week of intervention, the researcher taught basic competencies under the following units: Polynomial and Equations and inequalities (Figure 3.7.1.3).

(c) Polynomials	
• know how to manipulate polynomials	• carry out operations of addition, subtraction and multiplication of polynomials • carry out division of a polynomial by a binomial expression, and identify the quotient and the remainder
(d) Equations and inequalities	
• know how to solve equations and inequalities	• solve simple linear equations with one unknown • solve simultaneous linear equations with two unknowns • solve quadratic equations given in factorized form, e.g. $(x-2)(x+3)=0$ • find solutions to quadratic equations by factorising, by formula and by completing the square • solve equations with fractions, e.g. $\frac{x-2}{x-1}=1-\frac{x-8}{x+14}$ • solve simultaneous equations; one linear and one quadratic • solve linear inequalities in one unknown

Figure 3.7.1.3: Equations and inequalities

Thirdly, immediately after the innervations, the selected class group CA at the control School SA and class group CB at the experimental School SB wrote their post-test (See Appendix J) at the same time. The researcher marked the test scripts and recorded the marks and started with data analysis. In addition, the researcher also identified some algebraic learning difficulties as experienced by learners.

3.8 Data Analysis

Data analysis is the process of cleaning, inspecting, transforming, modelling and interpreting collected data with the goal of discovering useful information, to make conclusion and support decision making (Awasthi, 2022). The researcher used two tools to gather data, pre-tests and post-tests.

The data collected by using pre-test and post-test, was analysed and interpreted quantitatively using a two-sample t-test method to compare the performances of learners of the two groups CA and CB by determining their averages/means to identify if these were significantly different. Independent sample t-test was used to analyse the data. According to Kim (2019) independent-samples *t*-test is used to evaluate whether the means of variables differ significantly across to groups. The independent-samples t-test is dichotomous and is used to identify membership in one of two groups using scores.

The exemplary instructional strategies and learning difficulties were descriptively deduced from the systematic analysis of the pre- and post-test results to identify the characteristics, frequencies, trends, and categories at the participants (McCombes, 2023).

3.9 Research Ethics

Prior to the start of the study, the researcher obtained an ethical clearance from the University of Namibia Research Ethics Committee (UREC) (Appendix A). The researcher was also granted permission from the Executive Director of the Ministry of Education, Arts and Culture (Appendix C), Educational Regional Director of Erongo (Appendix E). The researcher also wrote letters to the school principals of selected schools (Appendix F). Consent forms were obtained from teachers, learners (Appendix H), and their parents (Appendix G).

After permission was granted by the two selected schools, the researcher organized meetings to meet with the participants before the commencement of the study. During the meetings the researcher explains the purpose and nature of the study to the participants. Additionally, the researcher also made it clear that the participants' information which included their names, ages, gender will be kept confidential and that their participation in the study was voluntary. Therefore, participants were at liberty to withdraw from the study anytime, at any stage should they not wish to continue taking part in the study.

Furthermore, during the study, the researcher used pseudonyms to ensure the anonymity of the participants and research sites, to protect the participants' identity. The data is stored in a lockable cabinet to which only the researcher has access to it, and the soft copies are stored in a protected folder of the researcher's personal computer. After the study is completed, the researcher destroys all the raw data, soft copies and hard copies after several publications in journals are done as well as the participants' information within five years.

3.10 Summary

In this chapter the researcher described the methodology, population, sample and sampling methods used were described as well as the research instruments, the pilot study the data analysis strategies as well as the ethical considerations for the study. The next chapter focuses on the presentation and discussions of results.

CHAPTER FOUR

DATA PRESENTATION AND ANALYSIS

4.1 Introduction

In this chapter, the researcher presents and discusses the data. The study sought to identify the NSSCO learners' learning difficulties in solving Algebraic equations and teachers' exemplary instructional strategies used to assist learners with learning difficulties. The data was obtained through pre and post-tests from three secondary schools, pilot study groups, control and experimental schools. The results are presented by using research questions of the study as outlined in chapters 1 and the themes that emerged from the data are also arranged as per the research questions.

Participants and schools are given fictitious names to protect them their identities. Learners were coded such as, learner L1A, learner L21A and so forth at school SA (control school) and learners at school SB (experimental school) were coded as L1B to learner, L15B and so forth, respectively. The results are presented according to research questions, starting with the research question one. Two teachers who took part in the study, one was a male teacher of 31 years old and had six years of teaching experience in Mathematics at senior secondary phase. He holds an honours degree in Mathematics and Biology and has taught Mathematics at School C for six years. The teacher also teaches Life science at grade 9 and Biology grade 10. Another teacher was a female teacher of 27 years old with four years of teaching experience in Mathematics and holds an honours degree in Mathematics and Physical Science. She taught Mathematics at School C for four years at senior secondary phase. She also teach Physical science grade 8 and 9.

4.2. Research Question One: What learning difficulties do learners have in solving Algebraic equations?

The following subheading presents the results under research question one and the discussion of results is presented thereof.

4.2.1. Learning difficulties experienced by NSSCO

Here the data for learning difficulties was collected through pre-tests and post-tests with three items that focused on Algebraic representations and formula, four items focused on Algebraic manipulation and two items focused on solving equations and inequalities. Learning difficulties in accordance with the themes that emerged from algebraic representations and formula, algebraic manipulation and solving algebraic equations and inequalities. Figure 4.2.1 below indicates the frequency results of experience difficulties as experienced in algebra as mentioned the given subheadings/themes.

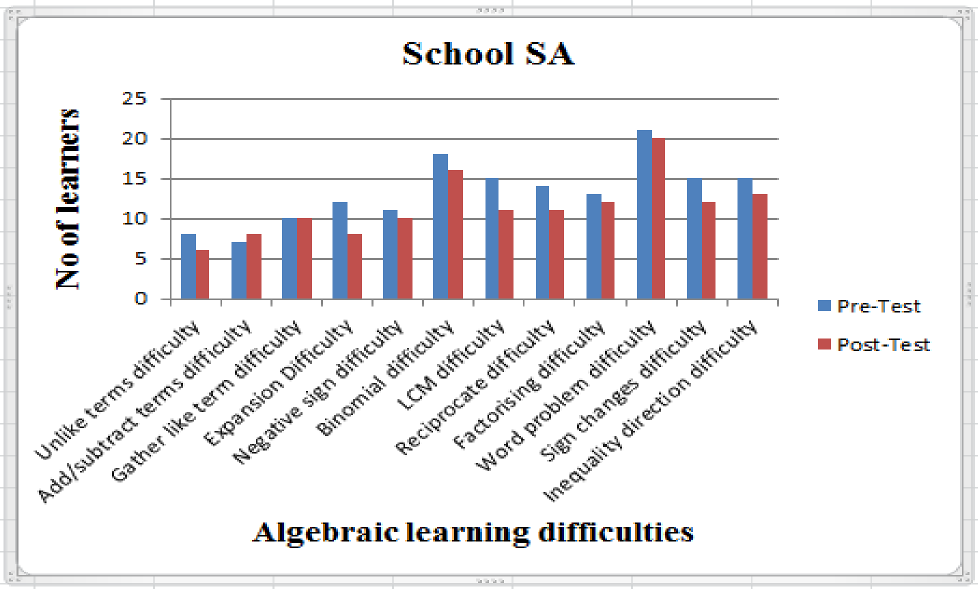


Figure 4.2.1.1: Difficulties in Algebraic equations for School SA

Figure 4.2.1.1 shows different learning difficulties as identified through pre-test and post-test items at School A. Figure 4.2.1.1 shows the first highest learning difficulties experienced by NSSCO learners at school SA to be laying tin solving *word problems difficulties*, (21 out of 21 learners in pre-test and 20 out of 21 learners in post-test), followed by learning difficulties in solving *binomial learning difficulties*, 18 out of 21 learners in the pre-test, and 16 out of 21 learners in the post-test, then difficulties with *sign change* and *inequality direction difficulties*, 15 out of 21 learners and 12 out of 21 learners in pre-and post-test respectively. In addition, NSSCO learners also experienced learning difficulties in solving *reciprocal* (14 out of 21 and 11 out of… 21), *factorizing* (13 out of 21 and 12 out of 21), *expansion* (12 out of 21 and 8 out of 21) *and negative sign* (11 out of 21 and 10 out of 21) difficulties in various test items in both pre- and post-tests. NSSCO learners also experienced learning difficulties the in solving algebraic equations with *correct like terms* (10 out of 21 and 10 out of 21), *'add/subtract terms* (7 out of 21 and 8 out of 21) and the use of *unlike terms* (8 out of 21 and 6 out of 21). Overall, there is also a decline in the learners' learning difficulties when comparing the items of pre-test to that of the post-test results with an exception to *addition and/or subtraction of terms* where there is a slight increase in the post-test to pre-test results. There seems to be more learners who experienced learning difficulties in the pre-test as compared to the post-test results, although the overall difference in the results of the NSSCO learners is not drastically huge.

The frequency results of the pre- and post-test of school SB, experimental school are presented in figure 4.2 below.

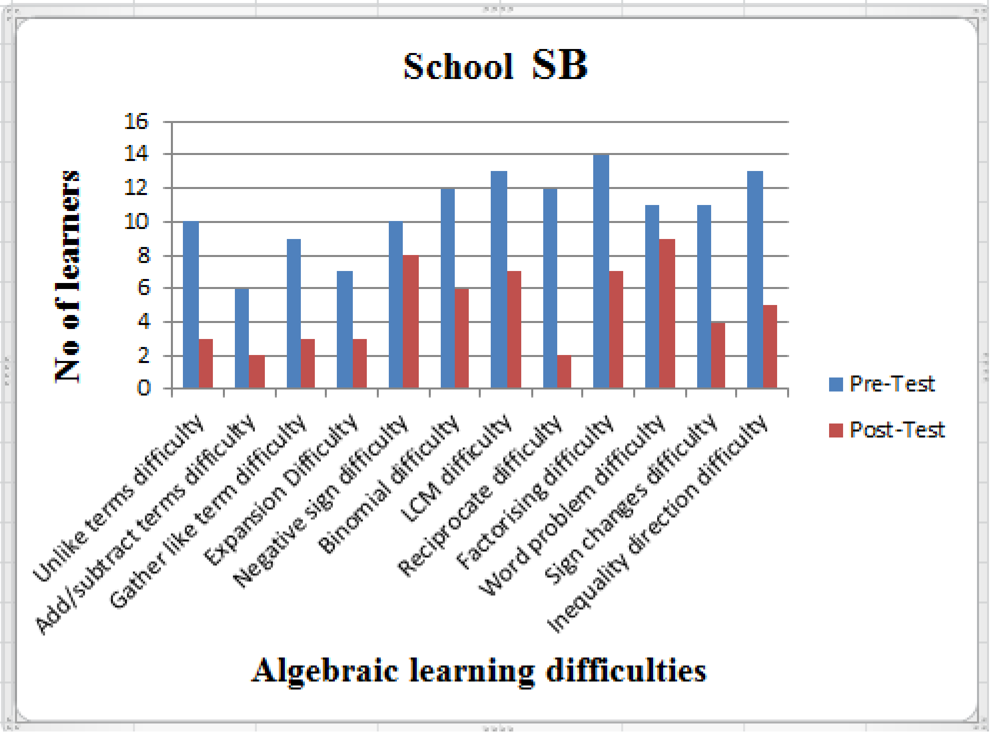


Figure 4.2.1.2: Difficulties in Algebraic equations for School SB

Figure 4.2.1.2 shows that most NSSCO learners experiences a lot of learning difficulties in the pre-test as compared to the post-test School SB. Figure 4.2.1.2 shows that most learners experienced learning difficulties in the first highest difficulties as per Figure 4.2.1.2, with *factorising* (in pre-test, 14 out of 15 learners and in post-test, 7 out of 15 learners) having the highest number of learners. Then, this is followed by learning difficulties in *inequality direction* and *LCM terms*, both with 13 out of 15 learners in pre-test and 5 out of 15 learners and 7 out 15 learners in post-test) and difficulties with *binomial* and *reciprocate terms* both with 12 out of 15 in pre-test and in post-test 6 out of 15 and 2 out of 15 learners. Then, followed by *sign changes* and *word problems* both with 11 out of 15 in pre-test and in post-test 4 out of 15 and 9 out of 15 learners.

On lower side, NSSCO learners experienced difficulties in solving algebraic equations using *negative sign* and *unlike terms* with both 10 out of 15 learners followed by fewer learners experiencing difficulties in solving algebraic equations with *correct like terms*, 9 out of 15 learners and even lesser number of learners experiencing difficulties with *expansion*, 7out of 15 learners and lastly, *add/subtract terms,* 6 out of 15 learners. Interestingly, there is a significant difference in the result of the pre- and the post-test.

4.2.2. Learning difficulties in Algebraic representations and formulae

The following learning difficulties were observed when learners worked on items that focused on Algebraic representations and formulae, such as addition and subtractions of unlike terms, multiplication law during addition and subtractions of polynomials, gather like terms difficulty and word problems difficulty.

4.2.2.1 Learning Difficulty with addition/subtraction of unlike terms

Figure 4.2.2.1 below indicates that some learners added and subtracted unlike terms as illustrated in figure 4.2.2.1.

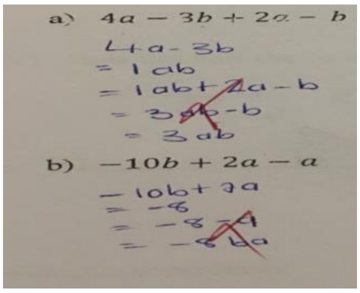


Figure 4.2.2.1: Difficulties in addition/ subtractions of polynomials, unlike terms

Figure 4.2.2.1 shows that some NSSCO learners experienced leaning difficulties in the addition/subtractions of polynomials, 14 out of 21, that is 66.7% at School SA (Control Group), and at School Sb, 13 out of 15 learners, that is, 86.7% at school SB (Experimental Group). For example, Learner L4A subtracted (a) subtracted *4a-3b* and in example (b) added *-10b + 2a* without even noticing that terms are like or unlike terms.

4.2.2.1.1 Discussion

The results show that the NSSCO learners are not different from other learners who experience learning difficulties with Algebraic manipulation solve binomial questions. Kojii et al. (2016) mentioned that learners experience problems in manipulating Algebraic equations such as expansion, solving inequalities, and not to change the direction of the inequality after division with negative coefficient.

4.2.2.2 Understanding of Multiplication law in adding and subtracting polynomials

It is also observed that learners had difficulties in adding and subtracting the powers of terms with the same variables when adding or subtracting like terms. For example, Figure 4.2.2.2 below illustrates this difficulty.

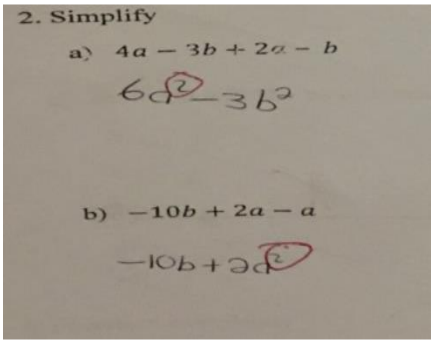


Figure 4.2.2.2: Learning Difficulties in adding and subtracting like and unlike terms

Figure 4.2.2.2 shows that some NSSCO learners experienced difficulties in adding like-terms as indicated in Figure 4.2.2.2 (a) above. For example, 13 out of 21 (61.9%) learners at School SA (Control Group), and 8 out of 15 (53.3%) learners at

School SB, Experimental. For example, Learner L12B added *4a* and *2a, where* the power of these two terms is added to obtain $6a^2$ while in example (b), Learner L12B also subtracted *2a* and – *a,* to obtain $2a^2$.

4.2.2.3 Difficulty in gathering like terms with their signs (operations)

The example provided in Figure 4.2.2.3 illustrates the difficulty experienced by learners in gathering 'like-terms' with their operations.

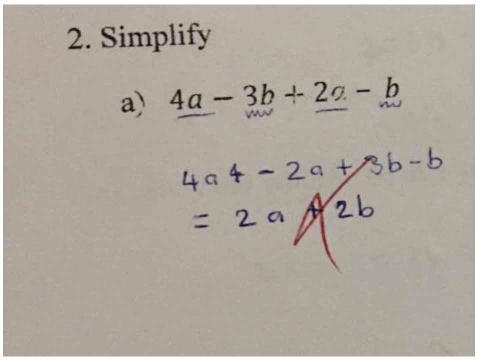


Figure 4.2.2.3: Difficulty in gathering 'like terms' with their operations

Figure 4.2.2.3 shows that some NSSCO learners, 20 out of 21(95.2%) at school SA, (Control Group) and 12 out of 15 (80%) learners at school SB (Experimental Group) moved term *2a* to term *4a* without using the sign that is in front of term *2a* but rather used the sign that is after term *4a*. Thus, making term *2a* negative. For example, Learner L6A moved term -*3a* and term *–b and* failed to consider the negative sign which is in front of term *3b* but rather used the positive sign which is behind term -*3b*. Furthermore, Learner L6A simplified an expression as *4a-2a+3b-b* as the multiplication rule says: *"when moving terms during collection of like terms, move them with the sign that precedes them."*

4.2.2.3.1 Discussion

It was found that many NSSCO Learners added and/or /subtracted unlike terms and seem not to realize that only terms which are alike can be either added or subtracted. Similarly, Sugiarti and Retnawat (2019) who argued that applying multiplication rule and to gather like terms in adding and/or subtracting rule, should be considered as capital to enable learners to solve algebraic equations correctly. If learners do not master and understand the simple process of addition and subtraction rules in Algebra, may lead to learning difficulties. Thus, learners who seem to have no understanding of when to add/ subtract variables without looking at the signs, may have such problems. It seems that NSSCO learners did not have an idea of what like terms and unlike terms are in applying the polynomial rule in learning algebra.

In addition, most NSSCO learners added or subtracted the power of variable instead of applying the multiplication law as found by Egodawatte (2011) that learners have misunderstanding of the unitary concept when multiplying a variable with a constant in algebra and they had no idea when to add/subtract the powers of variables including when multiplying or adding/subtracting of such terms. Samuel, Mulenga and Angel (2016) also highlighted the same problem that many learners with such problems, fail to group terms and manipulate of positive and negative signs appropriately when solving Algebraic equations as seen in what Learner L6A did to simplify this algebraic expression: $4a + 2a - 3b - b = 6a - 4b$. *The learner did not follow the rule that* says: *"when moving terms during collection of like terms, move them with the sign that precedes them"*.

4.2.2.4 Word problem equations in Algebraic equations

Figure 4.2.2.4 illustrates the difficulties that learners experience in analyzing and providing solutions to *word problems* in Algebraic equations.

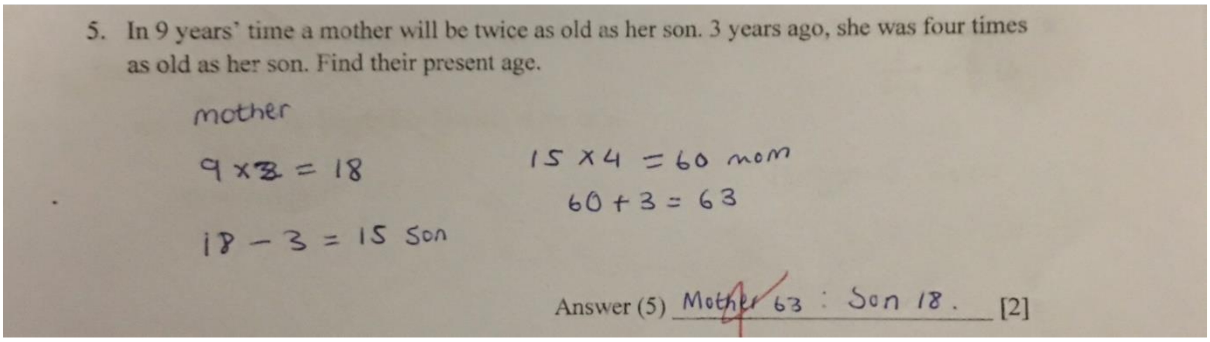


Figure 4.2.2.4.1: Difficulties in translating *word equations* in Algebraic equations

Figure 4.2.2.4.1 shows that some learners, 21out of 21 (100%) at school SA (Control Group) and 11 out of 15 (73.3%) learners at school SB (Experimental Group) have difficulties in translating *word problems* in Algebraic expressions. For example, Learner L2A tried to translate the 'word problem' in an Algebraic expression/equation but only took the numbers and multiplying them together and failed to form any Algebraic term from the given scenario.

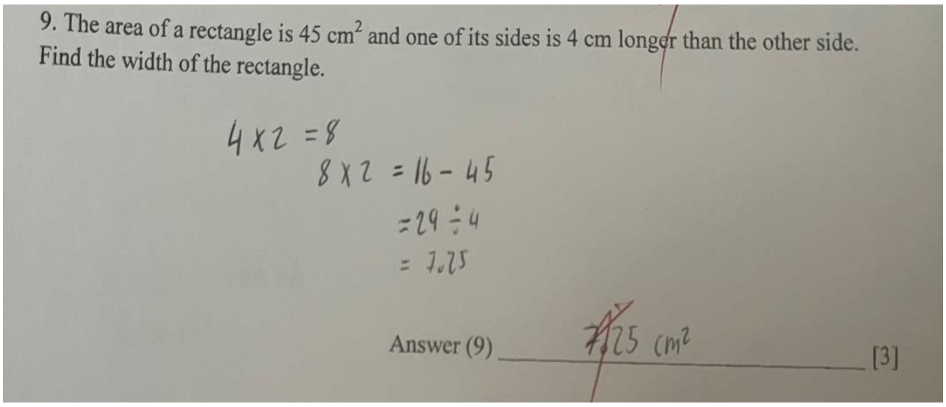


Figure 4.2.2.4.2: Difficulty when translating word equations

4.2.2.4.1 Discussion

Overall impression from the above-identified themes (Algebraic representation and formulae) the results shows that most learners had experienced learning difficulties not only with *word problems* but with *interpretation* of the meaning of algebraic expressions, misunderstanding and/or incorrectly *substituting signs,* and using wrong *algebraic concepts* to solve common Algebra problems. This notion agrees with Samuel, Mulenga and Angel (2016) and Ying et al. (2020) who highlited that learners have similar learning difficulties. In addition, Arnawa, Yerizon and Nita (2019) and, Samuel, Mulenga and Angel (2016) noted that reading and interpreting questions in Mathematics, proof to be problematic due to language challenges. It was found that most learners are challenged and seem not to identify the keywords in each *word problem*. Identification of keywords should be considered as important to translate the problem meaningfully. Translating *word problems* into Algebraic expressions or equations was found to be experienced by most NSSCO learners in the current study as many had problems in using and understanding the official English language.

4.2.3. Learners' difficulties in Algebraic manipulation

The following themes: expansion difficulty, negative sign difficulty, binomial difficulty, LCM difficulty, reciprocal difficulty and factorizing difficulty, were identified.

4.2.3.1 Difficulties with expansion operations

The first theme identified under this category is *expansion difficulty*. Some learners, 20 out of 21(95.3%) in pre and post-test at school SA (Control Group), and 10 out of 15 (66.7%) learners in pre and post at school SB (Experimental Group) experienced difficulties applying the multiplication law during expansion of Algebraic expressions. Some learners only multiplied the first term in the bracket and left out the other term as illustrated in Figure 4.2.3.1 below provides a very good illustration of such a problem.

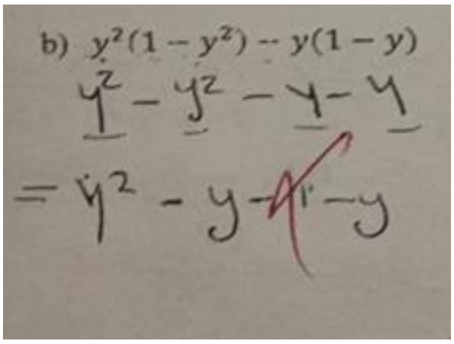


Figure 4.2.3.1.: Difficulties in solving *expansion* problems in Algebraic equations

Figure 4.2.3.1 shows that some learners, 20 out of 21 (95.3%) at school SA (Control Group) and 10 out of 15 (66.7%) learners at school SB (Experimental Group) failed to add the powers of the terms together when the bases of such an equation are the same. For example, Learner L7B did not apply the multiplication law correctly which says: *"when multiplying terms with the same bases we add the power".*

4.2.3.2. Multiplication of all terms in brackets with negative (-) sign

During expansion application, say, when a term that is outside the bracket is negative, many learners ignore or omit the negative sign as shown in Figure 4.2.3.2 below.

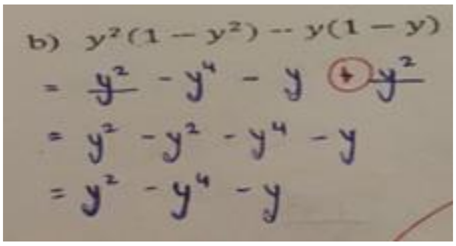


Figure 4.2.3.2: Difficulty in multiplying terms inside brackets with negative (-) sign

Figure 4.2.3.2 indicates that some 11out of 21 (52.4%) learners in pre-test and 10 out of 21 (47.7%) in the post-test at school SA (Control Group) and 10 out of 15 (66.7%) learners in pre-test and 8 out of 21 (38.1%) in post-test at school SB (Experimental

Group). For example, Learner L8A, failed to multiply with the negative sign when expanding for the second bracket.

4.2.3.2.1 Discussion

NSSCO learners were found to have learning difficulties to apply distributive law. The distributive law states that multiplying a number by a group of numbers added together, is the same as doing multiplication with each term separately. Most NSSCO learners were noticed to have learning difficulties in applying multiplication law during expansion of Algebraic equations as found by Egodawatte (2011) who considered invalid distribution to be a misconception in Algebra as many learners mistakenly distributed exponentiation over addition. For example, many learners simplify "$(A + B)^2 = A^2 + B^2$" as to be "$B^2 or\ (AB)^{2}$" $which\ is$ over-simplification.

Another example, NSSCO learners were observed to simply ignore or omit the negative sign in front of the brackets in simplifying expressions and this agrees with Pournara et al. (2016) who argue that many learners do not pay attention to signs and operations in solving algebraic expressions. Such learning difficulties are found to be common when questions involve subtraction and negatives (Pournara, et al., 2016) and learners found to focus only on letter and numbers but not on signs.

4.2.3.3 Difficulty with multiplying binomial with binomial or polynomial

Learners had some difficulties with multiplying when there are two brackets together in an equation, in other words, multiplying a binomial with a binomial or polynomial as illustrated in figure 4.2.3.3.1 below.

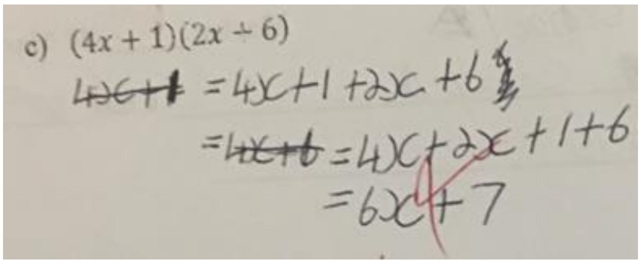


Figure 4.2.3.3.1: Difficulty when multiplying binomial with binomial

Figure 4.2.3.3.1 shows that some learners, 18 out of 21 (85.7%) in the pre-test and 16 out of 21 (76.1%) learners in post-test) failed to apply the rule as expected. For example, learner L3A failed to apply the rule of expanding binomial terms.

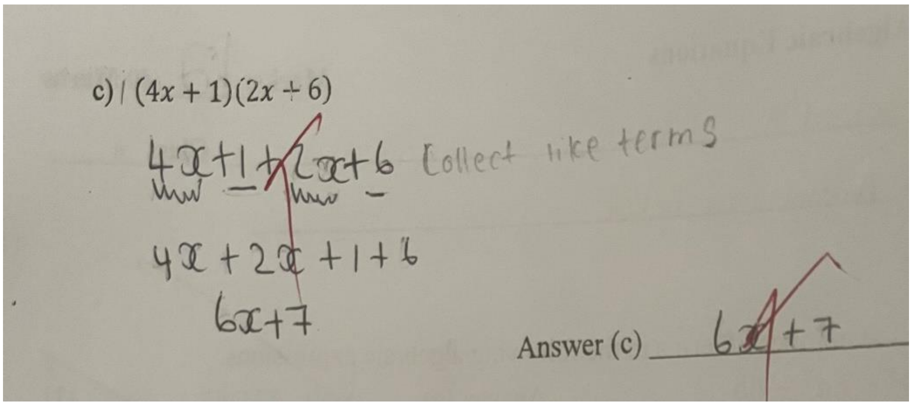


Figure 4.2.3.3.2: Difficulty when expanding binomial terms

4.2.3.4 Difficulty with denominator

The majority of learners 26 out of 42 (61.9%) for both pre- and post-test at school SA (Control Group) and 20 out of 30 (66.7%) learners at school SB (Experimental Group) experienced difficulties making denominators the same. More specifically, NSSCO learners found it hard to find the LCM of the denominators and a considerable number of learners had no idea of how to simplify expressions which involved additions or subtractions of Algebraic fractions. The example provided below in figure 4.2.3.4 illustrates such a difficulty.

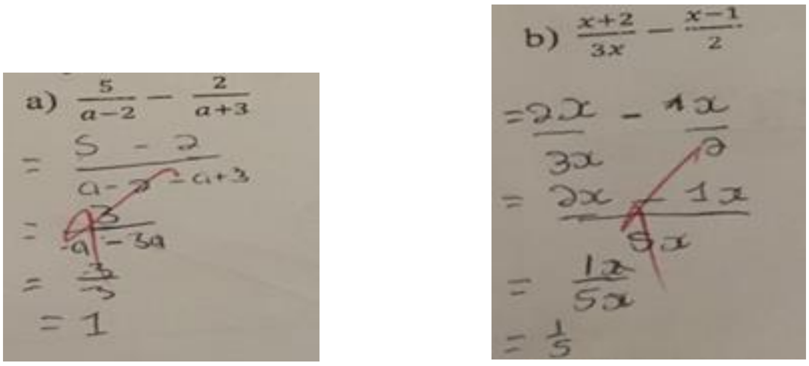


Figure 4.2.3.4: Difficulties in solving Algebraic fraction

Figure 4.2.3.4 shows some of the difficulties that NSSCO learners experienced in solving given algebraic fraction expressions and some learners, 15 out of 21 (71.4%) at school SA (Control Group) and 12 out of 15 (80%) of learners at school SB (Experimental Group). For example, Learners L9A and L11B, in Figure 4.4.5 (a) and (b) above indicate the difficulties experienced in simplifying the numerators and the denominators separately.

4.2.3.5 Difficulty with reciprocating

Another difficulty under Algebraic manipulation was that some learners 25 out of 72 at school SA, and 14 out of 30 (46.7%) learners at school SB (Experimental Group) for both pre- and post-test respectively, failed to reciprocate the second fraction after changing the division sign to multiplication sign. Figure 4.2.3.5 indicates the difficulty in applying reciprocation in the second fraction.

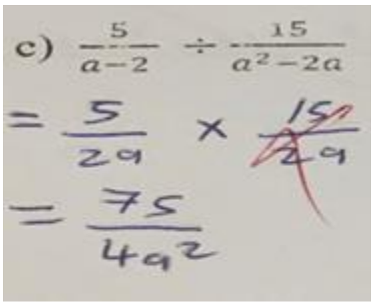


Figure 4.2.3.5: Difficulty in reciprocating the second fraction

Figure 4.2.3.5 indicates that a considerable number of learners, 12 out of 21 (57.1%) at school SA (Control Group) and 9 out of 15 (60%) learners at school SB (Experimental Group) managed to change the division operation to multiplication law but the second fraction remained unsolved. For example, Learner L7A changed division-sign to multiplication but failed to reciprocate the second fraction and instead, Learner L7A multiplied $\frac{5}{2a}$ by $\frac{15}{2a}$.

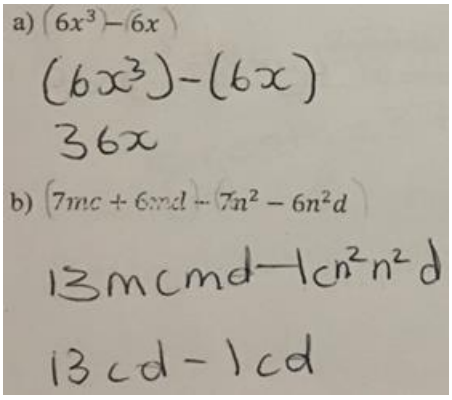
4.2.3.5.1 Discussion

NSSCO learners were found to experience learning difficulties with multiplying two brackets together (multiplying binomial with a binomial or polynomial). Learners were noticed only to remove the brackets from the expression *4x+1+2x+6* and then started simplifying without multiplying terms in the bracket first with all terms in the second bracket as said by D'Emilio, 2016). Learners were found not to apply the rule of binomial expansion which states: "*when expanding binomial by binomial, multiply each term in the first bracket with all terms in the second bracket"*.

In addition, most NSSCO learners have shown difficulties simplifying the numerators and the denominators separately. Many learners had no idea on how to solve algebraic fractions as stated by Ojose (2015) that learners cannot simply move from addition of fractions with numbers to addition of Algebraic concepts without making sense of the different scenarios and adjust accordingly and failure to clarify the difference between those scenarios, might lead to learning difficulties. All in all, many NSSCO learners seem to fail to follow the steps and interpret the question correctly and thus, majorities of them got wrong answers as argued by Nasser (2015) that learners do fail to interpret the Algebraic questions correctly and they end up using wrong rule or formula due to misunderstanding of the meaning of the word.

4.2.3.6 Difficulty with factorizing

Some learners at school SA (Control Group) and learners at school SB (Experimental Group) for both pre- and post-test, have difficulties with factorizing operation. Figure 4.2.3.6 below shows the difficulty experienced by learners with factorizing.



4.2.3.6 Difficulty with factorization of Algebraic expressions

Figure 4.2.3.6 above shows that some learners, 11 out of 21 (52.3%) at school SA (Control Group) and 12 out of 15 (80%) learners at school SB (Experimental Group) failed to apply the rule of factorization, which says: *pick a Highest Common Factor and then divide through with that HCF*. For example, Learner L15B subtracted $6x^3$ - 6x = 36x in Figure 4.4.6 (a) and in Figure 4.4.6 (b) 7mc +6md – $7n^2$ - $6n^2d$ = 13cd – 1cd.

4.2.3.6.1 Discussion

Most NSSCO learners experienced learning difficulties in factorizing, LCM, inequality direction, binomial expansion and reciprocal. Majorities of NSSCO learners had no clue on how to answer those questions. For example, learners might be asked to expand or simplify an expression, but they have no clues on what to do and/or some learners might have an idea but fail to apply the collect steps or rules in solving the problem (Rompas et al., 2023).

Most NSSCO learners were found to be unable to reciprocate the second fraction after changing division sign to multiplication sign. Some learners changed division to multiplication law but the second fraction remained the same while other did not even have an idea on how to solve algebraic fraction when multiplying as describe by Pramesti and Retnawati (2019) and Awasthi (2022) that some of the learning difficulties are experienced due to lack of understanding the problem in general, that is, difficulties in making generalizations, lack of understanding arithmetic operations, lack of knowledge about process of solving problems, lack of applying various steps in solving problems, lack of conceptual understanding of the meaning of variables as well as lack of knowledge on the operation of algebraic forms.

Most learners were found to either simplify or solve a problem instead of factorizing out the HCF when asked to factorize expressions. It seemed that most learners did not even understand the meaning of the concept of "factorization" and thus, failed to pick out the highest common factor from any given algebraic expression as espoused by Nasser (2015) and, Rompas, Wenas, Sambuaga and Mangelep (2023) due to lack of technical vocabulary.

4.4.4 Difficulties in solving Algebraic equations and inequalities

The following two learning difficulties were identified in NSSCCO learners when solving Algebraic equations and inequalities.

4.2.4.1 Difficulty with changing signs in Algebraic equations

Some of the learners at school SA (Control Group) and learners at school SB (Experimental Group) for both pre- and post-test respectively) failed to change the signs as they moved terms from different sides of the equal sign or inequality. The examples given in Figure 4.2.4.1 (a) and (c) below illustrate this difficulty.

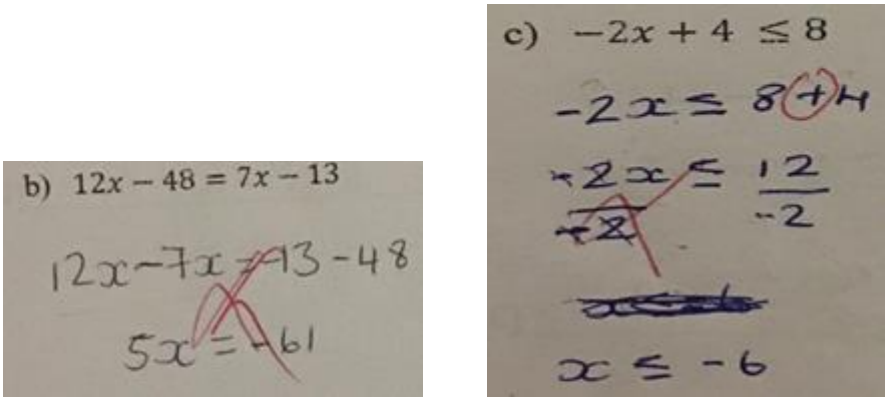


Figure 4.2.4.1: Difficulties in solving Algebraic equations and inequalities

Figure 4.2.4.1 (a) illustrates that Learner L19A failed to change *-7a* to positive *7a* and *-48* to positive *48* during gathering/collection of like terms. The same goes to Learner L11B in figure 4.2.4.1 (c) also failed to change from *4* to *-4* as per the Algebraic rule of solving equation that says: *the sign-change to positive or negative when terms are moved from one side of the 'equal to or inequality', to another side.*

4.2.4.2 Difficulty with changing the direction of the inequality sign

Some learners at school SA (Control Group) and learners at school SB (Experimental Group) for both pre- and post-test respectively, were unable to change the sign of the inequality after dividing both terms on each side with a negative number. Figure 4.2.4.2 at School A (Control Group) 15 out of 21 (71.5%) learners in pre-test and 13 out of 21 (61.9 %) learners in post-test.

Figure 4.2.4.2 at School SB (Experimental Group) 13 out of 15 (86.7% learners in pre-test and 5 out of 15 (33.3%) learners in post-test failed to apply the rule of solving inequalities correctly.

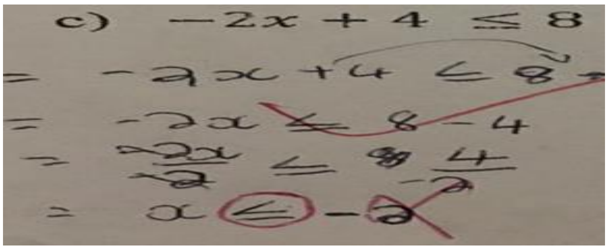


Figure 4.2.4.2: Difficulties in solving Algebraic inequality

Figure 4.2.4.2 shows that some learners experienced difficulties in solving Algebraic inequality problems. Most learners, 14 out of 21 (66.7%) at school SA (Control Group) and 10 out of 15 (66.7%) learners at school SB (Experimental Group) had difficulties in solving Algebraic inequality problems. For example, Learner L1A divided with -*2* on both sides of the inequality and the direction of the ≤ sign was not considered in solving this equation problem while the rule says: *after you divide with a negative number, the direction of the inequality sign changes the direction.*

4.2.4.1.1 Discussion

It was found that most NSSCO learners have difficulties in changing the signs when collecting terms from one side of the equal to or inequality, to another side. Learners were seen following the rule of collecting like terms correctly, but they did not apply it correctly as signs of moved terms remained the same even after the terms over the 'equal to or inequality' signs. Some learners were found to add/subtract unlike terms while others had no idea on how to answer such questions. According to Darmayanti et al., (2023) learners' failure to understand the concept of the "equal to" sign/symbol in solving algebraic equations, leads to many learners to be confused when solving algebraic equations which involves variable.

It was found that most NSSCO learners have difficulties with changing the direction of the inequality after divided both side with negative terms. However, they seem to have a better understanding in solving inequality equations. Most NSSCO learners were found not to pay attention to the inequality direction as in agreement with Darmayanti et al. (2023. This clearly indicates that learners mastered the steps on how to solve inequalities but, they were found to be easily confused or not remembering only the last step.

The next section deals with research question two about exemplary instructional strategies which were used in assisting learners with learning difficulties.

4.3. Research Question Two: What exemplary instructional strategies did teachers use to assist NSSCO learners with learning difficulties in solving algebraic equations?

The following exemplary instructional strategies were used to assist learners with learning difficulties in Algebra at school SB (Experimental Group): Peer Exemplary Instructional Strategy (PEIS), Cooperative Exemplary Instructional Strategy (CEIS) and the Viewing Exemplary Instructional Video Strategy (VEIVS) in teaching Algebra to learners about the (i) addition/subtraction of like and unlike terms, (ii) multiplication law to add/subtract polynomials, (iii) moving of like terms along with their signs and (iv) word problem equations in Algebraic equations/expressions, The presentation of the results is described under these three themes.

4.3.1. Peer Exemplary Instructional Strategy (PEIS)

During the classrooms instructions NSSCO learners were given class activities to complete individually under the supervision of the teacher. Some learners were found to experience difficulties in answering some questions correctly while other were not struggling at all. The teacher paired learners who were struggling with learners who did not have major challenges to work together under guided practice.

Learners who understood explained and showed to other learners how to arrive at the correct answers instead of the teacher doing the explanations all over. After every learner was completed the given exercise activity, the teacher picked some of the learners who seem to understand the item better and ask them to explain to the whole class on how they arrived at their answers. This was done to provide opportunity to learners with learning difficulties to ask questions freely to their peers since some learners felt free when asking their peers rather than asking the teacher.

Both pre-test and post-test consisted of 9 items of which are item 1, item 2, item 5 and item 9 which focused on Algebraic representation and formulae. The pre-test had a total of 17 marks while post-test had a total of 14 marks. Figure 4.3.1.1 below indicates the number of learners who experienced difficulties in different items in both pre-test and post.

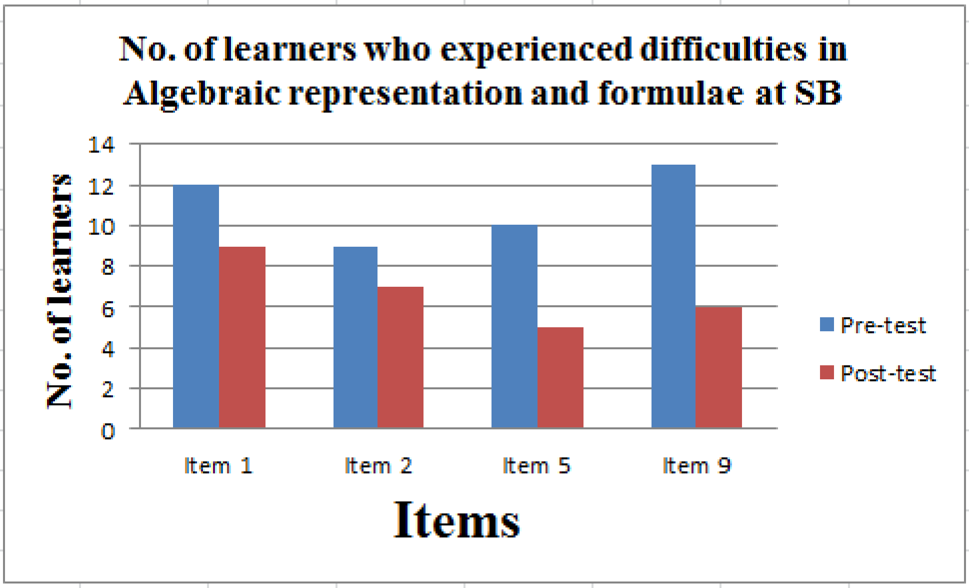


Figure 4.3.1.1: No. of learners who experienced learning difficulties in Algebraic representation and formulae

The results from figure 4.3.1.1 above indicates that 13 learners experienced difficulties when answering item 9 in pre-test but after intervention only 6 learners had difficulties. That is also the same in item 5 where the number of learners dropped from 10 learners to five learners. However, in item 2 and item 1 there was a slight difference from 11 to nine learners and 12 to nine learners in pre and post-tests respectively. This indicates that learners perform better when assisted with Peer Exemplary Instructional Strategy (PEIS).

Figure 4.3.1.2 below indicates the NSSCO learners' performances in pre and post-test at school SB (Experimental Group) using various exemplary teaching strategies.

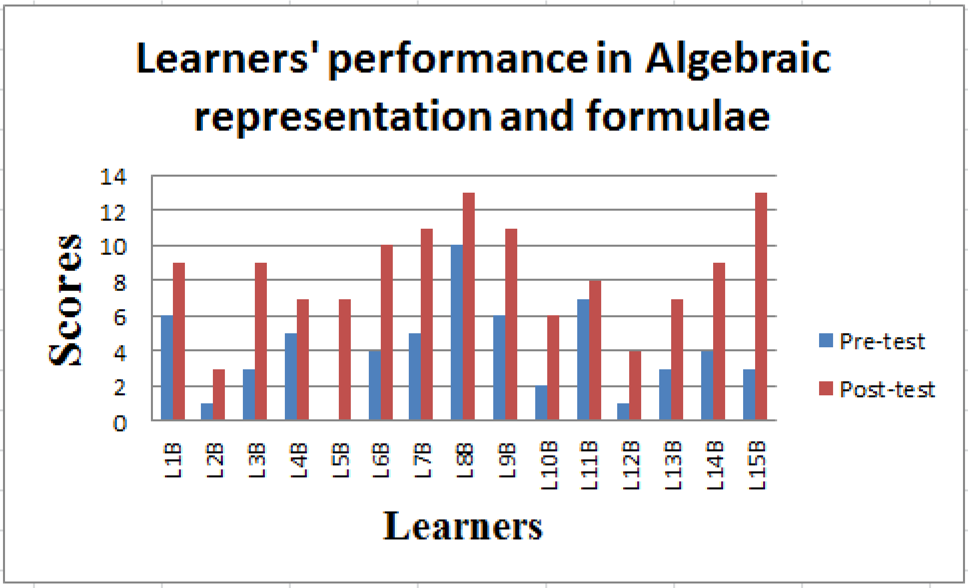


Figure 4.3.1.2: Learners' performances in Algebraic representation and formula

Figure 4.3.1.2 indicates that the overall performance of the NSSCO learners has improved when comparing the results of the pre-test to the results of the post-test after the intervention to assist learners who were challenged with learning difficulties through PEIS. Some learners, for example, Learners L5B, L3B, L6B, L7B, 9B, L14B and L15B have improved drastically, although it is difficult to remove all learning difficulties in a short period of time. PEIS shows promising results as the results of all learners have increased after assisting learners with these types of learning difficulties.

4.3.1.3 Discussion

This seems to agree with what Abramovich, Arcadii, and Milligan (2019) said that an appropriate way of teaching Mathematics at all levels is to do it through applications where all learners are allowed to do an exercise activity individually and then have a discussion with their peers to solve algebraic equations. PEIS allows almost every learner an opportunity to take part and promote active learning.

During PEIS, learning activities should be prearranged in a manner that allows learners to deal with such activities independently as well as finalising such activities through peers' discussions (Berta and Hoffman, 2020). In addition, PEIS seems to allow learners to uncover patterns and/or compare and connect strategies and steps that help them to strengthen and deepen their Algebraic solving skills and reducing their learning difficulties. For the current study, the number of learners who managed to perform better in the post-test, was 12 out of 15 (80%) learners as compared to 2 out of 15 (13.3%) learners in pre-test.

It was observed from Figure 4.4.9 that learners performed better after they were assisted using peer teaching. For example, learner L5B got a zero mark in pre-test and received a higher mark in the post-test. This shows that peer teaching as an exemplary instructional strategy assist learners with learning difficulties better as compared to the traditional strategies, such as a lecture teaching strategy.

4.3.2 Performance of NSSCO learners assisted through Cooperative Exemplary Instructional Strategy (CEIS)

Another exemplary teaching strategy that was used to render assistance to NSSCO learners, who experienced learning difficulties in solving algebraic formula, was the use of Group Work or Cooperative Exemplary Instructional Strategy (CEIS). During the current study, the teacher placed learners in three groups of 5 learners each where each group was given the same question items to work on. When learners were working on the items, the teacher was moving around, checking how learners were working in each group, thereby managing and controlling the reduction of monopolisation by some individuals. In the process, the teacher was assisting learners whenever mistakes were noticed by explaining and showing learners the correct way of answering the questions.

In both pre-test and post-test the items that focused on Algebraic manipulation were four. In pre-test, Item 3, Item 4, Item 6 and item 7, with a total mark of 18 and in post-test, Item 3, item 4, Item 6 and item 7 with a total mark of 19.

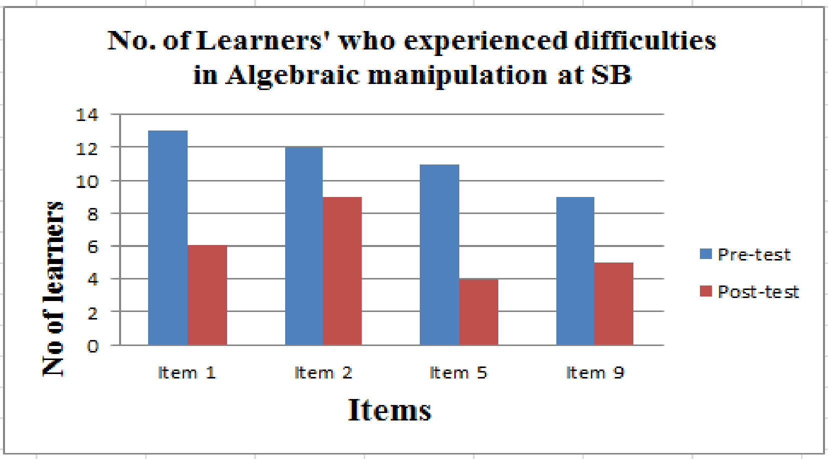


Figure 4.3.2.1: Numbers of learners who experienced difficulties in Algebraic manipulation

It can be seen from the graph above that in item five 11 learners had more difficulties in pre-test but after the intervention the number of learners dropped drastically to four. This can also be observed in item 1 where 13 learners had difficulties in pre-test and only six learners in the post-test. However, in item 2 there was a slightly difference from 12 learners in the pre-test to nine learners in the post test. Overly, the number of learners in all items had dropped which shows that Group Work or Cooperative Exemplary Instructional Strategy (CEIS) assisted learners with algebraic learning difficulties.

Figure 4.3.2.2 below shows below the performances of individual learners on items, 3, 4, 6 and 7 of the pre- and post-test about the Algebraic manipulation at SB.

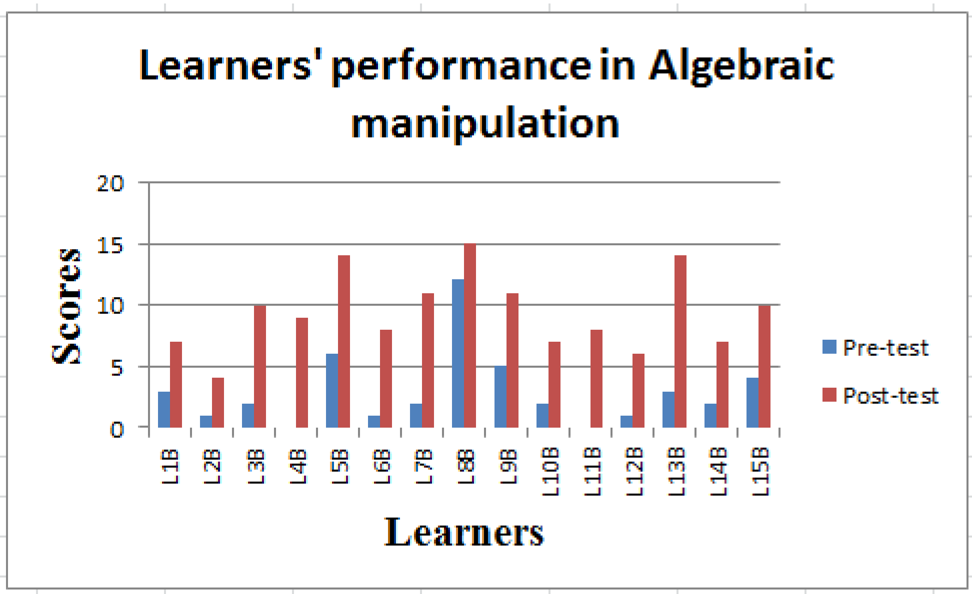


Figure 4.3.2.2: Learners' performances in Algebraic manipulation

The overall performance of all the learners has improved with an exemption of Learner L8B, with less difference in the pre- and post-test results. Figure 4.3.2.2 also shows that most learners performed well when comparing the results of the pre- to the results of the post-test. For example, Learners L2B, L3B, 5B, 6B, L9B, L10B, L13B and L15B performed well in the post-test as compared to pre-test. It can also be observed from the figure 4.3.2.2 that L4B and L11B performances improved drastically from getting a zero in the pre-test to nine marks for L4B and 14 marks for L11B in the post test. This indicates that assistance that was provided through the Cooperative Exemplary Instructional Strategy (CEIS) was highly effective in assisting learners with learning difficulties Algebra.

4.3.2.3 Discussion

Syarifuddin & Atweh, (2022) mentioned that cooperative instructions allow learners to utilize measurement, manipulate or construct activities to find relationships that they will later symbolically represent as law or rule. In addition, they highlighted that it was important for the teachers to act as a facilitator to manage inter-group led discussions which are designed to give learners an opportunity to reflect on given tasks as they have been working on.

According to Baig (2015), Star et al, (2015) and Berta & Hoffmann (2020) cooperative instructions paired with discussions, assist learners to be active to master the intended skills through the provision of mentoring and to foster meaning making process as one of the principles of the constructivism instructional theory.

Cooperative instruction helped learners to understand Algebraic manipulation better due to the identification of those learners who were not focusing on given activities who are usually hard to be noticed in lectures. More importantly, during CEIS, learners tend t help each other and showing each various ways on how to solve the problem. During the CEIS, many learners tend to develop own understanding, that may lead to change of their previous learned concepts (Shah, 2019).

4.3.3. Video Viewing Exemplary Instructional Strategy (VVEIS) to assist learners with algebraic equations and inequalities

Videos were used to assist learners with learning difficulties in solving equations and inequalities, such as difficulty with changing signs in an Algebraic equations and difficulty with changing the direction on the inequality sign. There was only 1 item in the pre-test and 1 item in the post-test and altogether, two items focusing on algebraic equations and inequalities. The teacher helped learners with learning difficulties by allowing them to watch videos with various repeated instructions about algebraic equations and inequalities exercises. The teacher first wrote projector questions on the chalkboard and instructed all the learners to copy them, before showing them videos about how to solve the project questions. Learners were requested to make notes of the steps involved in solving the equation or the inequalities in their notebooks while watching the videos. The videos were paused several times to explain each step before the video proceeding to the next step. Learners watched the videos more than once until everyone learners got the steps right and this process seemed to motivate them as feedback was given immediately to strengthen their procedural skills.

In both pre-test and post-test only one item (Item 8) focused on algebraic equations and inequalities. Figure 4.5.3.1 below indicates that 12 learners had difficulties in pre-test, however after the learners were assisted with Video Viewing Exemplary Instructional Strategy (VVEIS) the number of learners dropped drastically to four learners. That shows that learners taught with VVEIS performed better.

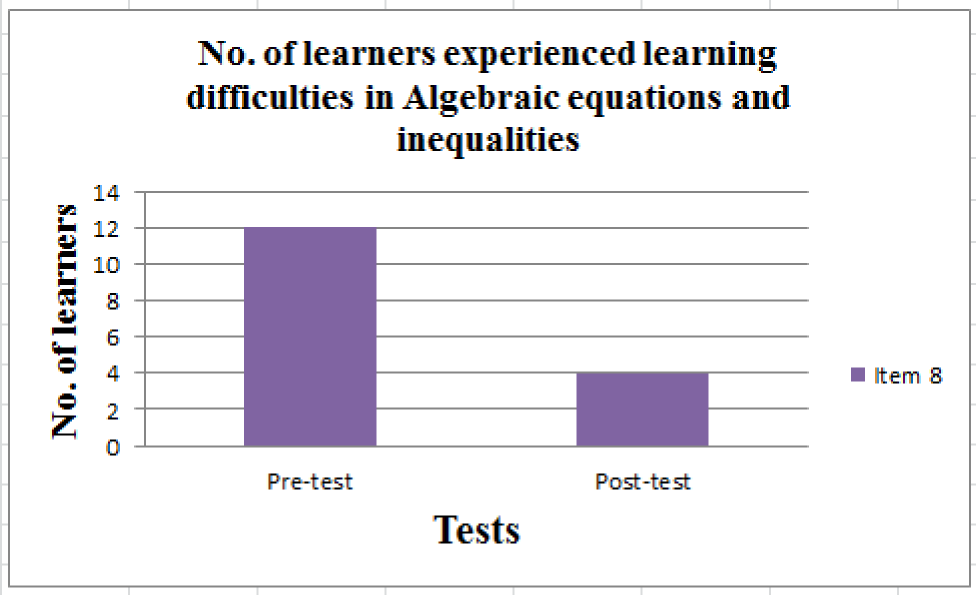


Figure 4.3.3.1: No of learners who experienced learning difficulties in item 8 in Algebraic equations and inequalities

Figure 4.3.3.2 below indicates the performances of individual learners in solving Algebraic equations and inequalities as per the mentioned test item.

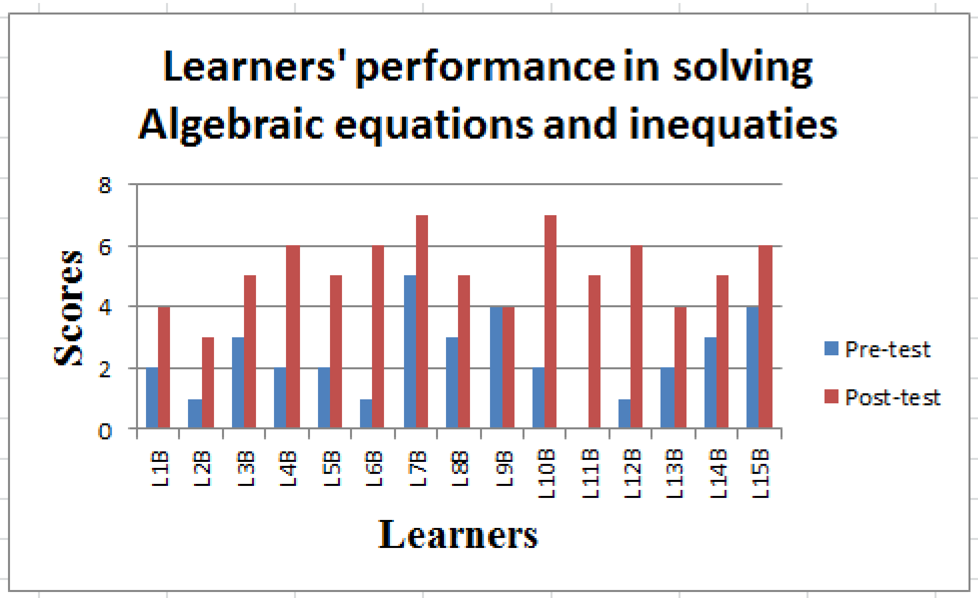


Figure 4.3.3.2: Learners' performances in solving Algebraic equations and inequalities

In both pre-test and post-test only 1 item focused on solving Algebraic equations and inequalities. In the pre-test item 8 consisted of five marks while in the post-test item 8 consisted of seven marks. Figure 4.3.3.2 indicates that NSSCO learners' performance has improved when comparing the results of the pre-test to the results of the post-test after the intervention with VEIVS. It can be observed from figure 4.3.3.2 that L7B and L10B managed to get all the marks in the post-test. Furthermore, L11B performance improved drastically from getting a zero in the pre-test to five marks in the post-test. It can also be observed that L4B, L6B, L10B and L12B performances improved drastically in the post-test as opposed to pre-test after the intervention. However, L9B's performance remained constant during the pre-test and post-test. .

4.3.3.3 Discussion

The viewing of videos showing various ways of solving given problems in algebra seem to provide measurable assistance in a similar manner to those noticed by Algani (2019) about integration of technology in teaching and learning environment. This instructional strategy seems to be enjoyable and thereby assisting learners to develop intellectual capabilities by analysing how others are providing solutions to various practical problems. Iyamuremye et al., (2021) also claims that the real objects allow learners to model their understanding of the abstract concept. Learners watching Algebraic videos allowed them to see how equations and inequalities were solved practically. Learners could connect what they saw in videos to their prior knowledge as well as what the teacher taught, and this seem assist them to remember such concepts and steps in future.

4.3.4 Lecture Tradition Instructional Strategy (LTIS)

The performance of learners at SA in both pre-test and post-test was low. It was observed that in some cases learners did not even improve their performances after intervention. Figure 4.3.4.1 below shows the numbers of learners who experienced difficulties in Algebra.

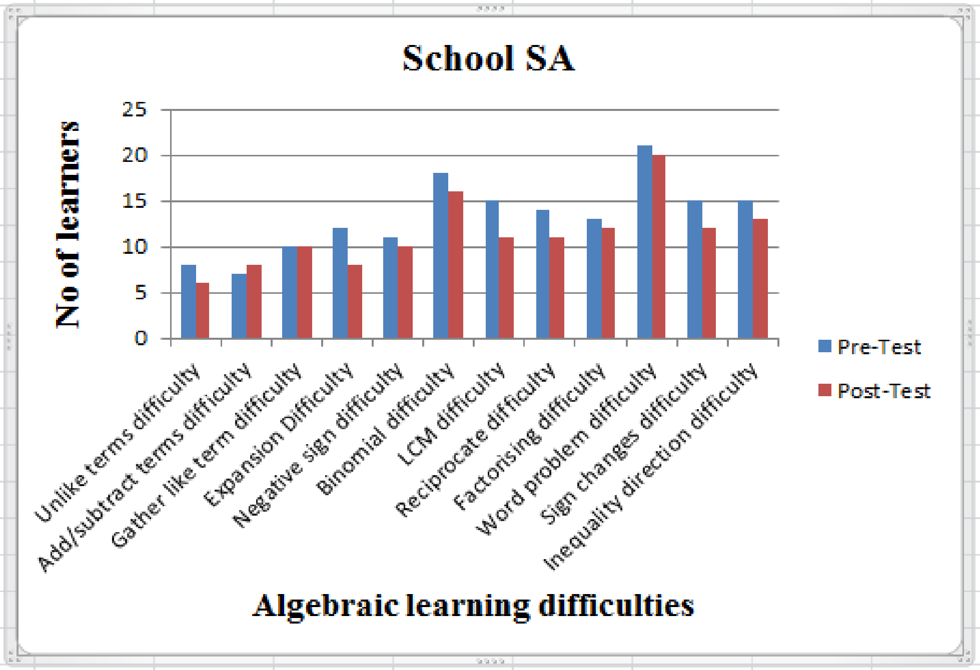


Figure 4.3.4.1: Learners' difficulties at SA

It can be observed from figure 4.3.4.1 that majorities of learners had more difficulties in words problem. In the pre-test all the learners did not get any of the collect answer in the word problems. After the intervention, only one managed to get that question collect. Majority of learners also experienced difficulties in binomial expansion. In the pre-test 18 learners experienced the difficulties while in the post-test 16 learners experienced the same difficulties. Moreover, it can also be observed from the graph that the number of learners who experienced difficulties in gathering of like terms did not change after the intervention with Lecture Tradition Instructional Strategy (LTIS). Therefore, this indicates that LTIS does not assist learners with algebraic learning difficulties.

The next section deals with the presentation of results for research question three.

4.4. Research Question Three: Is there a significant deference between teachers' exemplary instructional strategies in assisting NSSCO learners with learning difficulties in solving Algebraic equations/expressions?

At School SA, 21 learners were taught Algebra using lectures as traditional instructions strategies for a period of two weeks. Algebraic representation was taught over three days, Algebraic manipulation took four days and solving Algebraic equations and inequalities took three days. At School SB (Experimental Group), 15 learners were taught using peer teaching, group discussion and viewing videos on Algebra for a period of three weeks. Algebraic representation was taught over four days, Algebraic manipulation was taught over six days and solving Algebraic equations and inequalities was taught over four days. Learners were pre-tested to identify the learning difficulties and then they received an intervention treatment before they took a post-test to see whether there was a significant difference between the lecture traditional instructional strategy and the exemplary instructional strategies. Table 4.4.1 shows the results of the mean of learners' performance in pre-tests at school SA (Control Group) and school SB (Experimental Group).

PRE-TEST		
	No. of Participant	**Mean**
Exemplary Instructional Strategies		
Peer Exemplary Instructional Strategy	15	8.5
Cooperative Exemplary Instructional Strategy	15	10.5
Video Viewing Exemplary Instructional Strategy	15	17.5
Lecture Instructional Strategy	21	34.5

Table: 4.4.1 Mean of learners' performance from pre-tests on Algebra

On the comparison of group's mean averages of pre-tests at both schools (SA and SB) administered, it indicates that learners at School SB (experimental school) had more difficulties in pre-test with 36.5 compared to learners at School SA (control school) with 34.4. Figure 4.4.1 above indicates the group means of instructional strategies. It can be seen that the means for Peer Instructional Strategies with 8.5 and Cooperative Exemplary Instructional Strategies with 10.5 were the lowest. This shows that only few numbers of learners improved after the intervention. Learners assisted with Video Viewing Exemplary Instructional Strategies tend to improve with 17. 5. This shows that algebraic learning difficulties can be improved if learners are allowed to watch algebraic videos. Generally, the group mean for exemplary instructional strategies is higher than of lecture instructional strategy which means exemplary instructional strategies assist learners with algebraic learning difficulties than lecture instructional strategies.

The Interventions took place immediately after the administration of the pre-test at both schools. Upon completion of the intervention, both the groups sat for the same post-test. Figure 4.4.2 below shows the results for the post-tests for School SA and School SB.

t-Test: Two-Sample Assuming Unequal Variances		
	Traditional method	*Exemplary method*
Mean	12,05	17,5
Variance	48,99736842	29,19230769
Observations	21	15
Hypothesized Mean Differe	0	
df	32	
t Stat	2,559212004	
P(T<=t) one-tail	0,007710316	
t Critical one-tail	1,693888748	
P(T<=t) two-tail	0,015420632	
t Critical two-tail	2,036933343	

Figure 4.4.2: t test results summary for post-tests

Figure 4.4.2 indicates the mean scores for both the pre-test and post-test which is 17.2 for the experimental group and 12.05 for the control group that indicates that learners in the experimental group (PEIS, CEIS and VVEIS) outperformed learners in control group (Lecture Traditional Instructional Strategies (LTIS)). The means, t-score of the groups were determined and compared.

The null hypothesis states that, there is no statistical difference between learners' performance, assisted through exemplary instructional strategies as compared to being assisted through lecture traditional instructional strategies. The t-scores (stat) = 2.56 which is more than t (critical) = 2.04, therefore, the null hypothesis was rejected as there. This means that there is a significant difference in the mean scores of learners' performance who were taught through Peer, Cooperative and Video Viewing Exemplary Instructional Strategies, and learners taught through lecture traditional instructional strategy. The probability value (p) for two tails was equal to 0.02. The probability value p = 0.02 is less than the significance level at $\propto = 0.05$ *and h*ence, the null hypothesis was rejected.

Therefore, the study revealed that the peer, Cooperative and Video viewing exemplary instructional strategies assisted learners with learning difficulties in Algebra as opposed to lecture traditional Instructional Strategy. The selected strategies seem to promote active learning and meaning making through mentoring is promoted as espoused by the principles of the Cognitive Constructivist Learning Theory and assisted learners to remember Algebraic rules and steps.

4.5 Summary

In this chapter, the researcher presented the results, analysed, interpreted data and discussions. The findings of the study are summarized as per the research questions as follows:

4.5.1 Research Question One: What learning difficulties do learners have in solving Algebraic equations?

It was found that many learners experienced the following learning difficulties:

- In solving Algebraic representation and formulae, such as to solve as algebraic representations with unlike terms, add/subtract algebraic terms, gathering of like term and word problem.
- In manipulating Algebraic equations and inequalities such as:
    - The expansion difficulty, negative sign difficulty, binomial difficulty, LCM difficulty, reciprocal difficulty and factorization difficulty.
- Solving identified sign changes difficulty and inequality direction difficulty.

4.5.2 Research Question Two: What exemplary instructional strategies do teachers use when assisting learners with learning difficulties in solving algebraic equations?

- It was also found that the used exemplary instructional strategies assisted learners with learning difficulties as their performance in the post-test was found to be high and, in some cases, where individual learners did not receive any mark in the pre-test, scored very well in the post-test.
- It was also found that the Video Viewing Exemplary Instructional assisted learners better as compared to the Cooperative and peer exemplary Instructional Strategy.

4.5.3 Research Question Three: Is there a significant difference between lecture and exemplary instructional strategies in assisting learners with learning difficulties in solving Algebraic equations?

- It was found that PEIS, CEIS and VVEIS assist learners with learning difficulties in Algebra as opposed to LTIS.

The next chapter focus on the summary, conclusions of the findings and the recommendations of the study.

CHAPTER FIVE

SUMMARY, CONCLUTIONS, AND RECOMMENDATIONS

5.1. Introduction

In this chapter, the researcher described the summary, conclusions, and recommendations of the study as per the three research questions, starting with the summary of the study.

5.2. Summary

The summary of the study is presented as per research questions, starting with the first question.

5.2.1. Research Question One: What learning difficulties do learners face in solving algebraic equations/expressions?

The NSSCO learners experienced a variety of learning difficulties in solving Algebraic equations such as:

1. Learning difficulties in Algebraic representation and formulae: It was found that most NSSCO learners experienced difficulties in solving algebraic like and unlike terms, and addition/subtractions of algebraic terms.

2. In addition, most learners showed a lack of understanding on how to gather algebraic like terms step by step before providing the final solution for the equation/expression and word problem difficulty. The study findings revealed that majorities of learners experienced more difficulties in word problems as compared to other three difficulties.

3. Learning difficulties in Algebraic representation and formulae: It was found that most NSSCO learners experienced difficulties in solving algebraic like and unlike terms, and addition/subtractions of algebraic terms. In addition, most learners showed a lack of understanding on how to gather algebraic like terms before providing the solution for the equation/expression as a.

4. Also, NSSCO learners experienced difficulties in understanding and translating word problems as algebraic expressions due to lack of understanding the language of instruction or the official language.

5. The study revealed that NSSCO learners experienced difficulties in using Algebraic manipulation such as expansion, negative sign, binomial, LCM, reciprocal and factorizing difficulties.

6. In addition, NSSCO learners had problems solving algebraic equations and inequalities such as changing signs and applying inequalities direction to algebraic equations and expressions.

5.2.2 Research Question Two: What exemplary instructional strategies did teachers use to assist NSSCO learners with identified learning difficulties in solving Algebraic equations?

The study revealed that NSSCO learners in the Experimental Group outperformed the learners in the Control Group. In other words, NSSCO Experimental Group learners performed highly in the post-test as compared to NSSCO Control Group learners in pre-test. Thus, the intervention activities proof to be successful due to high performance of the NSSCO learners in the post-test.

However, one should not forget that perhaps the intervention period was not long enough to decrease or count out the effect of remembrance of what was taught during the intervention period.

5.2.3 Is there a significant difference between teachers' exemplary instructional strategies in assisting NSSCO learners with learning difficulties in solving Algebraic equations?

The study revealed that here is a significant difference in the mean scores of learners' performance who were taught through Peer, Cooperative and Video Viewing Exemplary Instructional Strategies, and learners taught through lecture traditional instructional strategy.

5.3 Recommendations

The following recommendations are made as based on the findings of the study:

1. Of all the Algebraic learning difficulties identified, the researcher noticed that majorities of learners experienced difficulties with translating word problems. It is, therefore, recommended that Mathematics teachers should give more Algebraic *word* problems to learners to help them to become more efficient in solving such problems. Additionally, Mathematics lessons should allow learners to explore *word problems* solving skills and discover new mathematical concepts related to *word problems* on their own as homework by viewing videos
2. The study found out that majorities of NSSCO learners with difficulties in Algebraic manipulations should be allowed to view Algebraic videos to boost their understanding of Algebra better.

3. Exemplary instructional strategies such as viewing educational videos, the use computers web sites, are recommended to aid in assisting learners to learn algebra. Therefore, the Ministry of Education, Innovation, Youth, Sports, Arts and Culture is recommended to equip schools with better technological to allow teachers to use these tools as additional strategies to enrich and assist learners.

5.4. Further research

The current study only focused on learning difficulties in Algebra. However, other studies in Mathematics could be conducted on various topics in NSSCO level syllabus to identify further learners' learning difficulties.

The research was carried out only in Erongo educational region, Omaruru circuit with a sample of few schools. Thus, further studies on the analysis of exemplary instructional strategies could be conducted with a large sample consisting of other educational regions educational regions in the whole country at large or other educational circuits in Erongo region.

Further studies could also be conducted at primary phases and junior secondary phases in order to identify the causes of learners' learning difficulties in Algebra and close the gaps that exist in learning basics Algebraic terms. Once the gaps are identified the researcher believes that it can help teachers to select the best instructional strategies in the teaching and learning of Algebra education in the country.

5.5. Summary

In this chapter, the researcher described the summary and conclusions of the study. The presentations were based on the findings of the study under each research questions. The recommendations as well as suggestions for further studies were also described in this chapter.

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APPENDICES

APPENDIX A: Ethical clearance certificate from UNAM

UNAM
UNIVERSITY OF NAMIBIA

ETHICAL CLEARANCE CERTIFICATE

Ethical Clearance Reference Number: WKC0045 Date: 13 December 2023

This Ethical Clearance Certificate is issued by the University of Namibia Decentralized Ethics Committee (DEC) in accordance with the University of Namibia's Research Ethics Policy and Guidelines. Ethical approval is given in respect of undertakings contained in the Research Project outlined below. This Certificate is issued on the recommendations of the ethical evaluation done by the **School of Education (Windhoek & Khomasdal Campuses) Decentralized Ethics Committee**.

Title of Project: An analysis of teachers' exemplary instructional strategies in relation to grade 10 learners' learning difficulties in solving algebraic equations in Omaruru circuit
Researcher: Beath Lumeta
Student number: 201203882

Take note of the following:

1. Any significant changes in the conditions or undertakings outlined in the approved Proposal must be communicated to the ethics committee. An application to make amendments may be necessary.
2. Any breaches of ethical undertakings or practices that have an impact on ethical conduct of the research must be reported to the ethics committee.
3. The Principal Researcher must report issues of ethical compliance to the ethics committee (through the Chairperson) at the end of the Project or as may be requested by the ethics committee.
4. The ethics committee retains the right to:

i) Withdraw or amend this Ethical Clearance if any unethical practices (as outlined in the Research Ethics Policy) have been detected or suspected,

ii) Request for an ethical compliance report at any point during the course of the research.

The ethics committee wishes you the best in your research.

Dr. Job U. Hengari (Chairperson, Windhoek & Khomasdal Campuses Decentralized Ethics Committee)

Prof. Davis Mumbengegwi
(Head, Multidisciplinary Research)

APPENDIX B: Permission from the Executive Director

P O Box 15618

Oshakati

16 November 2023

To: The Executive Director

Ministry of Education

Private Bag 13186

Windhoek

Re: Request for Permission to Conduct Research at two Selected Secondary Schools in Erongo Region

Dear Ms Steenkamp

I am a part-time student at the University of Namibia pursuing a Master's degree in Mathematics Education. In partial fulfilment of the requirements for the completion of this degree, I am required to conduct a research project based on the topic: **"An Analysis of Teachers' Exemplary Instructional Strategies in Relation to Grade 10 Learners' Learning Difficulties in Solving Algebraic Equations in Omaruru Circuit"**. The purpose of my research is to analyse teacher's instructional strategies and learners 'learning difficulties in solving algebraic equations. This study is intending to provide information to educators in general including teachers and learners who experienced learning difficulties in solving algebraic equations. Thus, learners will realize various ways on how to solve algebraic equations and hopefully this will assist in reducing their learning difficulties.

I kindly therefore request for permission from your good office to allow me to use two of the selected schools in Erongo region as my research sites. I will need to conduct the study with one class doing Mathematics ordinary level at each school. The data will be collected using a pre-tests, post-tests, classrooms observations and interviews. The lessons will be based on Algebra at ordinary level of Mathematics. I planned to complete the data collection process at the end of March 2023. The school and participants will be assured of confidentiality and anonymity both during the experiment and in the final research report. The normal class teaching time slots and intact classes will be used and will not be changed in any ways.

I am confident that the recommendations emanating from my research will possibly contribute to the improved performances in mathematics in secondary schools. Therefore, humbly I request permission to conduct the research at the selected schools.

This research study has been cleared by the UNAM Research Ethics Committee and the ethical clearance certificate is attached. Should there be any queries about this request, please contact me at +264 81 715 121 8 or my supervisor Dr. Kandjeo at +264 81 475 672 5

I look forward to hear from your good office.

Yours Sincerely,

____

Beath A. Lumeta

APPENDIX C: Approval from the Executive Director of the Ministry of Education, Arts and Culture

REPUBLIC OF NAMIBIA

MINISTRY OF EDUCATION, ARTS AND CULTURE

Enquiries: Mr. N. Eiman
Tel: +264 61 -293 3202
Fax: +264 61- 293 3922
Email: nickeyeiman@gmail.com
File no: 13/2/9/1

Luther Street, Govt. Office Park
Private Bag 13186
Windhoek
Namibia

Beath A Lumeta
Cell Nr: 081 4756725
Email Address: beathlumeta@gmail.com

Dear Ms Lumeta,

SUBJECT: PERMISSION TO CONDUCT ACADEMIC RESEARCH IN THE ERONGO REGION

The Ministry wishes to acknowledge receipt of your letter dated 11 January 2024 seeking for permission to conduct academic research in the Erongo region for your Master's in Mathematics Education. The research study which is titled: "*An Analysis of Teachers Exemplary Instructional Strategies in Relation to Grade 10 Learners Learning Difficulties in Solving Algebraic Equations in Omaruru Circuit.*"

Permission has been granted to you. However, you have to seek for further clearance from the Regional Director of Education, Arts and Culture of the Erongo region to ensure that the following conditions are met:

- Participation is voluntary,
- Staff members' normal work is not disrupted during your interviews,
- Parental consent should be granted by the parents / guardians of all participants below the age of 16 years.

Furthermore, you are kindly requested to share your research findings with the Ministry after completion of the research project. You may contact Mr N. Eiman on the above provided contacts at the Directorate: Programmes and Quality Assurance (PQA) for submission of your research findings at the above indicated details.

We wish you the best in conducting your research and the Ministry looks forward to hearing from you upon completion of your studies.

Yours sincerely,

24. 01. 2024.

Ministry of Education, Arts and Culture
Office of the Executive Director
2024 -01- 24
Tel: 061-2933523/4
Fax: 061-253671
Private Bag 13186, Windhoek, Namibia

Sanet L. Steenkamp
EXECUTIVE DIRECTOR

All official correspondence must be addressed to the Executive Director Page 1 of 1

APPENDIX D: Letter to the director of Erongo region

P O Box 15618

Oshakati

16 November 2023

To: The Regional Director

Erongo Regional Council

Directorate of Education

Private Bag 5024

Swakopmund

Re: Request for permission to conduct research at two Selected Secondary Schools in Erongo Region

Dear Mrs. Stefanus

I am a part-time student at the University of Namibia pursuing a Master's degree in Mathematics Education. In partial fulfilment of the requirements for the completion of this degree, I am required to conduct a research project based on the topic: "**An Analysis of Teachers' Exemplary Instructional Strategies in Relation to Grade 10 Learners' Learning Difficulties in Solving Algebraic Equations in Omaruru Circuit".** The purpose of my research is to analyse teacher's instructional strategies and learners 'learning difficulties in solving algebraic equations. This study is intending to provide information to educators in general including teachers and learners who experienced learning difficulties in solving algebraic equations. Thus, learners will realize various ways on how to solve algebraic equations and hopefully this will assist in reducing their learning difficulties.

I kindly therefore request for permission from your good office to allow me to use the two selected schools in Erongo region as my research sites. I will need to conduct the study with one class doing Mathematics ordinary level at each school. The data will be collected using a pre-tests, post-tests, classrooms observations and interviews. The lessons will be based on Algebra at ordinary level of Mathematics. I planned to complete the data collection process at the end of March 2023. The school and participants will be assured of confidentiality and anonymity both during the experiment and in the final research report. The normal class teaching time slots and intact classes will be used and will not be changed in any ways.

I am confident that the recommendations emanating from my research will possibly contribute to the improved performances in mathematics in secondary schools. Therefore, humbly I request permission to conduct the research at the selected schools.

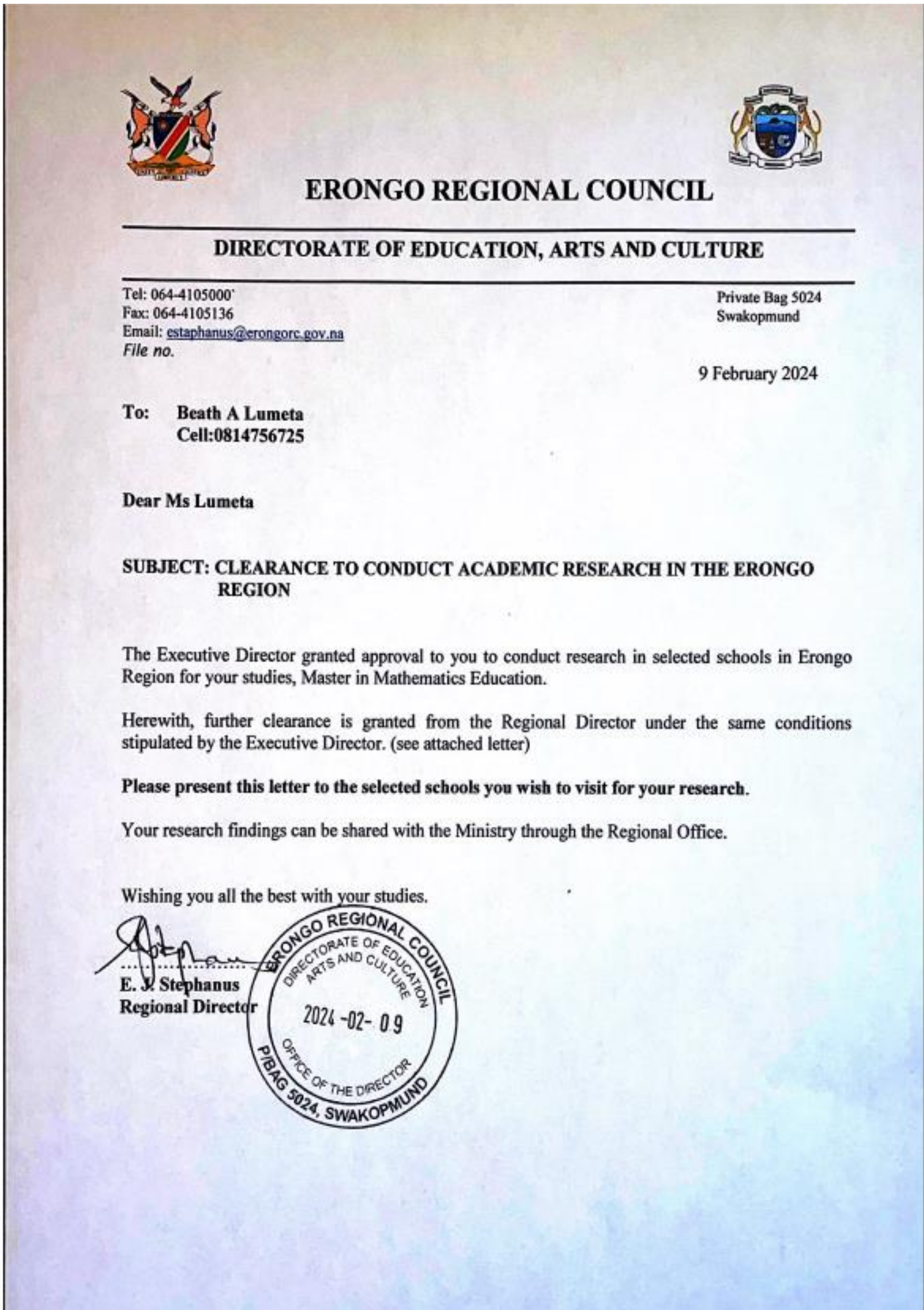
Should there be any questions about this request, please contact me at +264 81 715 121 8 or my supervisor Dr. Kandjeo at +264 81 475 672 5

I look forward to hear from your good office.

Yours sincerely

Beath A. Lumeta

APPENDIX E: Approval from the Director of Erongo



ERONGO REGIONAL COUNCIL

DIRECTORATE OF EDUCATION, ARTS AND CULTURE

Tel: 064-4105000
Fax: 064-4105136
Email: estaphanus@erongorc.gov.na
File no.

Private Bag 5024
Swakopmund

9 February 2024

To: Beath A Lumeta
Cell:0814756725

Dear Ms Lumeta

SUBJECT: CLEARANCE TO CONDUCT ACADEMIC RESEARCH IN THE ERONGO REGION

The Executive Director granted approval to you to conduct research in selected schools in Erongo Region for your studies, Master in Mathematics Education.

Herewith, further clearance is granted from the Regional Director under the same conditions stipulated by the Executive Director. (see attached letter)

Please present this letter to the selected schools you wish to visit for your research.

Your research findings can be shared with the Ministry through the Regional Office.

Wishing you all the best with your studies.

....
E. J. Stephanus
Regional Director

ERONGO REGIONAL COUNCIL DIRECTORATE OF EDUCATION ARTS AND CULTURE
2024 -02- 09
OFFICE OF THE DIRECTOR
P/BAG 5024, SWAKOPMUND

APPENDIX F: Letter to the school principal

P O Box 15618

Oshakati

16 February 2024

The Principal

The Selected Secondary School

Omaruru Circuit

Dear Mr Principal,

<u>Re: Research to be conducted at the Selected Secondary School</u>

I am a part-time student for a Master's degree in Mathematics Education at the University of Namibia. In order to complete my course, I am required to conduct a research. The title of the study is "**An Analysis of Teachers' Exemplary Instructional Strategies in Relation to Grade 10 Learners' Learning Difficulties in Solving Algebraic Equations in Omaruru Circuit**". The purpose of my research is to analyse teacher's instructional strategies and learners 'learning difficulties in solving algebraic equations. This study is intending to provide information to educators in general including teachers and learners who experienced learning difficulties in solving algebraic equations. Thus, learners will realize various ways on how to solve algebraic equations and hopefully this will assist in reducing their learning difficulties.

I kindly therefore request for permission from your good office to allow me to use your selected school as my research site. I will need to conduct the study with one class doing Mathematics at ordinary level. The data will be collected using a pre-tests, post-tests, classrooms observations and interviews. The lessons will be based on Algebra at ordinary level of Mathematics. I planned to complete the data collection process at the beginning of March 2024. The school and participants will be assured of confidentiality and anonymity both during the experiment and in the final research report. The pre-tests, post-tests and the interventions procedures will take place during the afternoon and evening studies, while classroom observations will take place during normal school time. However, the normal teaching time slots and intact classes will not be disrupted in anyway.

I am confident that the recommendations emanating from my research will possibly contribute to the improved performances in mathematics in secondary schools. Therefore, humbly I request permission to conduct the research at your school.

Should there be any queries about this request, please contact me at +264 81 715 121 8 or my supervisor Dr. Kandjeo at +264 81 475 672 5.

I look forward to working with you in this study.

Yours sincerely,

____

Beath A. Lumeta

APPENDIX G: Consent form for parents

P O Box 15618

Oshakati

16 February 2024

Dear Parent(s)/Guardian(s),

Re: Information letter and consent form for research study to be conducted

Title of Study

An analysis of teachers' exemplary instructional strategies in relation to grade 10 learners' learning difficulties in solving algebraic equations in omaruru circuit

Name, Qualification and Contact Details of Researcher

Beath A. Lumeta, Master of Education (Mathematics Education) a student at University of Namibia, Cell: 081 715 121 8

Background

Your child, …………………………………………………., is being invited to take part in a research study. Before you allow your child to participate in this study, it is important that you understand why the research is being done and what it will involve. Please take the time to read the following information carefully. For more information you may ask me on the contact details provided.

Purpose and Benefit

This study is intending to provide information to educators in general including teachers and learners who experienced learning difficulties in solving algebraic equations. Thus, learners will realize various ways on how to solve algebraic equations and hopefully this will assist in reducing their learning difficulties. The study will be conducted toward the attaining of the Masters of Education (Mathematics Education). Furthermore, the study will make proper recommendations for mathematics instructional strategies. Moreover, your learner active participation in this study will be appreciated.

Study Procedure

Learners will write a pre-test which will be followed by an intervention and then lastly a post-test. The expected time commitment for this study is 3 weeks. The study will take place during afternoon studies and evening studies.

Risks

There will be no risks or harm in participating in this study. If you find anything upsetting in the study, do not hesitate to bring it to my notice. In case of unforeseeable risks, efforts will be made to minimize such risks.

Confidentiality and anonymity

Confidentiality will be maintained throughout the study, since the study will only be used for academic, professional presentations as well as publications purposes. Data collected by this study will be anonymous through the use of pseudonyms. Your child’s name will not be reported, either on the data collection instrument, research reports or any final publication.

Costs to Subject and Compensation

There are no costs and no monetary compensation to you for your child’s participation in this study.

Consent

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions. I understand that the participation of my child/children is voluntary and that my child/children is/are free to withdraw at any time, without giving a reason and at no harm or cost. I understand that I will be given a copy of this consent form. I voluntarily agree on behalf of my child/children to take part in this study.

Thank you,

Parent's signature__________________________ Date ______________________

APPENDIX H: Consent form for learners

P O Box 15618

Oshakati

16 February 2024

Dear Learner,

<u>Re: Information letter and consent form for research study to be conducted</u>

Title of study

An analysis of teachers' exemplary instructional strategies in relation to grade 10 learners' learning difficulties in solving algebraic equations in omaruru circuit

Name, Qualification and Contact Details of Researcher

Beath A. Lumeta, Master of Education (Mathematics Education) a student at University of Namibia, Cell: 081 715 121 8

Background

You, …………………………………………………., being invited to take part in a research study. Before you decide to participate in this study, it is important that you understand why the research is being done and what it will involve. Please take the time to read the following information carefully. For more information you may ask me on the contact details provided.

Purpose and Benefit

This study is intending to provide information to educators in general including teachers and learners who experienced learning difficulties in solving algebraic equations. Thus, learners will realize various ways on how to solve algebraic equations and hopefully this will assist in reducing their learning difficulties. The study will be conducted toward the attaining of the Masters of Education (Mathematics Education). Furthermore, the study will make proper recommendations for mathematics instructional strategies. Moreover, your active participation in this study will be appreciated.

Study Procedure

You will write a pre-test which will be followed by an intervention and then lastly a post-test. The study will take place during the afternoon studies and evening studies. The expected time commitment for this study is 3 weeks.

Risks

There will be no risks or harm in participating in this study. If you find anything upsetting in the study, do not hesitate to bring it to my notice. In case of unforeseeable risks, efforts will be made to minimize such risks.

Confidentiality and anonymity

Confidentiality will be maintained throughout the study, since the study will only be used for academic, professional presentations as well as publications purposes. Data collected by this study will be anonymous through the use of pseudonyms. Your name will not be reported, either on the data collection instrument, research reports or any final publication.

Costs to Subject and Compensation

There are no costs and no monetary compensation to you for your participation in this study.

Consent

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving a reason and at no harm or cost. I understand that I will be given a copy of this consent form. I voluntarily agree to take part in this study.

Learner's signature___________________________ Date ______________________

APPENDIX I: Pre -Test in Algebra

Pre-Test in Algebraic Equations **Total: / 40 Marks**

School name: ____ Class ________

Learner name: _____________________________________

1. Write down the number of terms in the following algebraic expressions.

   a) $2a + bc - 5d$ Answer______________________________ [1]

   b) $(2a - ac) + 7d$ Answer______________________________ [1]

2. Simplify

   a) $4a - 3b + 2a - b$

   Answer (a) ________________________ [2]

   b) $-10b + 2a - a$

   Answer (b) ________________________ [1]

3. Expand and simplify

   a) $2x^2\ (2x - 3x^3y)$

   Answer (a) ________________________ [2]

   b) $\frac{1}{2}\ (6t - 2) - 2\ (t - 2)$

   Answer (b) ________________________ [3]

   c) $(2a - b)(2a + b)$

   Answer (c) ________________________ [2]

4. Express each of the following expressions as a single fraction in its simplest form

a) $\frac{3}{a+3} + \frac{2}{a-1}$

Answer (a) ________________ [3]

b) $\frac{4c}{2d} - \frac{c}{2d}$

Answer (b)________________ [2]

c) $\frac{2x}{y^3} \div \frac{4x^2}{y}$

Answer (c)________________ [3]

5. Marry is x years old.

a) How old will Marry be 8 years from now?

Answer (a) _______________________ [1]

b) How old was she 10 years ago?

Answer (b) ________________________ [1]

c) How old will she be in x year time?

Answer (c) ________________________ [2]

6. Factorize completely

a) $2x^2y + 16xy^3$

Answer (a) ________________________ [2]

b) $xy + y - xz - z$

Answer (b) ________________________ [2]

7. An equation is given by the formula $T = a + \frac{b}{V}$

a) Make V the subject of the formula

Answer (a) ________________________ [2]

b) Given that $a = 1$ and $b = -2$. Find

a) $a - b$

Answer (a) ________________________ [1]

b) $2a^2 + b$

Answer (b) ________________________ [2]

8. Solve the equations and inequalities

a) $(3x + 1) - (2x - 7) = 0$

Answer (a) ________________________ [3]

b) $6 - 3x = 5x + 1$

Answer (b) ________________________ [2]

c) $4x + 3 \ > 11$

Answer (b) ________________________ [2]

9. The area of a rectangle is 45 cm^2 and one of its sides is 4 cm longer than the other side. Find the width of the rectangle.

Answer (9) ________________________________ [3]

GOOD LUCK!!!!!!!!!!!!!

APPENDIX J: Post-Test in Algebra

Post-test in Algebraic equations **Marks: /40**

School name:____ Class____

Learner name: ______________________________________

1. Write down the number of terms in the following algebraic expressions.
   a) $2a + 4b^3 - 6a^2 - 8b$ Answer (a) __________________________ [1]
   b) $12d + (3a - b^2)$ Answer (a) __________________________ [1]

2. Simplify
   a) $-2x + 6y + 3x + y$

   Answer (a) __________________________ [2]

   b) $x^3 - 2x^2 - 3x^4 + 4x^3$

   Answer (b) __________________________ [2]

3. Expand and simplify
   a) $2a(5a^2 - 7a - 3)$

   Answer (a) __________________________ [2]

   b) $y^2(1 - y^2) - y(1 - y)$

   Answer (b) __________________________ [2]

   c) $(4x + 1)(2x + 6)$

   Answer (c) ______________________________ [2]

4. Express each the following expressions as a single fraction in its simplest form
   a) $\frac{5}{a-2} - \frac{2}{a+3}$

Answer (a) ___________________________ [3]

b) $\frac{x+2}{3x} - \frac{x-1}{2}$

Answer (b) _____________________________ [3]

c) $\frac{5}{a-2} \div \frac{15}{a^2-2a}$

Answer (b) ___________________________ [2]

5. In 9 years' time a mother will be twice as old as her son. 3 years ago, she was four times as old as her son. Find their present age.

Answer (5) ___________________________ [2]

6. Factorize completely

a) $6x^3 - 6x$

Answer (a) ___________________________ [1]

b) $7mc + 6md - 7n^2 - 6n^2d$

Answer (b) ____________________________ [2]

7. The formula for calculating Kinetic energy is given by the formula $KE = \frac{1}{2}MV^2$

a) Make V the subject of the formula

Answer (a) ____________________________ [3]

b) Find KE when $m = 4\ and\ v = 4$

Answer (b) ____________________________ [2]

8. Solve the following equations and inequalities.

a) $\frac{2}{y} = \frac{3}{y-2}$

Answer (a) ____________________________ [2]

b) $12x - 48 = 7x - 13$

Answer (b) ____________________________ [2]

c) $-2x + 4 \leq 8$

Answer (c) ______________________________ [2]

8. The distance around a rectangular field is 400 m. The length of the field is 26 m longer than the breadth.

a) Calculate the length and breadth of the field.

Answer (a) ______________________________ [4]

GOOD LUCK!!!!!!!!!!!!!!

APPENDIX K: Marking schemes for pre-test

Marking Scheme for Pre-Test

1. a) 3 terms

   b) 2 terms

2. a) $4a + 2a - 3b - b$

   $= 6a - 4b$

   b) $-10b - a$

   $= -a - 10b$

3. $a)$ $4x^3 - 6x^5y$

   b) $3t - 1 - 2t + 4$

   $= 3t - 2t - 1 + 4$

   $= t + 3$

   c) $4a^2 + 2ab - 2ab - b^2$

   $= 4a^2 - b^2$

4. a) $\frac{3}{a+3} + \frac{2}{a-1}$

   $= \frac{3\,(a-1)+2(a+3)}{(a+3)(a-3)}$

   $= \frac{3a-3+2a+6}{(a+3)(a-3)}$

   $= \frac{3a+2a-3+6}{(a+3)(a-3)}$

   $= \frac{5a+3}{(a+3)(a-3)}$

b) $\frac{4c}{2d}\ \frac{c}{d}$

$= \frac{4c - c}{2d}$

$= \frac{3c}{2d}$

c) $\frac{2x}{y3} \div \frac{4x^2}{y}$

$= \frac{2x}{y^3} \times \frac{y}{4x^2}$

$= \frac{2xy}{4x^2y^3}$

$= \frac{1}{2xy^2}$

5. a) $x + 8$

b) $x - 10$

c) $x + y$

6. a) $2xy\ (x + 8y^2)$

b) $y(x + 1) - z\ (x + 1)$

7. a) $1 - (-2)$

$= 1 + 2$

$= 3$

b) $2\ (-2)^2 + 1$

$= 2(4) + 1$

$= 8 + 1$

$= 9$

8. a) $3x + 1 - 2x + 7 = 0$

$3x - 2x + 1 + 7 = 0$

$x + 8 = 0$

$x = -8$

b) $6 - 1 = 5x + 3x$

$5 = 8x\ (divide\ both\ sides\ by\ 8)$

$x = \frac{5}{8} = 0.625$

9. a) $2x\ \leq 4\ (divide\ both\ sides\ by\ 2)$

$x\ \geq 2$

b) $2x + 3\ > 11$

$2x = 11 - 3$

$2x = 8\ (divide\ both\ sides\ by\ 2)$

$x = 4$

APPENDIX L: Marking scheme of post-test

Marking scheme for Post Test

1. a) 4 terms

   b) 2 terms

2. a) $-2x + 3x + 6y + y$

$= x + 7y$

b) $x^2 + 2$

2. a) $(-3)^2a^2b^4c^6)$

$= 9a^2b^4c^6$

b) $(x-5)(x-5)$

$= x^2 - 5x - 5x + 25$

$= x^2 - 10x + 25$

3. Quotient = $y^2 + 7y - 17$ $and\ the\ remainder = 44$

4. $6x(x^2 - 1)$

5. $a)$ $\frac{5\,(a+3)-2(a-2)}{(a+3)(a-2)}$

$$= \frac{5a + 15 - 2a + 4}{(a+3)(a-2)}$$

$$= \frac{5a - 2a + 15 + 4}{(a+3)(a-2)}$$

$$= \frac{3a + 19}{(a+3)(a-2)}$$

b) $\frac{5}{a-2} \times \frac{a^2-2a}{15}$

$$= \frac{5(a^2 - 2a)}{15(a-2)}$$

$$= \frac{a}{3}$$

6. $a)$ $2(y-2) = 3y$

$2y - 4 = 3y$

$2y - 3y = 4$

$y = -4$

b) $6x(x^2 - 1) = 0$

$6x = 0 \; or \; x^2 = 0$

$x = 0 \; or \; x = 1 \; or \; x = -1$

c) $-2x \; \leq 8 - 4$

$x \geq -2$

7. a) $KE = \frac{1}{2}MV^2$

$\sqrt{V^2} = \sqrt{\frac{2KE}{M}}$

$V = \sqrt{\frac{2KE}{M}}$

b) $KE = \frac{1}{2}\,(4)(4)^2$

$= 32$

8. a) $400 = 2(a + 26) + 2a$

$a = 87, Length = 226\, m \; and \; braedth = 87$

9.

	Present age	In 9 years' time	3 years ago
Mother	X	x + 9	x – 3
Her Son	Y	y + 9	y - 3

$x + 9 = 2(y + 9) \ldots\ldots\ldots\ldots\; 1$

$x - 3 = 4(y - 3) \ldots\ldots\ldots\ldots .2$

10. a) $p = 2l + 2b$

$25 = 2(x + 3) + 2(3x - 1)$

$25 = 2x + 6\; 6x - 2$

$25 = 8x$

$x = \frac{29}{8} = 3.625$