Use of Nonstandardised Measurements To Enhance Student Learning of Units

N. !Gaoseb and C. D. Kasanda

Department of Mathematics, Science and Sport Education

University of Namibia

Paper presented at the 2nd Mathematics Congress

Held at Namib High School, Swakopmund

13 – 16 May 2007

MEASUREMENT ACTIVITY SEQUENCE

Measurement is described in the directives of the National Council of Teachers of Mathematics (NCTM, 1989: 51) as "of central importance to the curriculum because of its power to help children see that mathematics is useful in everyday life and to help them develop many mathematical concepts and skills." At primary level, the instruction emphasized the importance of establishing a firm foundation in the basic concepts and skills of measurement. For example, understand the attributes of length, capacity, mass, weight, area, volume, time, temperature, and angle. However, when learners reach secondary level, still many do not have the sense of estimating one centimeter, one centimeter square, or hectare. This can be attributed to the way measurements are introduced and used in the schools. In fact metric units are just taken for granted.

We need to inculcate into our learners the sense of ownership of the units of measurements, even the metric units. This is only possible if we revisit the nonstandard units our ancestors used to measure objects as the basis of known (contextualizing) and move to the unknown, the metric units. The metric systems of units have evolved since the adoption of the first well-defined system in France in 1791 (Wikipedia, 2007). Learners need to see the necessity of using the metric systems and this can only be done if they work with nonstandard units and see the inconsistency in their measurements.

COMPARISON

The measurement activity sequence entails three components: comparison, use of units, and the use of instruments. Basically, the measurement activities need to start off with direct measurements such as measuring the length of a desk, the width of a door, the thickness of a book, area, volume of cuboids, etc. Once, these measurements are done and discussions around them satisfactorily carried out, then the activity sequence can now move to indirect measurements.

Indirect measurement is not obvious as it makes use of concepts learned in mathematics and other related subjects. For instance, the height of a flagpole can be measure indirectly by making use of similarity. In this case we can make use of the shadow or the reflection of light (see figure 1).

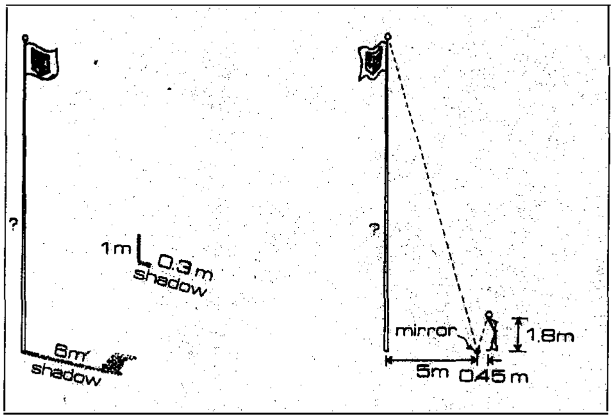

Figure 1: Height of a flagpole

Indirect measurement is one area where learners' questions such as, "Why do we need to learn this?" can easily be answered. By making use of the similarity concept we can now figure out the height of the flagpole in the drawing above as 20 meters.

Another example of indirect measurement would be the distance between the earth and the sun. It is not possible to take a rope or a string and go to the sun in order to measure the distance (even if we go at night). So, how did we figure out the distance? Astronomers have come up with some brilliant ways to determine the distances between planets. The orbits of the planets are elliptic; however, we can approximate it to a perfect circle at say its furthest points (see figure 2). With the technology of today, the distance from the earth to Venus can be measured using radar. We can also determine the angle between the Sun, Earth and Venus. What mathematics do we then need to solve the problem? Basically, tangent to the circle as well as the trigonometric ratios are applied.

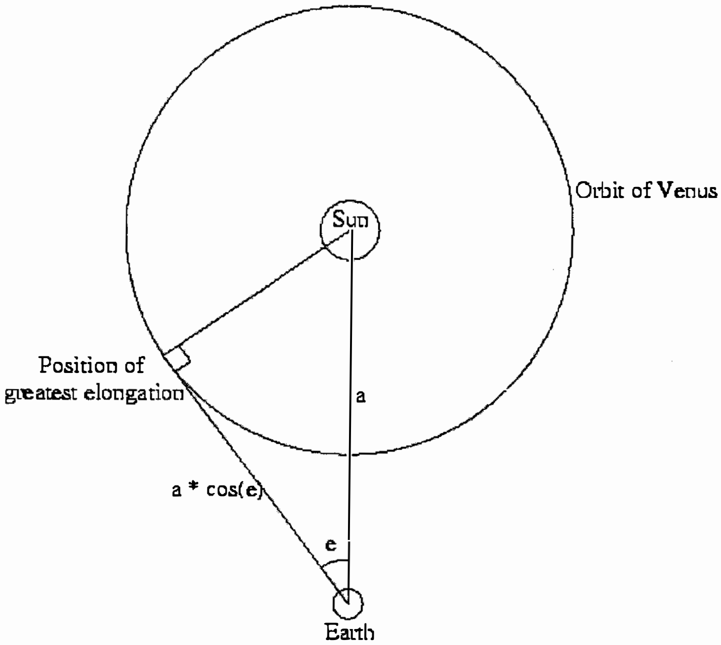

Figure 2: Indirect measurement of the distance to the sun

Another very simple indirect measurement would be about displacement. When measuring the volume of an irregular object such as a stone, one could make use of water and go through the procedures of displacement.

All in all, measurements must not stop with direct measurements, but learners need to be challenged to find ways to measure things they cannot measure directly. In so doing they will be forced to apply what they have learned in real life situations.

USE OF UNITS

The suggested measurement activity sequence for the use of units is starting with nonstandard units and then move to standard unit (van de Walle, 2005; Clements, 1999). The rationale for starting off with nonstandard unit is given in figure 3.

Rationale for informal units (nonstandard units)

- Informal units make it easier to focus directly on the attribute being measured. For example, instead of using square centimeters to measure area, an assortment of different units, some of which are not square, can be used to help understand the essential features of area and units of area.
- By selecting units carefully, the size of the numbers in early measurements can be kept reasonable.
- Learning to measure is different from learning about the standard units used to measure.
- Informal units provide a good rationale for standard units.
- Informal units can be fun.
- Learners must eventually develop a familiarity with the most common standard units. Also learn the appropriate relationships between standard units.
- Once a measuring concept is fairly well developed, it is frequently just as easy or even easier to use standard units.

Figure 3: Importance of the informal (nonstandard) units of measurement

Nonstandard measures were generally derived from an association with a part of the human body. For instance:

- the digit (width of the forefinger - in Latin *digitus* = finger)
- the inch (width of the thumb)
- the foot (from where 30 cm ruler originates)
- the cubit (*theoretically* the distance between the elbow and the middle finger)
- the pace (or double step)
- the fathom (finger-tip to finger-tip with arms outstretched)

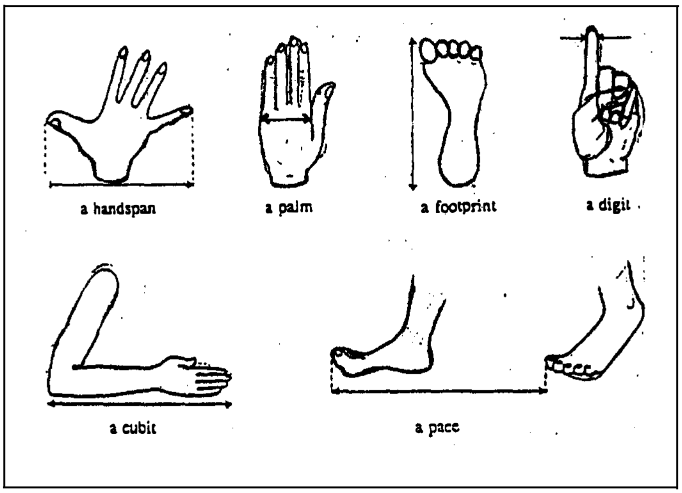

Figure 4: Nonstandard units of measure

Learners enjoy measuring things with their body parts. We need to introduce learners to all sorts of nonstandard units; length, area, volume, weight, time, and so forth. Learners must see the inconsistencies in using nonstandard units hence, the rationale for coming up with standard units; units that will be acceptable in the whole wide world. Learners need to develop ownership of the units of measurement and have the sense of why we needed to standardize the unit. Figure 4 is just one way of measuring the area with nonstandard units.

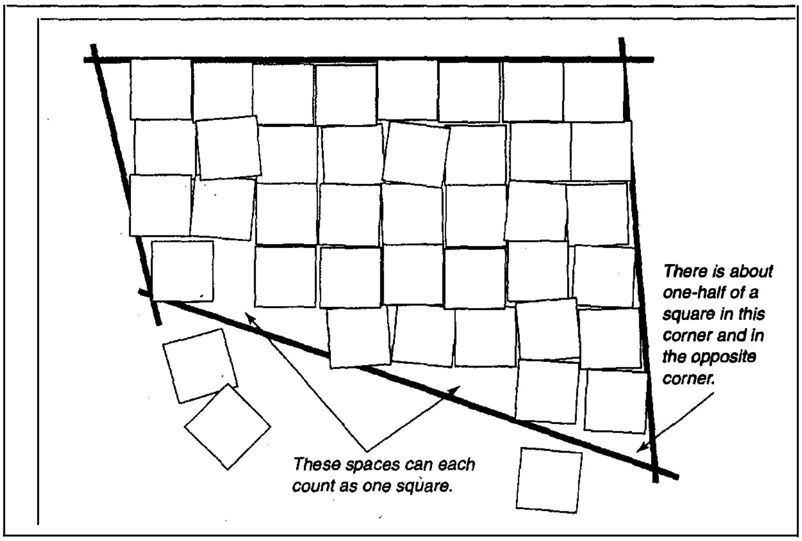

Figure 4: Measuring area using unit pieces of paper

USE OF INSTRUMENTS

An activity that would foster the ownership of the systems of measurement would be if we allow learners to create their own instruments. We need to encourage our learners to come up with their own instruments of measuring length, area, volume, weight, and time for instance. One such activity that pre-service teachers enjoy is the making of rulers with nonstandard units (body parts). Once they make the ruler, they give a name as well as the unit of measure (see figure 5). Activities of measuring given lengths are carried out to test the practicality of the instruments. Each student or group promotes their instrument.

Digit ruler

Figure 5: Digit ruler

Finally, learners can inspect the rulers of others and write a report on the quality of the rulers and their accuracy as compared to the standard ruler. Note the above Digit ruler does not have numbering. The name is excellent, because it informs us that the intervals on the ruler are determined by the digit. That means the width of the forefinger. However, students can be more innovative and come up with their own names and give the unit of measurement as a digit. A summary of the measurement activity sequence is given in figure 6 below.

Comparison:	Direct	→	Indirect measures
Use of units:	Nonstandard	→	Standard units
Use of instruments:	Nonstandard	→	Standard units
Student made		Conventional	

Figure 6: Measurement Activity Sequence

IMPLICATIONS TO TEACHING AND LEARNING MATHEMATICS

In school mathematics, measurements are applied in many topics. For example, estimation, directed numbers, standard form, limit of accuracy, ratio, proportion and rate, money, mensuration, geometric constructions, locus, graphs in practical situations, graphs of functions, trigonometry, etc. Practically, all topics make use of measurement. Let us look at an extract from a lesson on limit of accuracy. This extract came from the observations that were carried out in schools in 2001 (MASTEP).

Limit of Accuracy
(T = Teacher, L = A learner, C = Choir response)

T: So, last week we talked about the rounding of numbers. (Cough)
Then, for this week the topic is, today is the limits and accuracy.
(Pause and noise)
In the limit of accuracy is whereby we are going to, to try and find out because, when we were rounding you realize that most of the numbers they are already round, like if I ask you, "What is your height, how, how tall are you?" You may tell me that you are one hundred and fifty centimeters for instance, for instance. Which is a number which is a hundred number already. You might be 150 centimeters coming some millimeters for instance or I also ask you may be how, "What is your mass?", "What is your mass?"

L: Sixty nine kg.

T: Sixty nine kg? But if you look at the scale you might not be necessarily 69 exactly. May be comma some, some grams OK.
Let's look at aah, suppose you are fifty kg, then in the limits of accuracy, because you said that fifty kg, it may not be necessarily fifty kg. so then what we are going to look at is, what are the possible numbers, that you can round such that if you round all these numbers they will give you fifty.
If you round, if you round all these number correct, say you were given, aah, say to the nearest grams, it will give you fifty kgs, or to the nearest milligrams or so, it will give you exactly fifty kg. what are the possible numbers? So this is what we are going to look at in eeh, limits of accuracy.

T: Let's take one example (pause, writing on the chalkboard, cough)
That is the example, number one we are going to look at, so to find those numbers, we have to find what is called lower bound and upper bound. So, we are given the number is, given correct to the nearest unit. Then we want to find the set of number of the, the range of numbers that if you round those numbers for instance to the nearest unit, then we'll get fifty, exactly fifty.
So in simple terms this one says, if x is a number, then what is the highest number here, which you can round and then that number will be more than, over the, more than fifty and what is the smallest number that you can round, such that, that number will be , will give you exactly fifty.
So the lower bound and the upper bound; to get a lower bound of this number

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because, they said to the nearest unit, then we take, how do we write a unit? Hmm, anyone who knows what we mean by unit. So the unit is the same as a?

L: A one

T: A one, so then take a one, take a one divide by two, then you get how much .

L: Zero point five.

T: You get how much?

C: Zero point five.

T: Once you get a zero point five you have to get lower bound, you must take the number which is given, which is fifty, then you subtract what you got here, the zero point five. Then fifty minus zero point five. What do you get?

C: 49.5

T: Forty nine?

C: point five

T: point?

C: five

T: Point five. And for the lower bound, for the upper bound, take that number again which is given and then you and what we got here. Which is zero point five. You get how much?

C: 50.5

T: Fifty?

C: Point five

T: Point five, which aah, upper bound is 50.5 and our lower bound is 49.5, so therefore our number, this one, numbers of ours which is x could between fifty point five and forty nine point five. Fifty point five is the lower bound which is not included and 49.5, forty nine point five is also included and 50.5, if you round it correct to nearest unit, then you'll always get?

L: 50

T: You'll always get?

C: Fifty

T: You will always get fifty. Let's try to, to take some numbers which are between. Anyone who can give me any number which is between this, this number 49.5 and 50.5 whereby this is not included but 49.5 included. Any, any, any, anyone who can give me number. Just give me any number which is between there, hmm?

L: Zero point five

T: Zero point five? Is 0.5 between 49.5 and 50.5? give me any other number, yes please they are so many. Give me any number, give me as many as possible.

L: 50.1

T: Fifty?

L: point one

T: Point one. Anyone who can give me any other number? They are so many. Give me as many as possible.

L: Fifty point three ...

The above lesson would have provided an ideal situation for measurements. This is a lesson where learners would have carried out simple measurement, some mentioned in the lesson such as the height or the weight of the learners. Teachers jump too quickly into procedural knowledge before concepts are formed (Cangelosi, 2003). Consequently, because, they said to the nearest unit, then we take, how do we write a unit? Hmm, anyone who knows what we mean by unit. So the unit is the same as a?

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L: 50

T: You'll always get?

C: Fifty

T: You will always get fifty. Let's try to, to take some numbers which are between. Anyone who can give me any number which is between this, this number 49.5 and 50.5 whereby this is not included but 49.5 included. Any, any, any, anyone who can give me number. Just give me any number which is between there, hmm?

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The above lesson would have provided an ideal situation for measurements. This is a lesson where learners would have carried out simple measurement, some mentioned in the lesson such as the height or the weight of the learners. Teachers jump too quickly into procedural knowledge before concepts are formed (Cangelosi, 2003). Consequently, learners find it difficult to calculate the upper bound and the lower bound of numbers due to the fact that they do not have ownership of the measurements and discussion around the measurements (upper and lower bounds). Given for example a problem $\frac{4.2 \times 5.1}{1.5}$ to find the bounds, learners will only carry out procedures without understanding. Most of the learners do not realize that these are measurements hence; each value (4.2, 5.1, and 1.5) has a limit of accuracy as determined by the instrument of measurement.

CONCLUSION

If learners' initial explorations use nonstandard units, they will develop some understanding about units and come to recognize the necessity of standard units in order to communicate. Estimation activities should be integrated throughout measurement. Measurement activities sequence should guide teachers in carrying out measurement activities. That implies that in senior secondary classes learners should be encouraged to carry out indirect measurements or do investigations of how we came up with certain measurements.

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