STRUCTURAL CREDIT RISK MODELING USING MERTON MODEL AND ITS DEFAULT PROBABILITY: A CASE STUDY OF COMMERCIAL BANKS IN NAMIBIA

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Abstract

This research work presents a comprehensive study on commercial banks in Namibia, focusing on three main banks over the period from December 2011 to December 2021. The primary objective is to assess the credit risk position in the light of the Merton Structural credit Risk Model. The financial statements of these banks are analysed, specifically the balance sheets and statements of income, to extract relevant information for the computation of various ratios. The ratios examined include the working capital, total assets, retained earnings before interests and taxes ratio, and sales over total assets ratio. These ratios serve as risk factors for both the Merton Model and within the logit model framework. The Merton approach is utilized to estimate the default risk for the three commercial banks in Namibia, and the accuracy of these estimates is assessed using a range of different techniques. The efficiency of the estimates is assessed by testing the extent to which the predictive power of the estimates could be improved by incorporating other information publicly available in company accounts. The event of default is determined by the market value of the bank's assets in conjunction with the liability structure of the bank. When the value of the assets falls below a certain default point, the firm is considered to default. Through this research, valuable insights into the financial performance and default risk of the commercial banks in Namibia are gained, contributing to a deeper understanding of the banking sector in the country.

Keywords: Credit risk, Financial statements, Merton structural Model, Logit model, Default risk.

Contents

Declaration

I hereby declare that this thesis, titled Structural credit risk modelling using Merton model and its default probability: A case study of commercial banks in Namibia, is my original work. The thoughts, ideas, and opinions expressed in this document are entirely my own, unless otherwise cited and referenced. This work has not been submitted for any other degree, diploma, or certification, either in part or in its entirety. Furthermore, I understand that any form of plagiarism or academic misconduct is strictly prohibited and that such behavior would result in severe consequences as outlined by the academic regulations and policies of the institution. I take full responsibility for the integrity of this thesis and assure its compliance with all ethical guidelines and academic standards.

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Aina Nuugwanga Shaanika Date

Dedication

This paper is dedicated to my husband Mr Iipumbu Sakaria, whose belief in me has been a constant source of inspiration, your encouragement, understanding, and sacrifices have been instrumental in my pursuit of knowledge throughout this academic journey. To my incredible children, Malakia, Jennifer and Ndeuli, you have been my motivation to strive for excellence. You inspire me to push beyond my limits and pursue my dreams. Thank you for making me see this adventure through to the end.

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List of Figures

List of Tables

Glossary of terms

A – Assets

AIC – Akaike Information Criterion

BFI – Banking Financial Institutions

BON – Bank Of Namibia

CRS – Centre Research Services

D – Debt

DD – Distance to Default

DEC – Decentralised Ethics Committee

DPT – Default point

D-SIB'S –Domestic Systematically Important Banks

E – Equity

EAD – Exposure At Default

EBIT – Earnings Before Interest and Taxes

EDF – Expected Default Frequency

EL – Eexpected Loss

FICO – Fair Isaac Co.

ITC – International Trade Centre

KMV – Kealhofer McQuown Vasicek

L – liability

LDA – Linear Discriminant Analysis

LGD – Loss Given Default

LTD – Long Term Debt

M – Multivariate

MDA – Multiple Discriminant Analysis

MV – Measured Variables

N – Cumulative Standard Normal Distribution

NAD – Namibia Dollar

NAMFISA – Namibia Financial Institution Supervisory Authority

NBFI – Non Banking financial Institutions

NPL – Non Performing Loans

PD – Probability Of Default

RE – Retained Earnings

S – Sales

STD – Short Term Debt

T –Time

TA – Total Asset

TL – Total Liability

UL – Unexpected Loss

WC – Working Capital

1 Chapter

1.1 Introduction

Financial institutions such as banks provide loans and earn revenues primarily on the difference in the interest rates charged for credit accounts and the rates paid to depositors. Researchers have long studied loans and related credit risk disclosures. In the wake of the 2007-2009 financial crisis, interest in the analysis of credit risk in banks surged [8]. Analyzing credit risk is vital. Regulators and commercial banks progressively understand the effectiveness of employing mathematical models to minimize credit risk. There are factors affecting credit risks, macroeconomic factors (external) and microeconomic factors (internal). Models of credit risk measurement have focused on the estimation of the default probability of firms, since it is the dominant source of uncertainty in the lending decision. When developing these models there are three main factors to be considered namely: probability of default, the exposure of credit and the estimated rate of recovery. These models are used to calculate the probability of default, credit spread, credit default swaps, diffusion equation for a firm based on the value of its assets and liabilities [5]. The Merton model developed by Robert C. Merton in 1974, has emerged as one of the most widely used structural credit risk models in the financial industry. This model provides a framework for assessing the likelihood of default by incorporating the interaction between a firm's underlying asset value, its debt structure, and the prevailing market conditions. While the Merton model has been extensively studied and applied globally, its practical implementation and efficacy in the context of commercial banks in Namibia warrant further investigation. This thesis aims to contribute to the existing body of knowledge by conducting a comprehensive case study focused on structural credit risk modelling using the Merton model, specifically tailored to three main commercial banks operating in Namibia. By utilizing this model, we seek to estimate, evaluate the default probabilities of these banks, and to assess the accuracy and reliability of the model's predictions. This study will employ a quantitative research approach, utilizing financial data from a selected sample of commercial banks in

Namibia over a defined period. By conducting a thorough analysis and interpretation of the results, this research aims to provide valuable insights into the credit risk profiles of commercial banks in Namibia and offer recommendations for enhancing risk management practices within the banking sector.

1.2 Background of the study

The global financial crisis of 2008, the increasing financial complexities and economic uncertainties highlights the critical need for accurate credit risk assessment and effective risk management practices within the banking sector [27]. This holds true particularly for commercial banks, which play a vital role in the economic development of nations, including the case of Namibia. Namibia's commercial banking sector serves as a unique case study due to its distinctive economic dynamics and regulatory environment. Namibia has a financially sound and well-developed bank-based economy, especially the commercial banks still play a critical role in the country's financial system. As banks continue to face the challenges of managing credit portfolios, ensuring stability, and safeguarding against potential defaults, the implementation of robust credit risk models becomes imperative. One of the most powerful and widely accepted measurement for calculating bank default risk among academics and practitioners is Merton model [18, 25]. It can estimate the probability of bank default on a daily basis by using market value of bank equity in the stock market. We can measure the probability of default by using the distance to default which is the difference between the values of the company's assets and its face value of debt. Despite its widespread use globally, there is a limited understanding of its implementation and effectiveness in the context of Namibian commercial banks. Before the introduction of the Merton model, the assessment of default probability for corporate bonds involved fundamental analysis [6]. This approach relied on examining a company's financial statements to gauge the risk faced by investors when purchasing bonds. The methodology employed stochastic calculus and risk-neutral pricing theory in contrast to the Merton model. Modifications to these models were introduced, including empirical models that operate under the assumption that accurately assigning the mathematical risk of debt is challenging. Moreover, the models for assessing the risk associated with corporate bonds evolved into two primary categories: the reduced form and the structural models. In 1900, Louis Bachelier introduced a model for determining the price of a call option contract, assuming that the price process of a stock could be represented by a Brownian motion process [6]. The significant advancement in option pricing occurred in 1973 when Fisher Black and Myron Scholes introduced their renowned model. Concurrently, independently of Black and Scholes, [18] developed a bond pricing model. The authour [30], introduced an alternative to the Merton model by employing jump-diffusion processes. Additionally, [9] as well as [17] have contributed some of the most recognized models utilizing jump-diffusion processes. Furthermore, various extensions are suggested to enhance these models. By conducting a case study focused on the three main commercial banks in Namibia, this research aims to address the existing knowledge gap and contribute to the field of credit risk modeling. The research findings will provide commercial banks in Namibia with a comprehensive understanding of their credit risk profiles and enable them to enhance their risk management practices. Additionally, regulatory authorities and policymakers will acquire a deeper understanding of the Merton model's efficacy in assessing credit risk.

1.3 Problem statement

According to [16]; empirical research support demonstrates that banking failures or banking crisis have been caused by non-performing loans (NPL). It has come to realization to financial institutions that in order to avoid default and prevent bank failures, credit risk should be managed effectively. There is a need to analyze banks structural credit model, assess their exposure to ensure the stability of the institutions, the financial system and a country's economy at large. This thesis primarily focuses on examining how commercial banks in Namibia can employ the Merton model and its default probability to effectively navigate and mitigate their credit risks. It seeks to identify the tools and techniques available to them and assess the extent to which their performance can be enhanced through the implementation of sound credit risk management policies and strategies. The need to improve overall performance and secure a competitive advantage in order to overcome significant credit risks that eventually lead to financial distress, is more than before heightened.

1.4 Objectives of the study

The main objectives of this thesis is to analyse the credit risks management of commercial banks in Namibia under the light of the structural Merton model approach.

Specific objectives are:

a) To estimate default risk.

b) To assess the reliability of these estimates.

c) To evaluate, illustrate findings and interpret simulations of probability of default.

d) To compare results between these commercial banks.

1.5 Significance of the study

The study is significant to commercial banks, customers of banks and other financial institutions, and the country's economy. All organizations that have an exposure to credit risks in Namibia will find some significance in the study. Credit of any kind has a negative impact on overall performance, if not assessed, planned and well managed in advance. Improper credit risk management reduces profitability, affects the quality of assets, increases loan losses and non-performing loans which may eventually lead to financial distress [21]. So far there is no research study on credit risk analysis of commercial banks in Namibia using the structural Merton model approach in the literature. Every sector of the economy at large may suffer significantly as a result of improper credit risk management. Since the major source of income for commercial banks comes from loans, it is therefore vital to analyse and manage these risks by determining the credit worthiness and quantifying the risk of loss to which the lender is exposed. The pivotal role played by the banking sector in economic development underscores the importance of credit risk analysis by banks. This proactive approach aims to prevent potential failures in the future. The study, prompted by the aforementioned concern, focuses on examining credit risks in commercial banks in Namibia.

1.6 Limitations of the study

The structural model approach used in this study does not observe the market value of a firm's assets, but uses an annual report which only provides an accounting version of its assets. The Merton model suffers from the fact that asset values cannot be observed in the market in the same way that stock prices are. In general, the fact that the credit health of counterparties and investors evolves constantly due to today's complex markets makes it difficult to monitor, keep up with and anticipate the pace of change. Secondly, the study only used secondary data to answer the research objectives, and not the current year. This is because we use available historical data to model the expected credit risk. Such a situation may lead to inappropriate estimates of default rates and correlations, with respect to the current financial standing of the company. Another limitation of the study, is that Merton model assumes that stock prices follow a log-normal distribution based on the principle that asset prices cannot take a negative value. Lastly, Ethical considerations pose a significant constraint in acquiring data from banks, as they maintain strict confidentiality. I had to depend on information available in financial reports, which proved challenging to pinpoint.

1.7 Delimitations of the study

As part of the study's delimitations, an examination of credit risk was conducted utilizing the Merton model, which incorporates credit scores. Information was sourced from the balance sheets of the primary financial reports of the three major commercial banks. Adjustments were made to the figures to align with the Merton model, facilitating the estimation of default probability based on actual data. This risk model does not have the ability to determine exactly the overall credit position of a bank but it is a useful tool in decision-making process. This research is built around publicly disclosed financial information, statements and data willingly provided by the targeted commercial institutions.

1.8 Structure of the study

This research is structured into six main sections. The initial section encompasses the abstract, table of content, declaration, dedication, acknowledgements, list of figures , list of table , glossary of terms, offering a comprehensive introduction, the background of the study , a problem statement, the study's objectives, the significance of the study , Limitations of the study , the delimitations of the study and an outline of its structure. The second section delves into a literature review focused on commercial banks and credit risk management. The third section explores the theoretical framework, centring on the Merton model and credit scoring. The fourth section is the Data analysis and the fifth section comprises both the results and the ensuing discussion. Finally, the sixth section presents the conclusion and recommendations.

2 Chapter

2.1 Literature review

Financial institutions and markets are faced with many risks, but the major risk banks encounter is credit risk. Past and recent literature show that the majority of failures are due to poor risk management, non-use of prudential classification and poor risk assessment methods. One of the most common indicators that is used to identify credit risk is the ratio of non-performing loans (NPL) [24]. Credit risks in commercial banks are caused by various macro and microeconomic factors, bondage, changes in economic law and regulations. There are structural weaknesses that need to be addressed to enable the financial sector to contribute meaningfully to the overall performance of the country's economy. Past analysis has shown that commercial banks loans risks can be reduced by; restructuring debt program, additional documentation and guarantees, reviving the work of management, the requirement payment of a guarantee and implementations of legal actions. Not much research on credit risk measurement and management is visible in Namibia. What is missing from the past studies is a comprehensive and structural approach in credit risk in the banking sectors. It is therefore important to expand research of this nature. Namibia's financial sector is closely tied to South Africa and is one of the most sophisticated and well-established financial systems in Africa. According to [26], Namibia's financial system is sound and well-functioning. It is composed of the banking and non-banking being the dominant financial institutions.

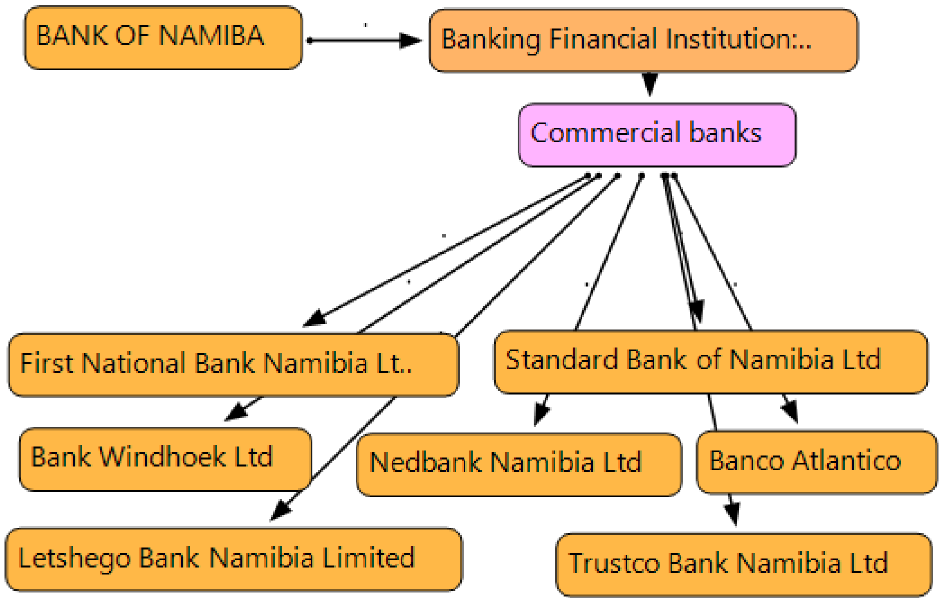

Figure 1: Banking Financial Institutions (BFI)

The banking financial institution (BFI) in Namibia consists of 9 fully-fledged commercial banks, one branch of a foreign banking institution and one representative office of a foreign banking institution [22]. The banking financial institution is however, dominated by the big four banks that are considered to be Domestic Systematically Important Banks (D-SIBs). The two largest banks are owned by South African banks, while the remaining two include a Swiss-owned bank and one that is jointly owned by Namibian and South African firms. The Namibian banking sector is regulated by the central bank, Bank of Namibia(BON), with the objectives to serve as the instrument to control money supply as well as to ensure financial stability, price stability and economic growth, among other mandates [22].

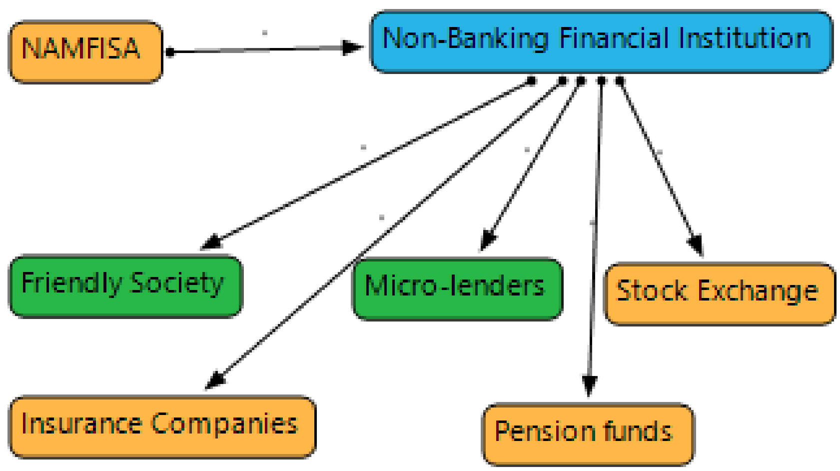

Figure 2: Non-Banking Financial Institutions (NBFI)

The non-banking financial institutions (NBFI) such as the micro-lenders, stock exchanges, friendly societies, pension funds institutions and insurance companies are regulated by the Namibia Financial institution Supervisory Authority (NAMFISA) [22], which was created in 2001. It aims to regulate and supervise financial institutions and financial intermediaries to foster a stable, fair non-banking financial sector and to promote consumer protection and provide sound advice to the Ministry of finance. Most of these institutions regulated by NAMFISA are private, with strong ownership links to South Africa.

There are structural weaknesses that need to be addressed to enable the financial sector to contribute meaningfully to the overall performance of the country's economy. Having efficient behavior monitoring models is crucial to ensure that bank staff exercise caution in minimizing operational risks. This involves furnishing customers with comprehensive information about financial instruments and the bank's imposed restrictions. Such measures are essential for safeguarding the interests of the financial institution. Researchers point out that risk modeling uses a variety of techniques incorporating internal and external data in order to analyze portfolios, produce forecasts and conduct stress testing and scenario analysis. Statistical techniques can be used to analyze or determine risk levels involved in credits, finances, and loans, thus default risk levels [13]. There are two main classes of credit risk models namely; the structural and reduced form models.

The structural approach aims to provide an explicit relationship between default risk and capital structure, while the reduced form approach models of credit defaults as exogenous events driven by a stochastic process. These models are used to calculate the probability of default for a firm based on the value of its assets and liabilities [5]. The structural models are useful in areas such as counterparty credit risk analysis, portfolio/security analysis and capital structure monitoring, while the difficulty in calibration limits their presence in front-office environments [29]. On the other hand, Reduced form models are widely used on credit security trading floors where traders require fast computation tools to help them react to market movements quickly. When developing these models there are three main factors to be taken into account namely: probability of default, the exposure of credit and the estimated rate of recovery. Since we are using banks internal credit models to devise capital adequacy standards, hence the structural credit risk models come to play. Structural models calculate the probability of default, credit spread, credit default swaps, diffusion equation for a firm based on the value of its assets and liabilities [5].

In particular the Merton model is used to assess the credit risk of a company's debt. It uses the Black-Scholes-Merton option pricing methods and is structural because it provides a relationship between the default risk and the asset (capital) structure of the firm. When calculating the price of defaultable bonds the Merton model relies on a number of assumptions; the assumption concerning the capital structure of the bond-issuing firm, the assumptions concerning the market efficiency and the assumption made on the underlying stochastic price process. Loan application processing involves making the accept or reject decision and determining the amount of money to be loaned to the customer, the interest rate that should be charged and the repayment terms of the loan. Certain measures are taken by financial institutions to minimize these risks in every capital borrowed out of its customer [5]. There has been a rapid growth of financial risk modeling in recent years as technology advances and supply of human capital increases. Risk modeling helps identify, analyze, and mitigate risks so you're prepared to deal with them should they occur. The models provide information on the level of a borrower's credit risk at any particular time to make sound decisions on whether to charge high interest rate for high-risk loans or reject the loan application altogether. There are factors that lenders should consider when assessing the level of credit risk; the probability of default (PD), loss given default and exposure at default.

Probability of default (PD) is the likehood that a loan will not be repaid. Loss given default (LGD) is the fraction of the exposure at default (EAD) that will not be recovered in the case of a default event. The exposure at default (EAD) is an estimation of the extent to which a bank may be exposed to a counterparty in the event of and at the time of , that counterparty's default. The components are calculated for the minimum of a year or the maturity of the loan.

$$ExpectedLoss(EL) = PD * LGD * EAD$$

While the unexpected loss (UL) represents the volatility of actual loss rates that occur around EL. The presence of UL creates the requirement for a capital cushion to ensure the viability of the bank during the year when losses are unexpectedly high. The basic purpose of analysis of credit risk as part of Basel II is to provide for adequate capital as a safety net against possible default, it is a method of quantifying the chance of default [1]. If a frequency function approach for estimation of probability is assumed, the probability density function needs to be estimated based on the data to be able to estimate the probability of default. This involves statistical science to find an empirically valid estimate of default probabilities representative of the population under consideration. Techniques such as discriminant analysis, neural networks and regression techniques can be used to find the estimate of the default probabilities. Multivariate analysis is all statistical methods that simultaneously analyze multiple measurements on each individual or object under investigation [7]. It is used to detect subtle changes that might not be detectable, except with very large, prohibitively expensive samples considering only one variable at a time. The objective is to predict the changes in the dependent variable, using the changes in the independent variables. When considering the statistical analysis of a group of correlated variables we shall find it convenient to adopt the terminology of the theory of regression and refer to these variables which we shall analyze in relation to other variates or classifications as correlated dependent variances.

3 Chapter

3.1 Theoretical framework

In this chapter, we introduce the theoretical foundation supporting our implementation of Merton's model and credit scores. The aim is to provide the reader with an intuitive grasp of the model and highlight its crucial properties relevant to comprehending this thesis. The initial section of this chapter discusses the fundamental assumptions of the Merton model, while the subsequent section delves into the pricing formulas and the method for determining the default probability.

3.1.1 Merton model

When a company's liability exceeds its assets there will be a default. Structural models are used to calculate the probability of default for a firm based on the value of its assets and liabilities [5]. It is a multivariate technique incorporating measured variables and latent constructs and explicity specifying measurement error. The aim for structural equation modeling is to understand the patterns of correlation/covariance among a set of variables and to explain as much of their variance. The purpose of the model account for variation and covariation of measured variables (MV) and regression analysis tests models and relationships among MV's. Modeling relies on several statistical tests in order to determine the adeguacy of the model fit to the data. Statistical tests are valid if certain assumptions are met.

Structural models were initiated by [18], they assess the credit risk of a company's debt and use the Black-Scholes option pricing framework to characterize default behavior. It ingeniously employs modern option pricing theory in corporate debt valuation. The Merton model estimates the probability of defaults of a corporate based on a simple structure of its balance sheet [19]. The probability of default of a firm is calculated based on the value of its assets and liabilities which can further be divided into debt and equity. The Merton model treats bankruptcy as a continuous probability of default, to some extent this answers the economic causes of default. It provides a relationship between the default risk and the asset (capital) structure of the firm.

In the equation

$$A = E + L$$

Where A is the Assets which is unobservable, E is Equity which is observable and L is the Liabilities also unobservable. Merton model assumes a single liability L, with maturity T, the firm's value to the shareholder

$$E = A - L.$$

Structural credit risk models view firm's liabilities as contingent claims issued against the firm's underlying assets [5]. Risky debt is the default-free value of the debt minus the expected loss and it is derived from the value of uncertain assets. Equity is a residual claim on assets after debt has been repaid. The value of the equity E_T of the call option at maturity time T, depends on the final value of the underlying, A_T

$$E_T = max(A_T - D, 0),$$

where D is the debt.

In practice firms have multiple maturities for their liabilities, hence L is chosen based on the whole liability structure of the firm and is commonly set to a value between the value of the short term liabilities and the value of the total liabilities.

3.1.2 Applied Merton model

Assuming a lognormal distribution for the asset returns, use Black-scholes Merton Equations to relate the observable market value of equity E, and unobervable market value of assets A.

$$E = AN(d_1) - Le^{-rT}N(d_2), \tag{3.1}$$

where r is the risk free interest rate, N cumulative standard normal distribution,

$$d_1 = \frac{\ln\left(\frac{A}{L}\right) + (r + 0.5\sigma_A^2)}{\sigma_A\sqrt{T}} \tag{3.2}$$

and

$$d_2 = d_1 - \sigma_A\sqrt{T}. \tag{3.3}$$

This approach solves for (A, σ_A) using a 2 by 2 system of nonlinear equations.

$$\sigma_E = \frac{A}{E}N(d_1)\sigma_A. \tag{3.4}$$

If the equity time series has n data points and asset values $A_1, A_2, ..., A_n$, then

$$E_1 = A_1 N(d_1) - L_1 e^{-r_1 T_1} N(d_2) \tag{3.5}$$

$$\vdots$$

$$E_n = A_n N(d_1) - L_n e^{-r_n T_n} N(d_2). \tag{3.6}$$

The function directly computes the volatility of assets σ_A from the time series $A_1, A_2, ..., A_n$ as annualized standard deviation of the log returns. This value is a single volatility value that captures the volatility of assets during the time period spanned by the time series.

After computing the values of A and σ_A then the distance to default is given by

$$d_1 = \frac{\log A + \left(\mu_A - \frac{\sigma_A^2}{2}\right) T - \log(L)}{\sigma_A \sqrt{T}}. \tag{3.7}$$

The drift parameter μ_A is the expected return for the assets, can also be equal to the risk-free interest R and has a value based on expectations for that firm. The probability of the asset value falling below default point at the end of the time (T),

$$PD = 1 - N(DD).$$

The probability of default provides an estimate of the likelihood that a borrower will be unable to meet its debt obligations. Evaluating the probability of default of a firm is the initial step, while serving the credit exposure and potential misfortunes faced by a firm.

The main challenge with this approach is the real value of a firm's assets is not directly observable since we only know the firm's accounting value of assets, which might differ significantly, the approach does not observe the market value of a firm's assets. Furthermore, the asset value is not observable daily, only when accounting reports are publicly disclosed which normally takes place on a quartely basis. The basic idea of the model is that the company defaults if the value of its assets is less or below the value of the debt of the company at the time of the maturity of the debt. Ever since the works of Black-Scholes and Merton started the literature of structural credit risk modeling, extension to merton model has been proposed by several researchers and these extensions have been criticized for a number of assumptions [19]. Among others the KMV model is an upgrade of the Merton model which has been proposed and used by Moody's [15]. It is developed to provide probabilistic assessment of firm's likelihood to default. KMV corporation has observed from a sample of several hundred companies that firms are generally more likely fo default when their asset values reach a certain critical level somewhere between the value of total liabilities and the value of short-term debt [11]. The default takes place as soon as the market value of these assets falls below a certain threshold. The Default Point (DPT) is roughly approximated by:

$$DPT = STD + 0.5LTD,$$

where STD is Short Term Debt and LTD is Long Term Debt.

Before computing the probability of default KMV implements an intermediate phase of computation of Distance to Default (DD) [11], defined as:

$$DD = \frac{E(A_T) - DPT}{\sigma_A} \tag{3.8}$$

There are two stages in calculating the Distance to default; Absolute Distance to Default expressed in percentages of expected assets and is the distance between expected assets and Default point. This can be displayed as a sum of initial distance and the growth of that distance within the period T.

$$DD' = \ln\left(\frac{A_0}{DPT}\right) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)T, \tag{3.9}$$

where μ_A is no longer risk-free rate but expected rate of the return of the firm's asset and DPT is a default point instead of nominal value D of the face value of the debt. The rate of return is normally distributed, consequently future value of investment is distributed lognormaly.

$$\ln\left(\frac{A_0}{DPT}\right) N\left[\left[\mu_A - \frac{\sigma_A^2}{2}\right]T, \sigma_A\sqrt{T}\right]. \tag{3.10}$$

Dividing absolute value DD' with volatility of assets, we can calculate DD in relative terms as a multiplier of standard deviation to get:

$$DD = d_2 = \frac{ln\left[\frac{A_0}{DPT}\right] + \left[\mu_A - \frac{1}{2}\sigma_A^2\right]T}{\sigma_A\sqrt{T}}. \tag{3.11}$$

In the above equation we replaced r with μ_A and D with DPT, hence similar to d_2. The similarity is the result of a relationship between the risk neutral probability and the actual probability. The actual probability uses the expected return of the assets in the drift term, while the risk-neutral probability uses the risk free rate r. A simplified situation of normally distributed assets value after period T the probability of default is given as

$$PD = 1 - N[d_2] = N[-d_2]. \tag{3.12}$$

The KMV model operates on the historical set of frequencies of default rather than on theoretical normal or log-normal distribution. The probability of default is replaced with expected default frequency. Several improvements of the Merton model are implemented mainly in the areas of structure of the debt, the threshold for default estimated as the short-term debt plus one half of the long term debt, and the loss distribution an empirical relationship between so-called distance to default (DD) and the expected default frequency (EDF) [19].

A fundamental premise of Merton models is that a default occurs if the value of assets falls below a critical value of liabilities. If the firm pays no dividends, the equity value can be determined with the standard Black-Scholes call option formula:

$$E_t = A_t \cdot \phi(d_1) - Le^{-r(T-t)}\phi(d_2) \tag{3.13}$$

where

$$d_1 = \frac{ln(A_t/L) + (r + \frac{\sigma^2}{2})(T-t)}{\sigma\sqrt{T-t}}, \tag{3.14}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{3.15}$$

and r denotes the logarithmic risk-free rate. Re-arranging Black-Scholes formula we get:

$$A_t = [E_t + Le^{-r(T-t)}\phi(d_2)]/\phi(d_1). \tag{3.16}$$

If we go back in time, say n trading days, we get a system of equations

$$A_t = [E_t + L_t e^{-r(T-t)}\phi(d_2)]/\phi(d_1) \tag{3.17}$$

$$A_{t-1} = [E_{t-1} + L_{t-1} e^{-r_{t-1}(T-(t-1))}\phi(d_2)]/\phi(d_1) \tag{3.18}$$

$$\vdots$$

$$A_{t-n} = [E_{t-n} + L_{t-n} e^{-r_{t-n}(T-(t-n))}\phi(d_2)]/\phi(d_1). \tag{3.19}$$

3.1.3 Credit scores

Credit scoring models are used to predict the creditworthiness of a customer and determine whether they will be able to meet a given financial obligation often by estimating default probability. Credit scoring methods use data on observed borrowers' characteristics either to calculate the probability of default or to sort borrowers into different default classes. The aim of the credit score model is to build a single aggregated risk indicator for a set of risk factors. These models use statistical or mathematical methods to classify customers between groups (quantitative) while qualitative methods are ones more judgmental and subjective in nature. Since there is no objective base for making decisions or judgements about a customer, screening between and among good and bad customers is difficult in qualitative methods. The models allow a financial institution to minimize the risk of loss by setting decision rules regarding which customers receive loan and credit card approvals. The first retail credit scoring model for credit cards in the US was proposed in around 1941 which are based on certain parameters for scoring credit card applications. The increase in the US credit card business mandated a reduction in the decision time. In 1956, Fair, Isaac and Co. (FICO) was established to help consumer credit evaluation and in the 1960's computers were bought to process credit card applications [23]. Technological advancement makes it possible to implement automated objective procedures that are based on formal statistical models of customer behaviour. These procedures permit the decision to take into account more potential factors than a human could. Automated decision making could go wrong hence, allowance errors is considered and should be taken with caution. Theoretically, there are many quantitative credit scoring methods including linear probability model, logit/probit, multiple discriminant analysis (MDA), recursive Partitioning algorithm, and neural network [20].

The discriminant and logit analyses approaches have been used to predict business failure or success, while MDA have been used for credit scoring and not many used MDA as a risk assessment model. Despite some shortcomings, MDA is considered the best credit classification method among all other parametric and non-parametric methods especially in the bank lending environment [20]. One can assume that for each application there are a specific number of explanatory variables available. In a simple case of two subsets, the goal is to find the linear combination of explanatory variables, which leave the maximum distance between means of the two subsets. Consider the distributions $p(y|M)$ and $p(y|N)$ which are multivariate normal distributions with common variance then,

$$A_G = x|\sum w_i x_i > c,$$

where explantory variables x_i and w_i are associated coefficients (weights) in the linear combination of explanatory variables and c is a constant. Taking $s(y) = \sum w_i x_i$ the problem reduces to only on a dimension. There is a prevalent misunderstanding regarding the requirement for multivariate normality. The optimal application of the linear discriminant rule occurs when variables adhere to a multivariate ellipsoidal distribution (with the normal distribution being a specific instance). If discriminant analysis is viewed as providing a linear combination of variables that maximizes a specific separation criterion, then its applicability is evidently broad.

The normality assumption only becomes important if significance tests are to be undertaken. Despite the short coming of Linear Discriminant Analysis (LDA), requiring normally distributed data which in any case are often non-normal and categorical there are several advantages which are that it is simple, easily estimated and works very well. This technique starts by identifying characteristics unique to each group. It derives a statistical function and scores to separate the groups with their distinguishing traits. The scores are then used to assign each observation to the appropriate categorization. The function is later used to validate a holdout/validation sample on its predictive/and classification accuracy, and determines which factors (ratios) give the function the most discriminatory power. These models allow a financial institution to minimize the risk of loss by setting decision rules regarding which customers receive loan and credit card approvals. In the past among others discrminant analysis and linear regression were the most widely used techniques for building scorecards and both have the merits of being conceptually straightforward and widely available in statistical software packages. Classically, the coefficients and the numerical scores of the attributes were combined to give single contributions which are added to give an overall score. Currently logistic regression is probability the most used technique for credit scoring [2]. Financial services companies continually create and update their custom scoring models that are created by credit-score-industry heavyweights FICO and VantageScore. Apart from customers information in credit reports alternative data points could be designed to assist companies better understand the risks and opportunities associated with their particular customers and prospects. Linear regression can model many variables of which some are on different measurement scales as mentioned early.

The most popular methods adopted in credit scoring are logistic regression and its variations. It was first introduced in 1980 by Martin as he analysed the bankruptcy probability interval distribution, two types of errors and the relationship between the split point [12]. His findings were that key indexes for the judgment were size, capital structure, and performance. Wiginton was one of the first researchers to report credit scoring results with a logistic regression model [3]. Following his work, the approach became more widely adopted, notably with the contributions of Hosmer and Lemeshow [10]. It can be used to predict default events and model the influence of different variables (a number of predictors) on a consumer's creditworthiness. It is used to analyze the data with qualitative response variable defined as

$$\pi(x_i) = \frac{e^{\sum_{p=1}^{k} x_{ip}\beta_p}}{1 + e^{\sum_{p=1}^{k} x_{ip}\beta_p}}, \tag{3.20}$$

where, x_i is default events (number of predictors), β_p the vector of the coefficients of the model, and p is the probability of number of default (proportion of ones) from 1 to k.

Logistic regression is an example of a generalized linear model whose main use it to estimate the probability that a binary response occurs based on several predictor variables. The goal of an analysis using logistic regression is the same as that of any model-building technique used in statistics, yet reasonable model to describe a relationship between a dependent and one or more independent variables. Hence the difference between normal regression and logistic regression is the use of a binary or dichotomous dependent variables with assumptions emerge as to why linear regression cannot be used for credit scoring model [28].

The mean of a binary distribution is denoted as p, which is the proportion of ones. The proportion of zeros is then $1-p$, often denoted as q. The variance of this distribution is pq. Having more than one observation in the independent variable, the dependent variable becomes a binomial variable. In logistic regression a binary dependent variable means that the conditional mean must be greater or equal to 0 and less than or equal to 1:

$0 \leq E(y|x) \leq 1.$

Logistic regression assumes that the sum of the weighted input variables is linearly correlated to the natural log of the odds that the outcome event will happen described as [4].

$$\log\left(\frac{p}{1-p}\right) = \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + \varepsilon = \beta^T X_k + \varepsilon \tag{3.21}$$

where $\beta = (\beta_1, \beta_2, ..., \beta_k)$ is the vector of the coefficients of the model, the maximum likelihood method can be applied to compute the estimate of $\beta_i,\ i = 1, 2, ..., k$.

Assuming that the regression model above is obtained then the estimated probability of no default is:

$$p = \frac{e^{\beta^T x}}{1+e^{\beta^T x}}.$$

The logistic regression above can take any value between $-\infty$ and $+\infty$ but the left side can only take values between 0 and 1. The method is intrinsically linear and cannot deal well with non linear effects in practice therefore there is a need to use the variable combinations and trial-and-error process to deal with non-linear effects. The method is sensive to redundancy or collinearity in the input variables, can give bad estimates of the coefficients. For better explanations, results also calls for the residual error to obey the logit normal distribution.

3.1.4 Applied Credit Score

A scoring model is an assessment of the default probability by combining in a specific way different pieces of information. We explain here how we will build a scoring model, using the logistic regression technique or logit technique following the Z-score approach [14]. From the balance sheet records and statement of income position of the firm, we collect information, compute ratios of working capital over total asset, retained earnings before interests and taxes, sales, each over total assets. These ratios are used as risk factors in the logit model. The logistic regression model equates the logit transform, the log-odds of the probability of a success, to the linear component:

$$\ln\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right) = \beta_0 + \beta_1 x_1 + ... + \beta_k x_k + \varepsilon. \tag{3.22}$$

Let $\ln\left(\frac{\pi(x_i)}{1-\pi(x_i)}\right) = y_i$. Then, after solving for $\pi(x_i)$ we have

$$\pi(x_i) = \frac{\exp(y_i)}{\exp(y_i)+1} = \frac{\exp(\beta_0 + \beta_1 x_1 + ... + \beta_k x_k)}{1+\exp(\beta_0 + \beta_1 x_1 + ... + \beta_k x_k)}. \tag{3.23}$$

or

$$\pi(x_i) = \frac{e^{\sum_{p=1}^{k} x_{ip}\beta_p}}{1+e^{\sum_{p=1}^{k} x_{ip}\beta_p}}. \tag{3.24}$$

Hence, the maximum likelihood method can be used to estimate the parameters of the logistic regression model. Y_i is random variable and assumed to be independent with $i = 1,2...,n$ and $Y_i \sim Bernoulli(\pi(x_i))$. A convenient way to express the contribution to the likelihood function for the pair (x_i, y_i) is through the expression

$$\prod_{i=1}^{n}\left(\pi(x_i)^{y_i}\left[1-\pi(x_i)\right]^{1-y_i}\right). \tag{3.25}$$

Since the observation of a single value x_i is assumed to be independent, the likelihood function is obtained as a product of the terms in Eq. (3.25) as follows:

$$\begin{aligned} l(\beta) &= \prod_{i=1}^{n} \pi(x_i)^{y_i} \left[1-\pi(x_i)\right]^{1-y_i} \\ &= \prod_{i=1}^{n} \frac{e^{(\beta_0+\beta_1 x_1+\ldots+\beta_k x_k)(y_i)}}{1+e^{(\beta_0+\beta_1 x_1+\ldots+\beta_k x_k)^{-(1-y_i)}}}, \end{aligned} \tag{3.26}$$

where $\beta = (\beta_0, \beta_1, ..., \beta_k)$. To simplify the derivative of Eq. (3.26), the log-likelihood function is used. The log-likelihood function is defined as follows:

$$\begin{aligned} l(\beta) = \ln(L(\beta)) &= \sum_{i=1}^{n} y_i (\beta_0+\beta_1 x_1+\ldots+\beta_k x_k) \\ &\quad -(1-y_i)\sum_{i=1}^{n} \ln(1+\exp(\beta_0+\beta_1 x_1+\ldots+\beta_k x_k)). \end{aligned} \tag{3.27}$$

Thus, differentiating Eq. (3.27) with respect to each β_p,

$$\begin{aligned} \frac{\partial l(\beta)}{\partial \beta_p} &= \sum_{i=1}^{n} y_i x_{ip} - (1-y_i)\frac{1}{1+e^{\sum_{p=0}^{k} x_{ip}\beta_p}} \times \frac{\partial}{\partial \beta_p}\left(1+e^{\sum_{p=1}^{k} x_{ip}\beta_p}\right) \\ &= \sum_{i=1}^{n} y_i x_{ip} - (1-y_i)\frac{1}{1+e^{\sum_{p=0}^{k} x_{ip}\beta_p}} \times e^{\sum_{p=0}^{k} x_{ip}\beta_p} \times x_{ip}. \end{aligned} \tag{3.28}$$

The maximum likelihood estimates for β can be found by setting each of the p+1 equations in Eq. (3.28) equal to zero and solving for each β_p.

The neural network can be seen as a generalization of the logit method, hence various methods are very comparable and there is no superior method for diverse data sets [11]. Classification methods such as regression are easy to understand, much more appealing both to users and to clients than other methods and they also permit more ready explanations of the sorts of reasons why the methods have reached their decisions. In credit scoring people have been constructing scorecards on similar data for several decades, hence it is very unlikely that new classification methodologies lead to other due to the fact that in credit score there is a solid undesrtanding of the problem domain. The logit method is the most favored method in practice, mainly because there are few or no assumptions imposed on variables, with the exception of missing values and multicollinearity among variables. Whereas non-parametric methods can deal with missing values and multicollinearity among variables and often computationally demanding [11]. Logistic regression produce methods that are easy to explain, implement and has been widely accepted in the banking industry as the method of choice. The Merton's and KMV model seems to be very convincing however, the question is whether the results given by this approach is really any better than the probabilities empirically derived by rating agencies and related to popular rating grades [11].

4 Data analysis

A comprehensive data collection process was undertaken from the financial reports of the three main commercial banks in Namibia namely, Bank 1, Bank 2 and Bank 3. Specifically, the data comprised information extracted from the financial statements, the balance sheet and the income statement over the period from 2011 to 2021. The data extraction process targeted key variables essential for calculating a range of financial ratios with significance in risk evaluation. We focused on five key financial variables: Working capital (WC), Retained Earnings (RE), Earnings Before Interest and Taxes (EBIT), Sales (S), and Market Value of Equity (ME). Each of these variables was divided by Total Assets (TA) or Total Liabilities (TL) to create meaningful financial ratios for our analysis. By meticulously computing these ratios, a holistic representation of bank's financial health and risk profile was attained. These ratios, serving as critical risk factors, were subsequently integrated into both a logit model framework and the Merton model. This analytical integration facilitated the quantification of potential credit default probabilities and an enhanced understanding of the bank's exposure to financial distress scenarios. The outcomes of this endeavour not only offered valuable insights into individual bank risk but also contributed to a broader understanding of the stability and resilience of Namibia's banking sector over the analysed timeframe.

5 Results and discussion

The Merton model

The purpose of this chapter is not to enumerate all the collected data but rather to elucidate how the data has been applied in implementing the Merton model. For a comprehensive list of the data, readers are encouraged to refer to appendix.

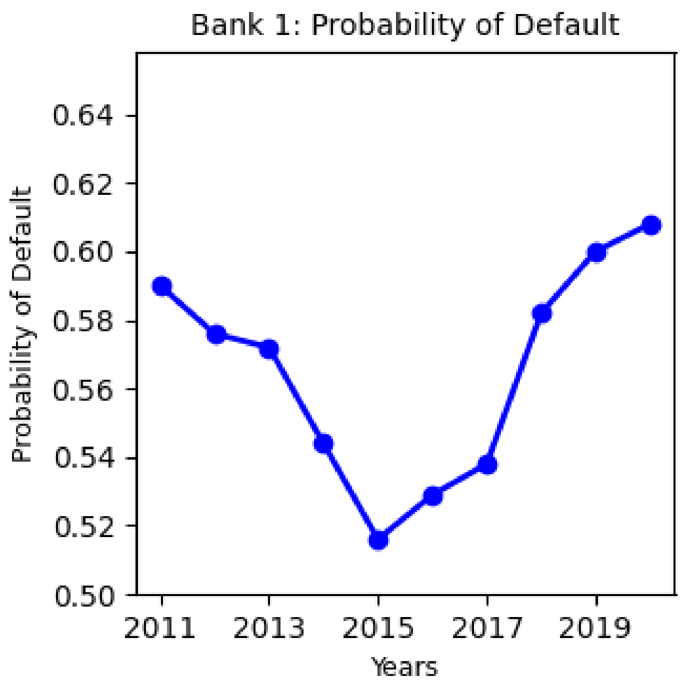

Figure 3: Merton's model Bank 1 (Probability of Default)

The PD values are probabilities of Default between 0 and 1. As the line moves along this axis, the varying points on the line graph correspond to different levels of probability of default. A lower point is experienced on the graph around the 5th year, and this indicates a lower likelihood of default. The lower Probability of Default (PD) during 2015 and 2016 was primarily due to Namibia's strong economic growth, particularly driven by robust performance in the mining and construction sectors, as highlighted in the Bank 1 annual report for 2015. While a higher point is on the last year indicating a higher likelihood of default. The line's trajectory and shape provide a visual representation of how the probability of default changes over the years from 2011 there was a decrease until around 2015 and 2016 then there was an increase of credit risk up to the final year. The increase in credit risk and higher Probability of Default (PD) can be attributed to the economic and financial disruptions caused by the COVID-19 pandemic, which led to reduced economic activity, increased unemployment, and financial strain on both businesses and consumers in Namibia, as reflected in the trajectory of the default risk. The Distance to Default (DD) close to zero was experienced on

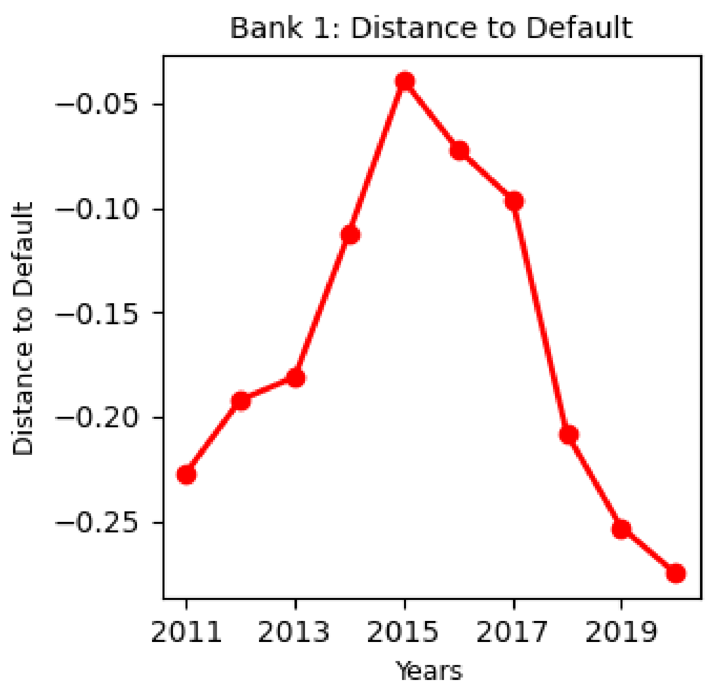

Figure 4: Merton's model Bank 1 (Distance to default)

the 5th year and this indicates a larger safety margin and suggests that the entity has a significant capacity to withstand financial stress or adverse market conditions before reaching the point of default. This could be attributed to improved economic stability and performance, potentially driven by factors such as strong GDP growth and favorable market conditions, as indicated in the FNB Namibia annual report for 2015.

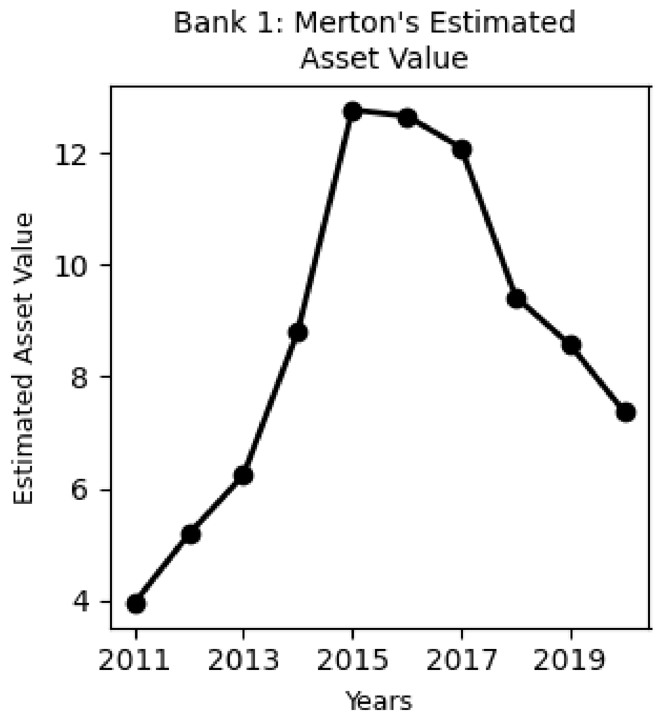

Figure 5: Merton's model Bank 1(Estimated Asset value)

The year 2011 to 2013 on the graph is closer to one indicating a lower estimated asset value, hence signals financial challenges and heightened likelihood default. Along the graph there is an increase, indicating a lower likelihood of default.

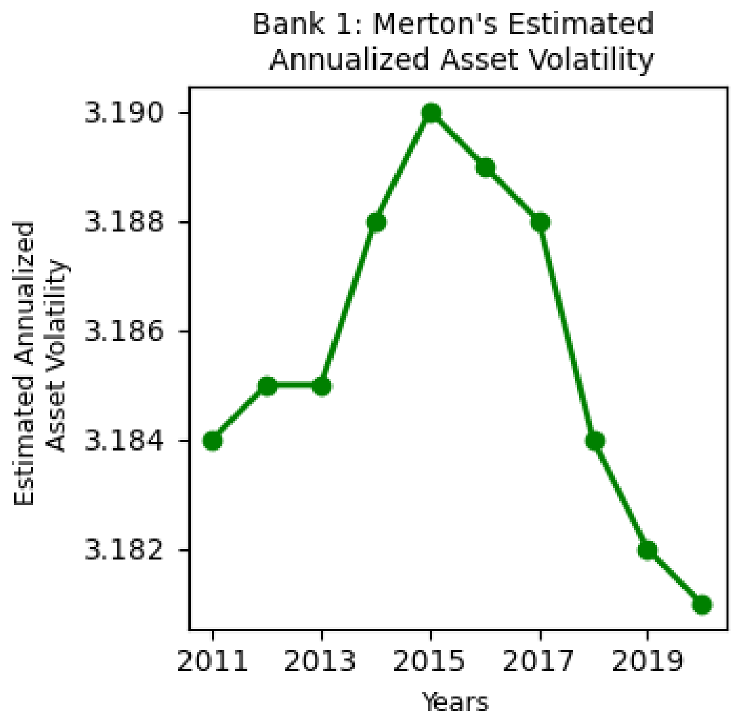

Figure 6: Merton's model Bank 1 (Estimated Annualized Asset volatility)

The graph illustrates the potential fluctuation of the value of an asset over the period from 2011 to 2021. In the initial years, the values are closer to one, indicating a higher level of expected volatility. However, the overall performance of the asset's volatility has exceeded one, suggesting that, on average, the fluctuations in the asset's value have been higher than expected throughout the entire time period.

Observing the graphs above of Bank 1, we observe an increase in Probability of Default over the years since probability is close to one. Bank 1 has the highest risk of default since it has the highest PD.

Table 1: Bank 1

PD	DD	A	Sa
0.590	-0.227	3.950	3.184
0.576	-0.192	5.184	3.185
0.572	-0.181	6.236	3.185
0.544	-0.112	8.817	3.188
0.516	-0.039	12.775	3.190
0.529	-0.072	12.664	3.189
0.538	-0.096	12.095	3.188
0.582	-0.208	9.427	3.184
0.600	-0.253	8.567	3.182
0.608	-0.275	7.363	3.181

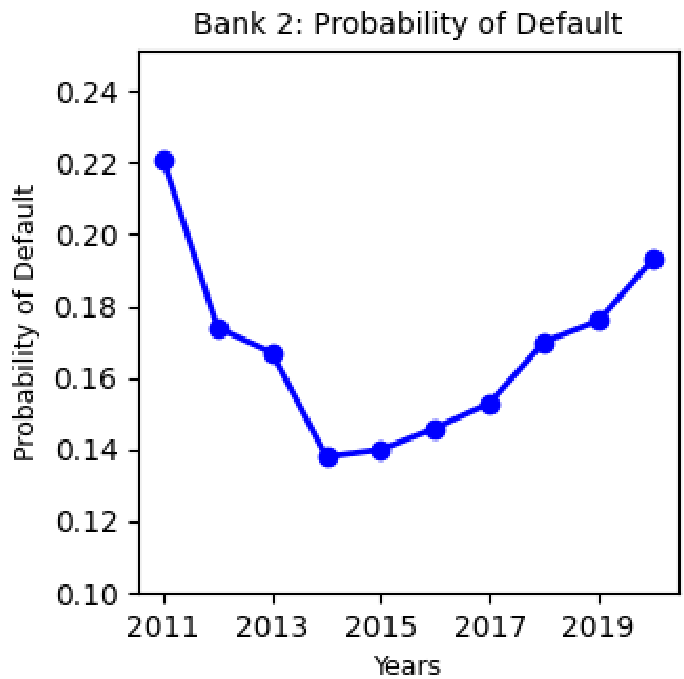

Figure 7: Merton's model Bank 2 (Probability of Default)

As the line moves along this axis, the varying points on the line graph are relatively close to one another, A lower point is experienced on the graph around the 4th year, and this indicates a lower likelihood of default. Meanwhile a higher point is experienced immediately on the 1st year and the final year, indicating a higher likelihood of default. The line's trajectory and shape provide an overall medium likelihood of default. The bank resisted COVID-19 effects by implementing comprehensive risk management strategies, including pre-acceptance measures in product development and pricing, and ongoing risk measurement, monitoring, and treatment post-acceptance, as detailed in Bank 2's risk and capital management practices.

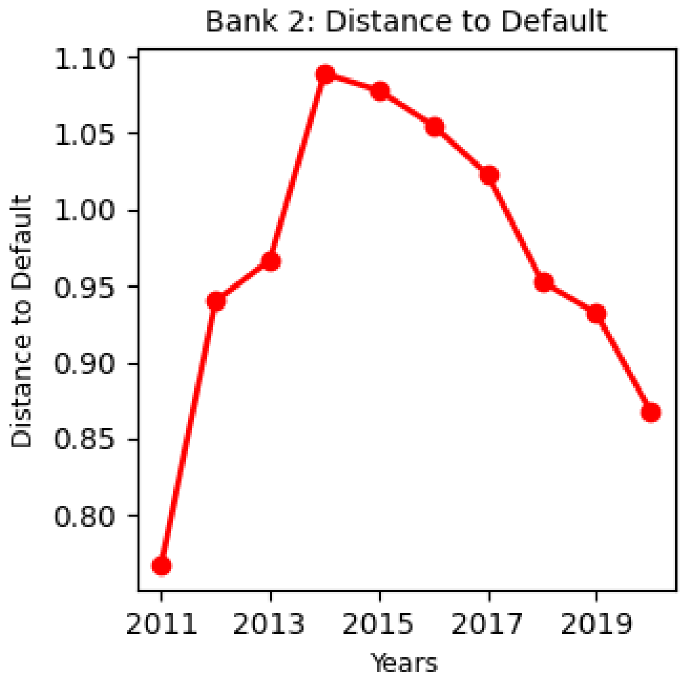

Figure 8: Merton's model Bank 2 (Distance to default)

A position close to zero was experienced from the 1st year and this indicates a smaller distance to default, implying a narrower safety margin and a diminished capacity to withstand financial stress or adverse market conditions before reaching the point of default. The overall period over the years, the distance to default is Closer to one and above signifies a larger distance to default, implying a more significant safety margin.

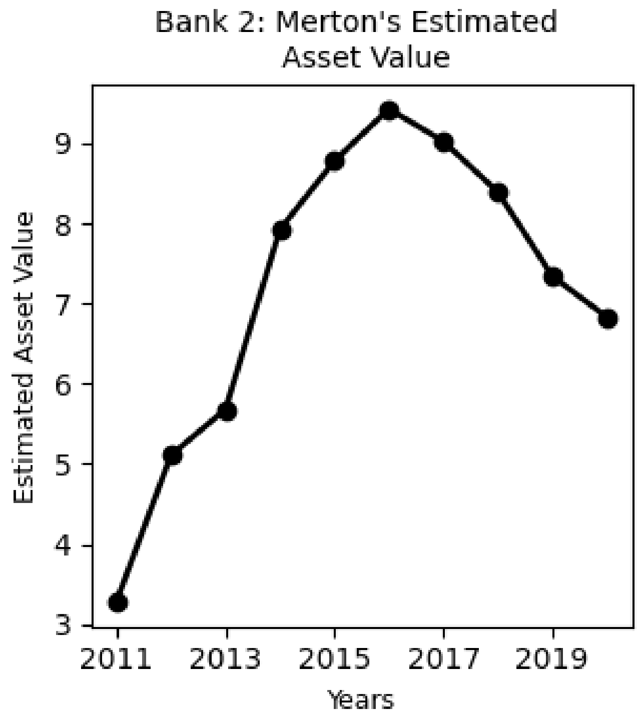

Figure 9: Merton’s model Bank 2 (Merton’s Estimated Asset value)

The potential fluctuation of value of an asset over a given time period from 2011 to 2021. The 1st year on the graph is closer to zero indicating a higher level of Estimated Asset value, hence indicates that the market value of the entity’s assets is reduced or perceived to be lower. The overall performance of the Estimated Asset value over the years is more than three.

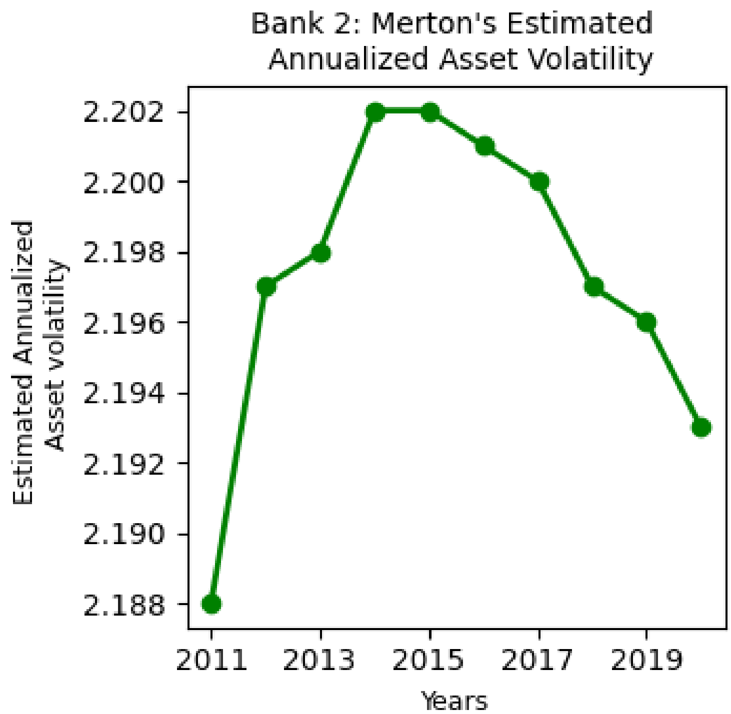

Figure 10: Merton's model Bank 2 (Merton's Estimated Annualized Asset volatility)

The graph indicates high volatility in the 1st year, with asset values fluctuating, reflecting a period of uncertainty likely influenced by global commodity price changes, market speculation, and economic events impacting investor sentiment. Over the period from 2011 to 2021, the overall asset volatility remained greater than one, suggesting sustained fluctuations due to Namibia's dependence on commodity prices, variable economic growth, inflation and interest rate changes, and external economic shocks. The significant disruption caused by the COVID-19 pandemic, particularly in the latter part of the period, further contributed to this high volatility, highlighting the Namibian economy's challenges and uncertainties throughout the decade.

Observing the graphs above of Bank 2, we observe a decrease in Probability of Default over the years since probability is lower compared to Bank 1. This indicates robust financial health and effective risk management, as seen in the analysis comparison with Bank 1.

Table 2: Bank 2

PD	DD	A	Sa
0.221	0.767	3.281	2.188
0.174	0.940	5.106	2.197
0.167	0.967	5.678	2.198
0.138	1.089	7.921	2.202
0.140	1.078	8.778	2.202
0.146	1.055	9.426	2.201
0.153	1.023	9.023	2.200
0.170	0.953	8.391	2.197
0.176	0.932	7.340	2.196
0.193	0.868	6.831	2.193

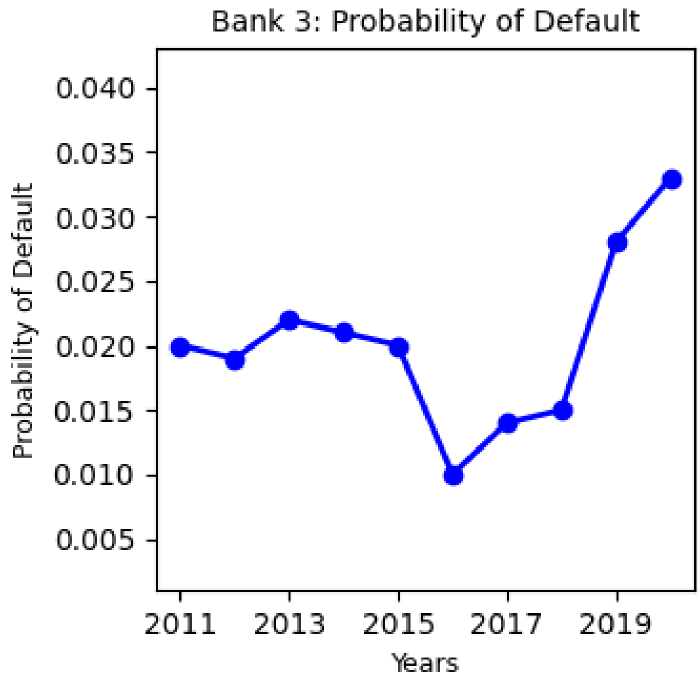

Figure 11: Merton's model Bank 3 (Probability of Default)

As the line moves along this axis, the varying points on the line graph correspond to different levels of probability of default. A lower point is experienced on the graph around the 6th year, and this indicates a lower likelihood of default, while a higher point is on the 10th year indicating a higher likelihood of default. The rise in credit risk and the higher Probability of Default (PD) from 2019 to 2021 can be attributed to the economic and financial disruptions caused by the COVID-19 pandemic, which resulted in decreased economic activity, higher unemployment, and financial strain on businesses and consumers in Namibia. The line's trajectory and shape provide an overall lower likelihood of default.

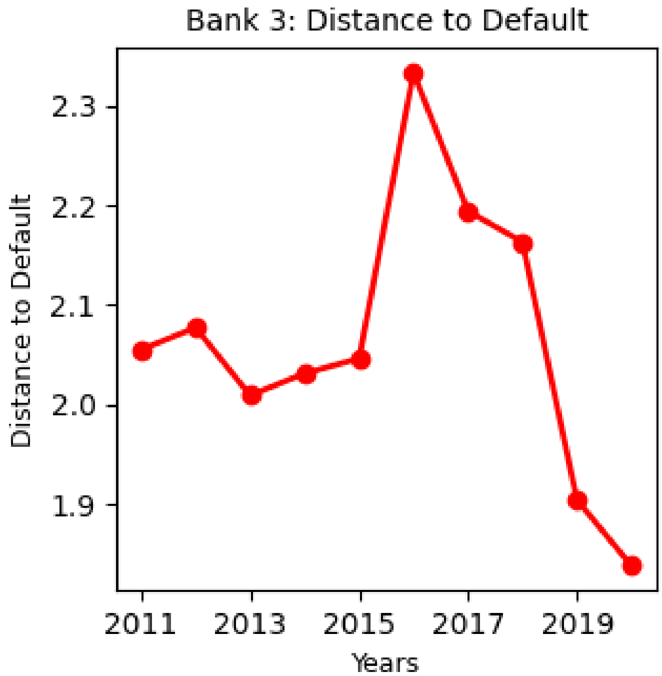

Figure 12: Merton's model Bank 3 (Distance to default)

The Distance to Default (DD) close to one was experienced on the final two years and this signifies a larger distance to default, implying a more significant safety margin. This could be attributed by improvement in economic stability and performance, potentially driven by factors such as strong GDP growth and favorable market conditions. As for the rest of the years we experience larger distance to default.

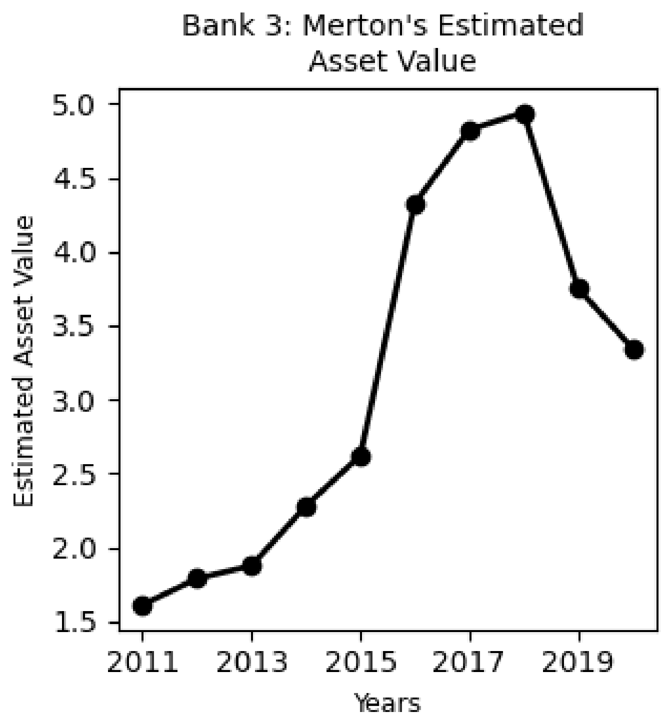

Figure 13: Merton's model Bank 3 (Merton's Estimated Asset Value)

The years from 2011 to 2013 on the graph are closer to one, indicating a lower level of expected volatility. This means the asset's value is expected to experience smaller and more stable changes during that period, suggesting sustained fluctuations due to Namibia's dependence on commodity prices, variable economic growth, inflation and interest rate changes, and external economic shocks. However, the overall asset volatility over the years is greater than one, indicating a higher likelihood of default.

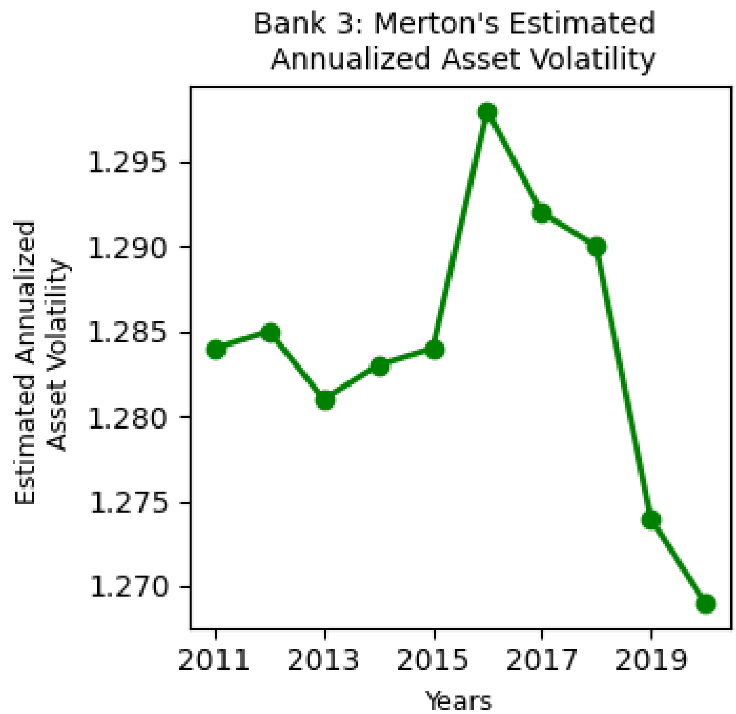

Figure 14: Merton’s model Bank 3 (Merton’s Estimated Annualized Asset volatility)

The final year on the graph is closest to one indicates a higher level of expected volatility. The overall performance of the asset volatility is closer to one indicates a higher level of expected volatility, suggesting that the asset is anticipated to undergo more significant price swings and fluctuations during the specified time period.

Observing the graphs above of Bank 3, we observe a decrease in Probability of Default over the years since probability is close to zero. this is the bank with the overall lowest risk. This signifies a substantial reduction in the likelihood of default, indicating that Bank 3 has effectively managed its credit risk and now exhibits the overall lowest risk among its peers. This trend suggests robust financial health, strong risk management practices, and the ability to maintain stability even during challenging economic conditions.

Table 3: Bank 3

PD	DD	A	Sa
0.020	2.055	1.612	1.284
0.019	2.077	3.790-	1.285
0.022	2.009	1.877	1.281
0.021	2.031	2.280	1.283
0.020	2.046	2.621	1.284
0.010	2.333	4.323	1.298
0.014	2.194	4.822	1.292
0.015	2.163	4.938	1.290
0.028	1.904	3.751	1.274
0.033	1.838	3.346	1.269

The Logit model

A comprehensive analysis of six distinct models was conducted to assess their suitability for predicting default risk based on the collected data. The data is extracted from the balance sheets of three commercial banks in Namibia over a period from 2011 to 2021. By selecting these models, we aimed to prioritize accuracy and precision in our default risk prediction efforts.

Applying the Logit function;

Model 1:

$$P(Default) = 1/\left(1 + \exp b_0 + b_1 \mathrm{WC/TA} + b_2 \mathrm{ME/TL} + b_3 \mathrm{EBIT/TA} + b_4 \mathrm{RE/TA} + b_5 \mathrm{SALES/TA}\right)$$

Model 2:

$$P(Default) = 1/\left(1 + \exp b_0 + b_1 \mathrm{ME/TL} + b_2 \mathrm{EBIT/TA} + b_3 \mathrm{RE/TA} + b_4 \mathrm{SALES/TA}\right)$$

Model 3:

$$P(Default) = 1/\left(1 + \exp b_0 + b_1 \mathrm{WC/TA} + b_2 \mathrm{ME/TL} + b_3 \mathrm{EBIT/TA}\right)$$

Model 4:

$$P(Default) = 1/\left(1 + \exp b_0 + b_1 \mathrm{ME/TL} + b_2 \mathrm{EBIT/TA} + b_3 \mathrm{RE/TA}\right)$$

Model 5:

$$P(Default) = 1/\left(1 + \exp b_0 + b_1 \mathrm{EBIT/TA} + b_2 \mathrm{RE/TA} + b_3 \mathrm{SALES/TA}\right)$$

Model 6:

$$P(Default) = 1/\left(1 + \exp b_0 + b_1 \mathrm{WC/TA} + b_2 \mathrm{EBIT/TA} + b_3 \mathrm{RE/TA}\right)$$

to the data and obtain the tables in the Appendix.

When using logistic regression models, such as the logit function, for default risk prediction, encountering issues such as non-convergence is not uncommon. From what we observe in the 6 models, logit model is not converging, it may due to several potential factors. Firstly, logistic regression models, particularly in the context of predicting default risk, are known to be sensitive to the quality and characteristics of the data. Non-convergence could be a result of data related issues, such as outliers, multicollinearity, or an insufficient sample size relative to the number of predictors. Additionally, logistic regression assumes a linear relationship between the predictors and the log odds of the outcome, and violations of this assumption might hinder convergence. Furthermore, the nature of credit risk prediction is complex, and factors influencing default may be inherently non-linear. We strongly believe that the models did not converge due to strong collinearlity.

As such, the non-convergence of the logit models might suggest that a more flexible modeling approach, such as non-linear models or ensemble methods, could be more suitable for capturing the intricate patterns within the data. The outcomes of this endeavor not only offered valuable insights into individual bank risk but also contributed to a broader understanding of the stability and resilience of Namibia's banking sector over the analyzed timeframe. To interpret the sign of the coefficient b, recall that a higher score corresponds to a higher default probability. Working capital over Total Asset captures the short-term liquidity of a firm. Retained Earnings over Total assets and EBIT over Total asset measure historic and current profitability respectively. The total number of observations is n=30 and the number of explanatory variables is 5.

Table 4: Descriptive statistics based on Bank 1

Statistic	Total Assets Total (in mil)	Total Liabilities Total (in mil)	Total Sales Total (in mil)	Equities (in mil)	Debts (in mil)	Working Capital (in mil)
Mean	29.1756	25.7046	3.813	3.256573	23.26858	17.382
Median	28.02001	24.9371	3.565	2.69865	22.737	11.466
St Deviation	13.4983	11.8697	1.659	1.4356	10.86265	15.412
Min	10.6737	9.4333	1.710	1.24044	7.817	0.05
Max	52.4423	46.1699	6.101	5.424305	38.656569	51.898

Table 5: Descriptive statistics based on Bank 2

Statistic	Total Assets Total (in mil)	Total Liabilities Total (in mil)	Total Sales Total (in mil)	Equities (in mil)	Debts (in mil)	Working Capital (in mil)
Mean	30.9250	27.1105	3.455	2.940736	17.61783725	142.09
Median	26.4636	23.0948	2.931	1.99271	21.137685	116.675
St Deviation	17.9781	15.5636	1.750	1.69163	12.137685	161.41
Min	9.6324	6.6755	1.509	1.105356	0.43419	10.543
Max	60.4396	52.6463	6.151	5.799344	35.482418	574.967

Table 6: Descriptive statistics based on Bank 3

Statistic	Total Assets (in mil)	Total Liabilities Total (in mil)	Total Sales Total (in mil)	Equities (in mil)	Debts (in mil)	Working Capital (in mil)
Mean	32.7674	21.3876	2.591	2.3156	19.04018	42.938
Median	23.9003	20.74375	2.108	2.0107	18.0250713	11.267
St Deviation	9.0669	7.6735	0.960	1.0905	7.266028	123.366
Min	11.0615	10.1455	1.508	0.9534	5.22067	1.265
Max	36.2156	31.6668	4.254	4.3128	28.671578	504.071

Analysing the data reveals notable trends and patterns in key financial metrics. Across the banks, there has been a consistent upward trend in assets, with substantial growth observed over the years. This suggests that the banks have been successful in expanding their operations and increasing their overall scale. The descriptive statistics for the banks are presented in Table 4 (Bank 1), Table 5 (Bank 2) and Table 6 (Bank 3).

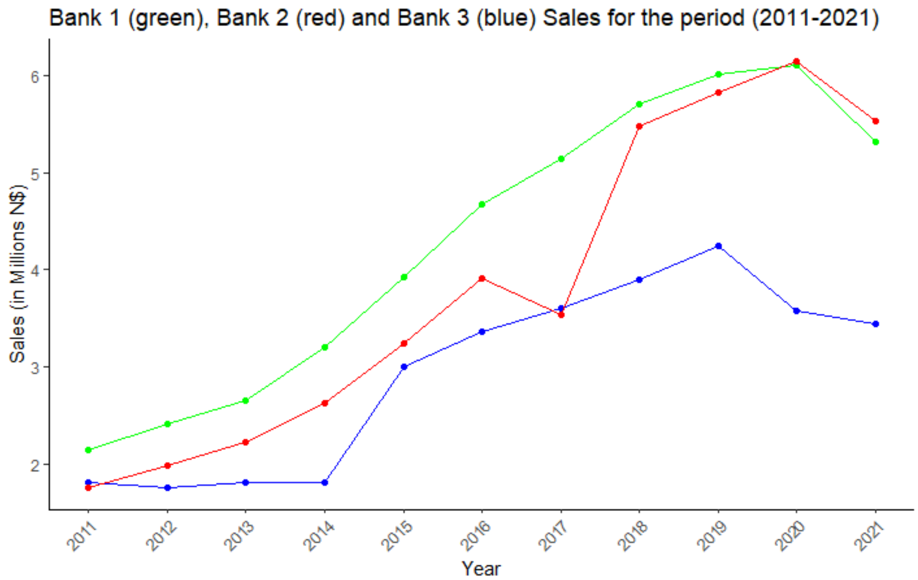

Figure 15: Trend Sales in million dollars

Sales or revenue, shows mixed patterns across the banks. Bank 1 experienced consistent growth up to 2020, surpassing the other banks in terms of revenue performance. Bank 2 also demonstrated steady sales growth, albeit at a slower pace. In contrast, Bank 3 faced challenges in generating substantial revenue growth, with fluctuations observed over the years.

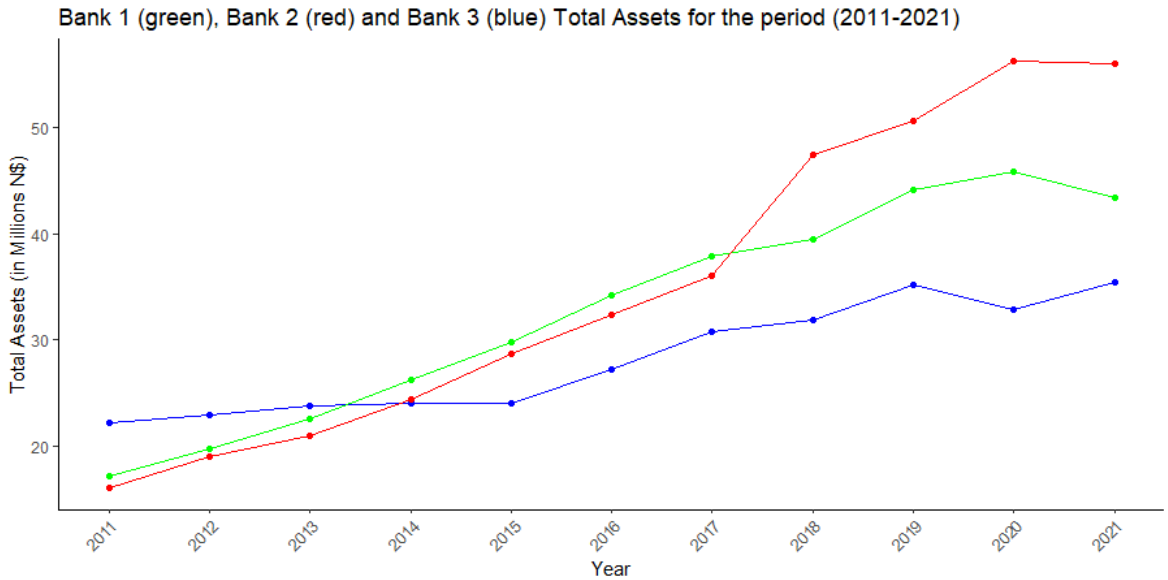

Figure 16: Trend Assets in million dollars

Notably, the growth rate varied between banks, with Bank 2 experiencing the highest asset growth from 2017 to 2021, followed closely by Bank 1. While Bank 3 showed comparatively slower growth from 2014 to the final year. The three banks have adopted distinct strategies to drive asset growth. Bank 1 has leveraged diversification, technological innovation, and a customer-centric approach, resulting in a broad and digitally integrated portfolio. Bank 2, on the other hand, has emphasized supporting the local economy, particularly SMEs, and personalized banking services, along with active community involvement, fostering strong customer loyalty and steady growth. Bank 3 has focused on corporate and investment banking, utilizing its regional and global presence to attract large-scale investments and ensuring robust risk management. Each bank's unique strategy reflects its strengths and market positioning, contributing to their respective asset growth trajectories.

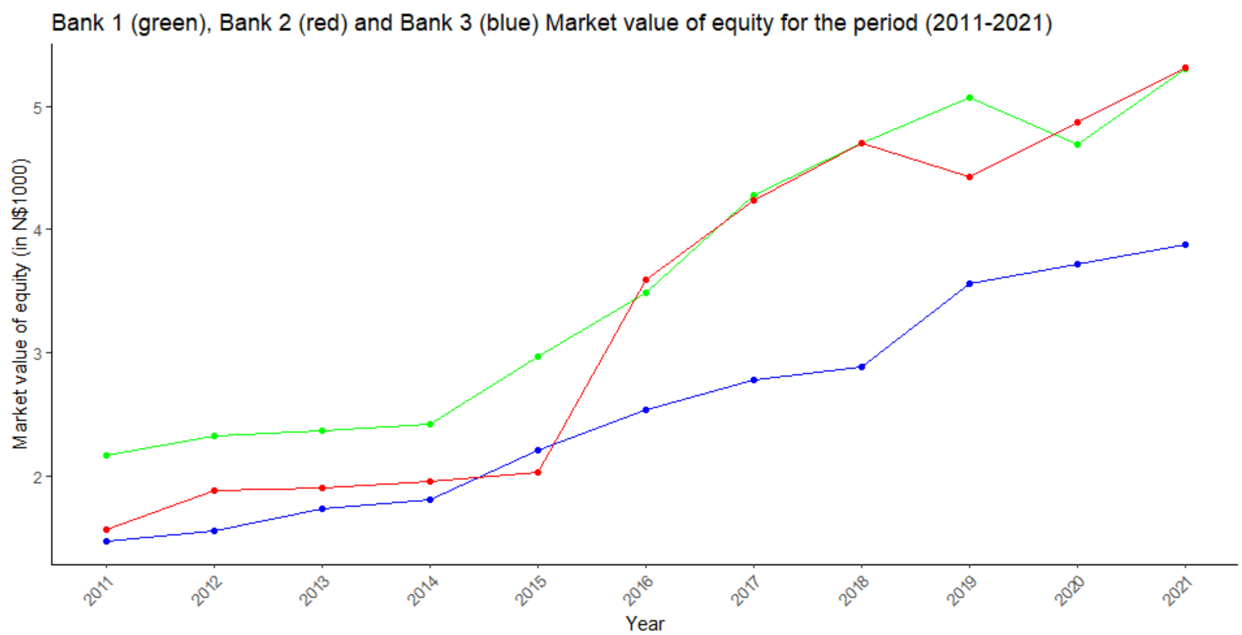

Figure 17: Trend Total Equities in million dollars

Equity, representing the shareholders' stake in the banks, generally showed positive growth trends for all three banks. Bank 2 experienced rapid equity growth from 2015 to 2021, while Bank 3 had comparatively lower equity. The positive equity growth across all three banks indicates an increase in their net worth, suggesting a favorable position in terms of capital adequacy and resilience.

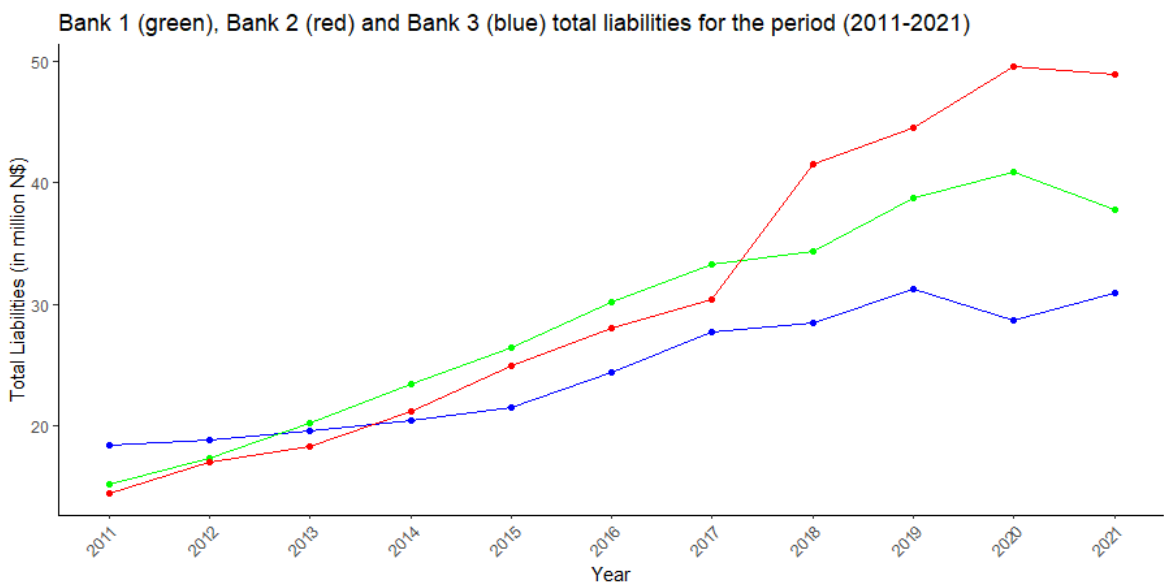

Figure 18: Trend Total Liabilities in million dollars

Liabilities exhibited a similar trend, mirroring the growth in assets. The banks have been able to attract funding from various sources to support their operations and lending activities. However, there were fluctuations in the growth rates of liabilities among the banks, indicating differences in their borrowing and financing strategies.

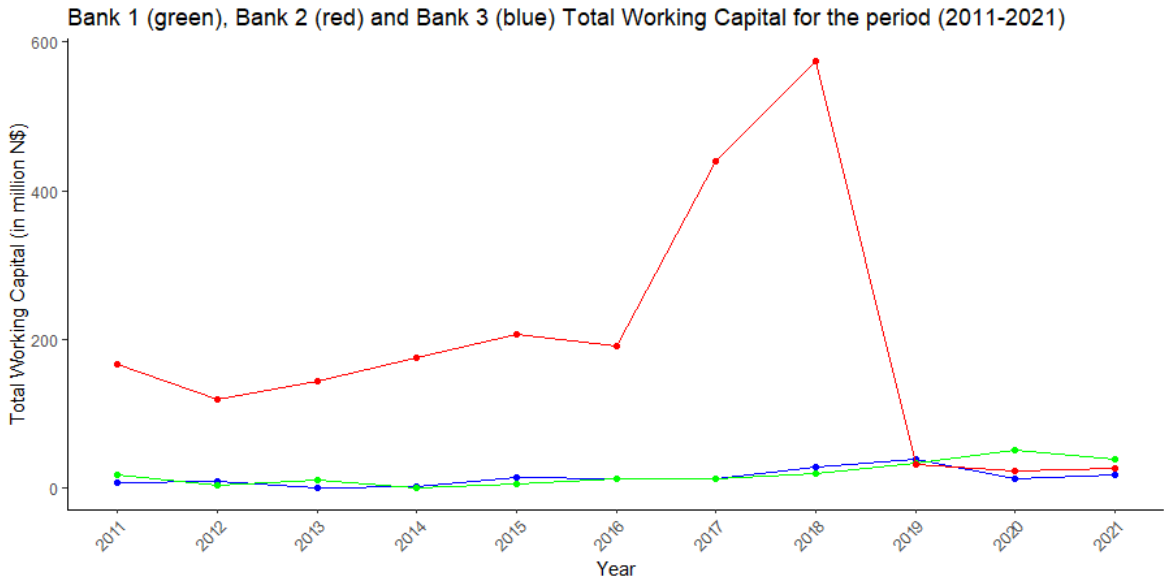

Figure 19: Trend working capital in million dollars

The working capital shows the bank's ability to manage its short-term financial obligations and its operational efficiency. Obtained by taking the current assets divided by current liabilities this ratio provide additional insights into a company's short term financial health.The graph indicates that the working capital of all three banks experienced a slight decline from 2019 to 2021, which can likely be attributed to the effects of the COVID-19 pandemic. The pandemic caused significant economic disruptions, leading to reduced business activity, increased financial strain on borrowers, and higher levels of loan defaults. As a result, the banks faced challenges in maintaining their liquidity and operational efficiency, reflected in the observed drop in working capital. Despite this decline, the banks' ability to manage their resources during such a turbulent period demonstrates their resilience and adaptability in navigating through financial stress. Bank 2 has an overly high working capital this could mean short term financial stability, providing liquidity and a buffer against financial challenges. However, it also raises questions about whether the company is making the most efficient use of its assets. Companies need to strike a balance between maintaining adequate working capital for short-term needs and optimizing asset utilization for long-term growth and profitability.

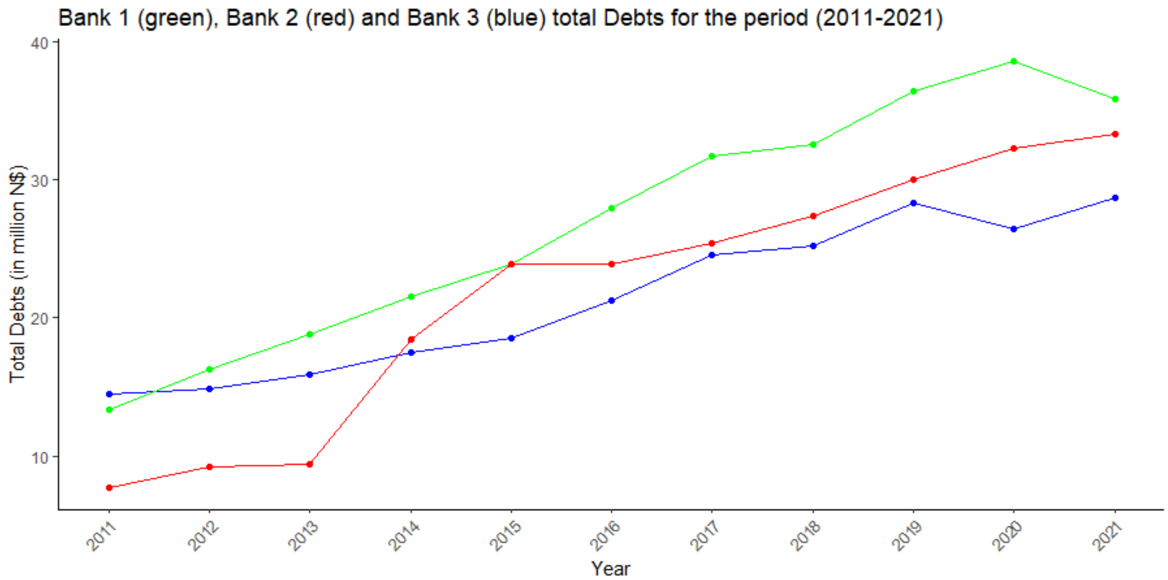

Figure 20: Trend Debt securities in million dollars

Debt levels varied among the banks, with Bank 1 demonstrating a high increase in debt over the years, potentially reflecting their strategic financing decisions. Bank 3, on the other hand, maintained a more conservative approach, keeping debt levels relatively stable. It is generally expected that the equity (also known as shareholders' equity or net worth) would be greater than the debt, but since the main Business for banks is to hold money and give loans to customers then the financial performance would be measured by the fact that total asset exceeds the liabilities due and this would indicate that the bank is in a solvent position.

6 Conclusion and Recommendations

6.1 Introduction

Risk models are constructed based on simplifying assumptions and inputs, hence should be treated with caution, because they are only as good as these assumptions and inputs. When choosing the method to use it is best to analyze the data structure, the details of the problem, the characteristics used, the extent to which it is possible to separate the classes by using those characteristics and the objective of the classification such as rate, cost-weighted misclassification rate, bad risk rate among those accepted, some measure of profitability, etc [11].

6.2 Conclusion

To sum up, commercial banks employ diverse tools, techniques, and evaluation models for credit risk management, aiming primarily to minimize instances of loan defaults, a leading contributor to bank failures. However, credit risk analysis faces several challenges, including assessing credit risk for a wide range of borrowers, allocating sufficient resources to credit risk management is crucial for commercial banks in Namibia to remain competitive, maintain their financial stability and comply with regulatory requirements.

Banks should use the Merton model to assess the creditworthiness of borrowers, monitor their credit risk on an ongoing basis, diversify their credit portfolio, set appropriate risk-based pricing, and conduct stress testing to identify potential vulnerabilities in their credit portifolio. A credit score provides an objective measure of a borrower's creditworthiness based on their credit history, payment behavior, and other factors. Banks can use credit scores to determine the creditworthiness of borrowers, monotor their credit scores on an ongoing basis, diversify their credit portifolio and set appropriate credit limits based on their credit score. Even though banks have all the risk management systems in place and sound credit risk models in place defaults are still increasing.

6.3 Recommendations

A review of our lending procedures, practices, and credit assessment mechanisms is essential to ensure the provision of credit to legitimate clients, especially directing attention to previously overlooked productive sectors. Historically, commercial banks primarily focused on financing properties and other personal items. The legislation related to collateral should be reconsidered, given the challenges that commercial banks face in enforcing contracts.

There is a need to expedite efforts in enhancing the credit bureau system for the exchange of credit information among borrowers. Currently, the International Trade Centre (ITC) serves as the primary credit bureau used by banks in Namibia. However, its functionality is limited to identifying borrowers with poor credit records, mitigating credit risk, and does not provide positive information about borrowers with a commendable repayment history. Establishing a credit bureau that facilitates the sharing of positive borrower information would particularly benefit small loans, enhancing their prospects for increased borrowings.

Since there are many more developed models, conducting a comparative analysis of various models to determine the strengths and weaknesses of each would be recomendable for further research.

7 Research ethics

Secondary and publicly disclosed data was used for this analysis. Nevertheless ethical clearance was obtained from UNAM Decentralised Ethics Committee (DEC) and research permission letter from the Centre for Research Services (CRS) of the University of Namibia. Clients'confidentiality will be ensured by de-linking data from identifiable information.

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Appendices:

Figure 21: Ethical clearence certificate

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ETHICAL CLEARANCE CERTIFICATE

Ethical Clearance Reference Number: SOS-0111 Date: 12 October 2022

This Ethical Clearance Certificate is issued by the University of Namibia Ethics Committee (REC) in accordance with the University of Namibia's Research Ethics Policy and Guidelines. Ethical approval is given in respect of undertakings contained in the Research Project outlined below. This Certificate is issued on the recommendations of the ethical evaluation done by the ethics committee.

Title of Project: STRUCTURAL CREDIT RISK MODELING USING MERTON MODEL AND ITS DEFAULT PROBABILITY: A CASE STUDY OF COMMERCIAL BANKS IN NAMIBIA

Student: AINA SHAANIKA

Student Number: 200833383

Supervisor(s): Dr. RODRIGUE GNITCHOGNA;
Dr. KAMGA PENE

Centre for Research Services

Take note of the following:

1. Any significant changes in the conditions or undertakings outlined in the approved Proposal must be communicated to the ethics committee. An application to make amendments may be necessary.
2. Any breaches of ethical undertakings or practices that have an impact on ethical conduct of the research must be reported to the ethics committee
3. The Principal Researcher must report issues of ethical compliance to the ethics committee (through the Chairperson) at the end of the Project or as may be requested by the ethics committee
4. The ethics committee retains the right to:

i) Withdraw or amend this Ethical Clearance if any unethical practices (as outlined in the Research Ethics Policy) have been detected or suspected,

ii) Request for an ethical compliance report at any point during the course of the research.

The ethics committee wishes you the best in your research.

Dr. Zivayi Chiguvare (Chairperson Ethics Committee)

Prof. Davis Mumbengegwi (Head, Multidisciplinary Research)

Figure 22: Research permission letter

CENTRE FOR RESEARCH SERVICES
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RESEARCH PERMISSION LETTER

Date: 03 /03/2023

Student Name: AINA SHAANIKA

Student Number: 200833383

Programme: Master of Science in Applied Mathematics

Approved Research Title: STRUCTURAL CREDIT RISK MODELING USING MERTON MODEL AND ITS DEFAULT PROBABILITY: A CASE STUDY OF COMMERCIAL BANKS IN NAMIBIA

TO WHOM IT MAY CONCERN:

I hereby confirm that the above-mentioned student is registered at the University of Namibia for the programme indicated. The proposed study met all the requirements as stipulated in the University guidelines and has been approved by the relevant committees.

The proposal adheres to ethical principles as per attached Ethical Clearance Certificate. Permission is hereby granted to carry out the research as described in the approved proposal.

Best Regards

Dr. AEE Shikongo
Head: Postgraduate Research Support Services
Tel: +264 61 206 3129
E-mail: aeshikongo@unam.na

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2023 -03- 03
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Table 7: Logit Model 1

Bank	Year	Default	WC/TA	ME/TL	EBIT/TA	RE/TA	SALES/TA	Logit	e^(Logit)	Probability	loglikelihood
BANK 2	2011	0	0.09839665	0.108592	0.008546	0.008546	0.109757	-55696.6	0	1	0
BANK 2	2012	0	0.09973331	0.110782	0.007922	0.007922	0.105134	-55683.4	0	1	0
BANK 2	2013	0	0.12532151	0.143277	0.009305	0.017007	0.106545	-56098	0	1	0
BANK 2	2014	0	0.12723595	0.145785	0.010809	0.016646	0.107926	-56137.5	0	1	0
BANK 2	2015	0	0.12735129	0.145936	0.010406	0.018041	0.113151	-56177.2	0	1	0
BANK 2	2016	0	0.13218547	0.15232	0.324745	0.019843	0.121135	-57603.5	0	1	0
BANK 2	2017	0	0.12419844	0.140787	0.031722	0.031722	0.108357	-56260.2	0	1	0
BANK 2	2018	0	0.12156722	0.144324	0.028976	0.028976	0.115306	-56282.1	0	1	0
BANK 2	2019	0	0.11945843	0.126184	0.02787	0.022424	0.114902	-56117.4	0	1	0
BANK 2	2020	0	0.11945843	0.128406	0.022947	0.028733	0.109173	-56109.4	0	1	0
BANK 2	2021	0	0.1268317	0.137157	0.020829	0.020464	0.098787	-56088.9	0	1	0
Bank 3	2011	0	-0.0261635	0.096796	0.026184	0.026184	0.093538	-54939.1	0	1	0
Bank 3	2012	0	0.19112263	0.081627	0.021582	0.021582	0.092938	-56117.7	0	1	0
Bank 3	2013	0	0.08635421	0.086354	0.018538	0.018538	0.083071	-55420.7	0	1	0
Bank 3	2014	0	0.09058283	0.090583	0.021358	0.021358	0.080396	-55482.3	0	1	0
Bank 3	2015	0	0.10414472	0.104145	0.010122	0.079411	0.125377	-56193.5	0	1	0
Bank 3	2016	0	0.10252295	0.102673	0.026063	0.079819	0.123369	-56230.5	0	1	0
Bank 3	2017	0	0.10098367	0.090342	0.022682	0.080133	0.116991	-56095.4	0	1	0
Bank 3	2018	0	0.10348818	0.091021	0.022566	0.08777	0.122469	-56189.5	0	1	0
Bank 3	2019	0	0.11337098	0.101593	0.023304	0.09296	0.120472	-56332.1	0	1	0
Bank 3	2020	0	0.12730669	0.111708	0.016517	0.104204	0.099974	-56384.1	0	1	0
Bank 3	2021	0	0.12428139	0.124281	0.014566	0.100692	0.097365	-56398.1	0	1	0
Bank 1	2011	0	0.11569891	0.112069	0.319872	0.319872	0.12465	-58896.6	0	1	0
Bank 1	2012	0	0.12351976	0.121011	0.312991	0.312991	0.122228	-58917	0	1	0
Bank 1	2013	0	0.10176733	0.101767	0.217847	0.217847	0.118075	-57732.2	0	1	0
Bank 1	2014	0	0.10577355	0.105372	0.233996	0.76319	0.12191	-60838.3	0	1	0
Bank 1	2015	0	0.11377972	0.11378	0.247623	0.103275	0.131951	-57462.2	0	1	0
Bank 1	2016	0	0.11819282	0.118193	0.324745	0.10902	0.136543	-57894	0	1	0
Bank 1	2017	0	0.12009194	0.00012	0.031722	0.112278	0.136059	-55998.4	0	1	0
Bank 1	2018	0	0.12727739	0.120084	0.028976	0.120813	0.144878	-56853.4	0	1	0
Bank 1	2019	0	0.12266107	0.109396	0.035382	0.120921	0.13621	-56734.7	0	1	0
Bank 1	2020	0	0.10901439	0.102528	0.026972	0.107441	0.133022	-56482.8	0	1	0
Bank 1	2021	0	0.12996039	0.129966	0.032883	0.126102	0.122205	-56834.2	0	1	0
											0
				P(X=0)	P(X=1)						
			Intercept	-53687.1	53687.09						
			WC/TA	-6064.2	6064.197						
			ME/TL	-6017.8	6017.801						
			EBIT/TA	-4133.78	4133.784						
			RE/TA	-5445.49	5445.49						
			SALES/TA	-6172.22	6172.222						

Table 8: Logit Model 2

Bank	Year	Default	WC/TA	ME/TL	EBIT/TA	RE/TA	SALES/TA	Logit	e^(Logit)	Probability	loglikelihood
BANK 2	2011	0	0.09839665	0.108592	0.008546	0.008546	0.109757	-551000	0	1	0
BANK 2	2012	0	0.09973331	0.110782	0.007922	0.007922	0.105134	-550787	0	1	0
BANK 2	2013	0	0.12532151	0.143277	0.009305	0.017007	0.106545	-553382	0	1	0
BANK 2	2014	0	0.12723595	0.145785	0.010809	0.016646	0.107926	-553661	0	1	0
BANK 2	2015	0	0.12735129	0.145936	0.010406	0.018041	0.113151	-554052	0	1	0
BANK 2	2016	0	0.13218547	0.15232	0.324745	0.019843	0.121135	-568037	0	1	0
BANK 2	2017	0	0.12419844	0.140787	0.031722	0.031722	0.108357	-555074	0	1	0
BANK 2	2018	0	0.12156722	0.144324	0.028976	0.028976	0.115306	-555453	0	1	0
BANK 2	2019	0	0.11945843	0.126184	0.02787	0.022424	0.114902	-553933	0	1	0
BANK 2	2020	0	0.11945843	0.128406	0.022947	0.028733	0.109173	-553853	0	1	0
BANK 2	2021	0	0.1268317	0.137157	0.020829	0.020464	0.098787	-553200	0	1	0
Bank 3	2011	0	-0.0261635	0.096796	0.026184	0.026184	0.093538	-550981	0	1	0
Bank 3	2012	0	0.19112263	0.081627	0.021582	0.021582	0.092938	-549590	0	1	0
Bank 3	2013	0	0.08635421	0.086354	0.018538	0.018538	0.083071	-548973	0	1	0
Bank 3	2014	0	0.09058283	0.090583	0.021358	0.021358	0.080396	-549333	0	1	0
Bank 3	2015	0	0.10414472	0.104145	0.010122	0.079411	0.125377	-555626	0	1	0
Bank 3	2016	0	0.10252295	0.102673	0.026063	0.079819	0.123369	-556095	0	1	0
Bank 3	2017	0	0.10098367	0.090342	0.022682	0.080133	0.116991	-554837	0	1	0
Bank 3	2018	0	0.10348818	0.091021	0.022566	0.08777	0.122469	-555627	0	1	0
Bank 3	2019	0	0.11337098	0.101593	0.023304	0.09296	0.120472	-556454	0	1	0
Bank 3	2020	0	0.12730669	0.111708	0.016517	0.104204	0.099974	-556129	0	1	0
Bank 3	2021	0	0.12428139	0.124281	0.014566	0.100692	0.097365	-556453	0	1	0
Bank 1	2011	0	0.11569891	0.112069	0.319872	0.319872	0.12465	-581988	0	1	0
Bank 1	2012	0	0.12351976	0.121011	0.312991	0.312991	0.122228	-581717	0	1	0
Bank 1	2013	0	0.10176733	0.101767	0.217847	0.217847	0.118075	-571177	0	1	0
Bank 1	2014	0	0.10577355	0.105372	0.233996	0.76319	0.12191	-602032	0	1	0
Bank 1	2015	0	0.11377972	0.11378	0.247623	0.103275	0.131951	-567742	0	1	0
Bank 1	2016	0	0.11819282	0.118193	0.324745	0.10902	0.136543	-571797	0	1	0
Bank 1	2017	0	0.12009194	0.00012	0.031722	0.112278	0.136059	-552711	0	1	0
Bank 1	2018	0	0.12727739	0.120084	0.028976	0.120813	0.144878	-560826	0	1	0
Bank 1	2019	0	0.12266107	0.109396	0.035382	0.120921	0.13621	-559919	0	1	0
Bank 1	2020	0	0.10901439	0.102528	0.026972	0.107441	0.133022	-558226	0	1	0
Bank 1	2021	0	0.12996039	0.129966	0.032883	0.126102	0.122205	-560472	0	1	0
											0
				P(X=0)	P(X=1)						
			Intercept	-536871	536870.9						
			ME/TL	-60179.3	60179.34						
			EBIT/TA	-41388.4	41388.43						
			RE/TA	-54522.1	54522.05						
			SALES/TA	-61725.7	61725.72						

Table 9: Logit Model 3

Bank	Year	Default	WC/TA	ME/TL	EBIT/TA	RE/TA	SALES/TA	Logit	e^(Logit)	Probability	loglikelihood
BANK 2	2011	0	0.09839665	0.108592	0.008546	0.008546	0.109757	-54972.6	0	1	0
BANK 2	2012	0	0.09973331	0.110782	0.007922	0.007922	0.105134	-54991.3	0	1	0
BANK 2	2013	0	0.12532151	0.143277	0.009305	0.017007	0.106545	-55347.7	0	1	0
BANK 2	2014	0	0.12723595	0.145785	0.010809	0.016646	0.107926	-55380.7	0	1	0
BANK 2	2015	0	0.12735129	0.145936	0.010406	0.018041	0.113151	-55380.6	0	1	0
BANK 2	2016	0	0.13218547	0.15232	0.324745	0.019843	0.121135	-56747.7	0	1	0
BANK 2	2017	0	0.12419844	0.140787	0.031722	0.031722	0.108357	-55418.6	0	1	0
BANK 2	2018	0	0.12156722	0.144324	0.028976	0.028976	0.115306	-55412.6	0	1	0
BANK 2	2019	0	0.11945843	0.126184	0.02787	0.022424	0.114902	-55286.1	0	1	0
BANK 2	2020	0	0.11945843	0.128406	0.022947	0.028733	0.109173	-55279.1	0	1	0
BANK 2	2021	0	0.1268317	0.137157	0.020829	0.020464	0.098787	-55367.7	0	1	0
Bank 3	2011	0	-0.0261635	0.096796	0.026184	0.026184	0.093538	-54219.2	0	1	0
Bank 3	2012	0	0.19112263	0.081627	0.021582	0.021582	0.092938	-55426.5	0	1	0
Bank 3	2013	0	0.08635421	0.086354	0.018538	0.018538	0.083071	-54807	0	1	0
Bank 3	2014	0	0.09058283	0.090583	0.021358	0.021358	0.080396	-54869.8	0	1	0
Bank 3	2015	0	0.10414472	0.104145	0.010122	0.079411	0.125377	-54987.2	0	1	0
Bank 3	2016	0	0.10252295	0.102673	0.026063	0.079819	0.123369	-55034.4	0	1	0
Bank 3	2017	0	0.10098367	0.090342	0.022682	0.080133	0.116991	-54936.9	0	1	0
Bank 3	2018	0	0.10348818	0.091021	0.022566	0.08777	0.122469	-54955.7	0	1	0
Bank 3	2019	0	0.11337098	0.101593	0.023304	0.09296	0.120472	-55082.3	0	1	0
Bank 3	2020	0	0.12730669	0.111708	0.016517	0.104204	0.099974	-55199.6	0	1	0
Bank 3	2021	0	0.12428139	0.124281	0.014566	0.100692	0.097365	-55248.9	0	1	0
Bank 1	2011	0	0.11569891	0.112069	0.319872	0.319872	0.12465	-56385.3	0	1	0
Bank 1	2012	0	0.12351976	0.121011	0.312991	0.312991	0.122228	-56458.1	0	1	0
Bank 1	2013	0	0.10176733	0.101767	0.217847	0.217847	0.118075	-55817.1	0	1	0
Bank 1	2014	0	0.10577355	0.105372	0.233996	0.76319	0.12191	-55929.9	0	1	0
Bank 1	2015	0	0.11377972	0.11378	0.247623	0.103275	0.131951	-56085.3	0	1	0
Bank 1	2016	0	0.11819282	0.118193	0.324745	0.10902	0.136543	-56457.4	0	1	0
Bank 1	2017	0	0.12009194	0.00012	0.031722	0.112278	0.136059	-54547.2	0	1	0
Bank 1	2018	0	0.12727739	0.120084	0.028976	0.120813	0.144878	-55301.3	0	1	0
Bank 1	2019	0	0.12266107	0.109396	0.035382	0.120921	0.13621	-55235.5	0	1	0
Bank 1	2020	0	0.10901439	0.102528	0.026972	0.107441	0.133022	-55076.7	0	1	0
Bank 1	2021	0	0.12996039	0.129966	0.032883	0.126102	0.122205	-55393.2	0	1	0
											0
				P(X=0)	P(X=1)						
			Intercept	-53687.1	53687.09						
			WC/TA	-6064.18	6064.179						
			ME/TL	-6017.8	6017.802						
			EBIT/TA	-4133.52	4133.52						

Table 10: Logit Model 4

Bank	Year	Default	WC/TA	ME/TL	EBIT/TA	RE/TA	SALES/TA	Logit	e^(Logit)	Probability	loglikelihood
BANK 2	2011	0	0.09839665	0.108592	0.008546	0.008546	0.109757	-544226	0	1	0
BANK 2	2012	0	0.09973331	0.110782	0.007922	0.007922	0.105134	-544297	0	1	0
BANK 2	2013	0	0.12532151	0.143277	0.009305	0.017007	0.106545	-546806	0	1	0
BANK 2	2014	0	0.12723595	0.145785	0.010809	0.016646	0.107926	-546999	0	1	0
BANK 2	2015	0	0.12735129	0.145936	0.010406	0.018041	0.113151	-547068	0	1	0
BANK 2	2016	0	0.13218547	0.15232	0.324745	0.019843	0.121135	-560559	0	1	0
BANK 2	2017	0	0.12419844	0.140787	0.031722	0.031722	0.108357	-548386	0	1	0
BANK 2	2018	0	0.12156722	0.144324	0.028976	0.028976	0.115306	-548335	0	1	0
BANK 2	2019	0	0.11945843	0.126184	0.02787	0.022424	0.114902	-546841	0	1	0
BANK 2	2020	0	0.11945843	0.128406	0.022947	0.028733	0.109173	-547115	0	1	0
BANK 2	2021	0	0.1268317	0.137157	0.020829	0.020464	0.098787	-547103	0	1	0
Bank 3	2011	0	-0.0261635	0.096796	0.026184	0.026184	0.093538	-545207	0	1	0
Bank 3	2012	0	0.19112263	0.081627	0.021582	0.021582	0.092938	-543853	0	1	0
Bank 3	2013	0	0.08635421	0.086354	0.018538	0.018538	0.083071	-543846	0	1	0
Bank 3	2014	0	0.09058283	0.090583	0.021358	0.021358	0.080396	-544370	0	1	0
Bank 3	2015	0	0.10414472	0.104145	0.010122	0.079411	0.125377	-547887	0	1	0
Bank 3	2016	0	0.10252295	0.102673	0.026063	0.079819	0.123369	-548480	0	1	0
Bank 3	2017	0	0.10098367	0.090342	0.022682	0.080133	0.116991	-547615	0	1	0
Bank 3	2018	0	0.10348818	0.091021	0.022566	0.08777	0.122469	-548068	0	1	0
Bank 3	2019	0	0.11337098	0.101593	0.023304	0.09296	0.120472	-549017	0	1	0
Bank 3	2020	0	0.12730669	0.111708	0.016517	0.104204	0.099974	-549958	0	1	0
Bank 3	2021	0	0.12428139	0.124281	0.014566	0.100692	0.097365	-550443	0	1	0
Bank 1	2011	0	0.11569891	0.112069	0.319872	0.319872	0.12465	-574293	0	1	0
Bank 1	2012	0	0.12351976	0.121011	0.312991	0.312991	0.122228	-574171	0	1	0
Bank 1	2013	0	0.10176733	0.101767	0.217847	0.217847	0.118075	-563888	0	1	0
Bank 1	2014	0	0.10577355	0.105372	0.233996	0.76319	0.12191	-594506	0	1	0
Bank 1	2015	0	0.11377972	0.11378	0.247623	0.103275	0.131951	-559597	0	1	0
Bank 1	2016	0	0.11819282	0.118193	0.324745	0.10902	0.136543	-563368	0	1	0
Bank 1	2017	0	0.12009194	0.00012	0.031722	0.112278	0.136059	-544312	0	1	0
Bank 1	2018	0	0.12727739	0.120084	0.028976	0.120813	0.144878	-551883	0	1	0
Bank 1	2019	0	0.12266107	0.109396	0.035382	0.120921	0.13621	-551511	0	1	0
Bank 1	2020	0	0.10901439	0.102528	0.026972	0.107441	0.133022	-550015	0	1	0
Bank 1	2021	0	0.12996039	0.129966	0.032883	0.126102	0.122205	-552928	0	1	0
											0
				P(X=0)	P(X=1)						
			Intercept	-536871	536870.9						
			ME/TL	-60179.4	60179.41						
			EBIT/TA	-41386.8	41386.77						
			RE/TA	-54520.2	54520.22						

Table 11: Logit Model 5

Bank	Year	Default	WC/TA	ME/TL	EBIT/TA	RE/TA	SALES/TA	Logit	e^(Logit)	Probability	loglikelihood
BANK 2	2011	0	0.09839665	0.108592	0.008546	0.008546	0.109757	-54446.4	0	1	0
BANK 2	2012	0	0.09973331	0.110782	0.007922	0.007922	0.105134	-54411.9	0	1	0
BANK 2	2013	0	0.12532151	0.143277	0.009305	0.017007	0.106545	-54475.8	0	1	0
BANK 2	2014	0	0.12723595	0.145785	0.010809	0.016646	0.107926	-54488.6	0	1	0
BANK 2	2015	0	0.12735129	0.145936	0.010406	0.018041	0.113151	-54526.7	0	1	0
BANK 2	2016	0	0.13218547	0.15232	0.324745	0.019843	0.121135	-55885.2	0	1	0
BANK 2	2017	0	0.12419844	0.140787	0.031722	0.031722	0.108357	-54659.8	0	1	0
BANK 2	2018	0	0.12156722	0.144324	0.028976	0.028976	0.115306	-54676.3	0	1	0
BANK 2	2019	0	0.11945843	0.126184	0.02787	0.022424	0.114902	-54633.6	0	1	0
BANK 2	2020	0	0.11945843	0.128406	0.022947	0.028733	0.109173	-54612.3	0	1	0
BANK 2	2021	0	0.1268317	0.137157	0.020829	0.020464	0.098787	-54494.4	0	1	0
Bank 3	2011	0	-0.0261635	0.096796	0.026184	0.026184	0.093538	-54515.2	0	1	0
Bank 3	2012	0	0.19112263	0.081627	0.021582	0.021582	0.092938	-54467.5	0	1	0
Bank 3	2013	0	0.08635421	0.086354	0.018538	0.018538	0.083071	-54377.4	0	1	0
Bank 3	2014	0	0.09058283	0.090583	0.021358	0.021358	0.080396	-54387.9	0	1	0
Bank 3	2015	0	0.10414472	0.104145	0.010122	0.079411	0.125377	-54935.2	0	1	0
Bank 3	2016	0	0.10252295	0.102673	0.026063	0.079819	0.123369	-54990.9	0	1	0
Bank 3	2017	0	0.10098367	0.090342	0.022682	0.080133	0.116991	-54939.3	0	1	0
Bank 3	2018	0	0.10348818	0.091021	0.022566	0.08777	0.122469	-55014.2	0	1	0
Bank 3	2019	0	0.11337098	0.101593	0.023304	0.09296	0.120472	-55033.2	0	1	0
Bank 3	2020	0	0.12730669	0.111708	0.016517	0.104204	0.099974	-54939.9	0	1	0
Bank 3	2021	0	0.12428139	0.124281	0.014566	0.100692	0.097365	-54896.6	0	1	0
Bank 1	2011	0	0.11569891	0.112069	0.319872	0.319872	0.12465	-57520.6	0	1	0
Bank 1	2012	0	0.12351976	0.121011	0.312991	0.312991	0.122228	-57439.7	0	1	0
Bank 1	2013	0	0.10176733	0.101767	0.217847	0.217847	0.118075	-56502.7	0	1	0
Bank 1	2014	0	0.10577355	0.105372	0.233996	0.76319	0.12191	-59562.8	0	1	0
Bank 1	2015	0	0.11377972	0.11378	0.247623	0.103275	0.131951	-56087.5	0	1	0
Bank 1	2016	0	0.11819282	0.118193	0.324745	0.10902	0.136543	-56465.9	0	1	0
Bank 1	2017	0	0.12009194	0.00012	0.031722	0.112278	0.136059	-55269.4	0	1	0
Bank 1	2018	0	0.12727739	0.120084	0.028976	0.120813	0.144878	-55359	0	1	0
Bank 1	2019	0	0.12266107	0.109396	0.035382	0.120921	0.13621	-55332.5	0	1	0
Bank 1	2020	0	0.10901439	0.102528	0.026972	0.107441	0.133022	-55204.7	0	1	0
Bank 1	2021	0	0.12996039	0.129966	0.032883	0.126102	0.122205	-55264	0	1	0
											0
				P(X=0)	P(X=1)						
			Intercept	-53687.1	53687.09						
			EBIT/TA	-4133.75	4133.753						
			RE/TA	-5445.49	5445.493						
			SALES/TA	-6172.21	6172.205						

Table 12: Logit Model 6

Bank	Year	Default	WC/TA	ME/TL	EBIT/TA	RE/TA	SALES/TA	Logit	e^(Logit)	Probability	loglikelihood
BANK 2	2011	0	0.09839665	0.108592	0.008546	0.008546	0.109757	-543658	0	1	0
BANK 2	2012	0	0.09973331	0.110782	0.007922	0.007922	0.105134	-543679	0	1	0
BANK 2	2013	0	0.12532151	0.143277	0.009305	0.017007	0.106545	-545783	0	1	0
BANK 2	2014	0	0.12723595	0.145785	0.010809	0.016646	0.107926	-545942	0	1	0
BANK 2	2015	0	0.12735129	0.145936	0.010406	0.018041	0.113151	-546008	0	1	0
BANK 2	2016	0	0.13218547	0.15232	0.324745	0.019843	0.121135	-559409	0	1	0
BANK 2	2017	0	0.12419844	0.140787	0.031722	0.031722	0.108357	-547445	0	1	0
BANK 2	2018	0	0.12156722	0.144324	0.028976	0.028976	0.115306	-547022	0	1	0
BANK 2	2019	0	0.11945843	0.126184	0.02787	0.022424	0.114902	-546491	0	1	0
BANK 2	2020	0	0.11945843	0.128406	0.022947	0.028733	0.109173	-546632	0	1	0
BANK 2	2021	0	0.1268317	0.137157	0.020829	0.020464	0.098787	-546540	0	1	0
Bank 3	2011	0	-0.0261635	0.096796	0.026184	0.026184	0.093538	-537795	0	1	0
Bank 3	2012	0	0.19112263	0.081627	0.021582	0.021582	0.092938	-550531	0	1	0
Bank 3	2013	0	0.08635421	0.086354	0.018538	0.018538	0.083071	-543886	0	1	0
Bank 3	2014	0	0.09058283	0.090583	0.021358	0.021358	0.080396	-544413	0	1	0
Bank 3	2015	0	0.10414472	0.104145	0.010122	0.079411	0.125377	-547935	0	1	0
Bank 3	2016	0	0.10252295	0.102673	0.026063	0.079819	0.123369	-548519	0	1	0
Bank 3	2017	0	0.10098367	0.090342	0.022682	0.080133	0.116991	-548303	0	1	0
Bank 3	2018	0	0.10348818	0.091021	0.022566	0.08777	0.122469	-548866	0	1	0
Bank 3	2019	0	0.11337098	0.101593	0.023304	0.09296	0.120472	-549779	0	1	0
Bank 3	2020	0	0.12730669	0.111708	0.016517	0.104204	0.099974	-550956	0	1	0
Bank 3	2021	0	0.12428139	0.124281	0.014566	0.100692	0.097365	-550501	0	1	0
Bank 1	2011	0	0.11569891	0.112069	0.319872	0.319872	0.12465	-574566	0	1	0
Bank 1	2012	0	0.12351976	0.121011	0.312991	0.312991	0.122228	-574380	0	1	0
Bank 1	2013	0	0.10176733	0.101767	0.217847	0.217847	0.118075	-563936	0	1	0
Bank 1	2014	0	0.10577355	0.105372	0.233996	0.76319	0.12191	-594580	0	1	0
Bank 1	2015	0	0.11377972	0.11378	0.247623	0.103275	0.131951	-559650	0	1	0
Bank 1	2016	0	0.11819282	0.118193	0.324745	0.10902	0.136543	-563423	0	1	0
Bank 1	2017	0	0.12009194	0.00012	0.031722	0.112278	0.136059	-551588	0	1	0
Bank 1	2018	0	0.12727739	0.120084	0.028976	0.120813	0.144878	-552376	0	1	0
Bank 1	2019	0	0.12266107	0.109396	0.035382	0.120921	0.13621	-552367	0	1	0
Bank 1	2020	0	0.10901439	0.102528	0.026972	0.107441	0.133022	-550456	0	1	0
Bank 1	2021	0	0.12996039	0.129966	0.032883	0.126102	0.122205	-552989	0	1	0
											0
				P(X=0)	P(X=1)						
			Intercept	-536871	536870.9						
			WC/TA	-60644.6	60644.64						
			EBIT/TA	-41386.4	41386.42						
			RE/TA	-54521.5	54521.55						