

**THE EFFECTS OF PROBLEM-BASED LEARNING ON GRADE 11 LEARNERS'  
RETENTION OF ALGEBRAIC KNOWLEDGE AT ONE SECONDARY SCHOOL IN  
OHANGWENA REGION, NAMIBIA**

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BY  
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**APPROVAL PAGE**

This research has been examined and is APPROVED as meeting the required standards for partial fulfilment of the requirements of the degree of Masters of Education.

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## **DEDICATION**

This thesis is dedicated to my grandmothers Olivia Shihepo, my mother Lavinia Nghishiko and my beloved daughter Avo Yvonne Olivia Kaufilua for the courage and positive attitudes toward my education.

## DECLARATION


I, Tuhafeni Ilenikwaame Kaufulua, hereby declare that “*The effects of problem-based learning on grade 11 learners’ retention of algebraic knowledge at one secondary school in Ohangwena region, Namibia*” is a true reflection of my own research, and that this work, or any part thereof has not been submitted for a degree at any other institutions of higher education.

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## ABSTRACT

This study assessed the effects of Problem-Based Learning (PBL) on Grade 11 learners' retention of algebraic knowledge at one secondary school in Ohangwena region, Namibia. The study employed a quantitative research approach, a non-equivalent comparison-group quasi-experiment research design in particular. Guided by John Sweller's theory, the Cognitive Load Theory, the study was carried out in the span of six weeks. Two out of twelve class groups at the school were sampled using simple random sampling, of which the two sampled classes were further assigned as experimental and control group using probability sampling. Experimental and control groups were treated using Problem Based learning (PBL) and the traditional instruction approach respectively. The population was all the 480 learners in the 12 Grade 11 class groups at the school. All the 78 learners from the two sampled classes participated in the study. Three tests, namely the pre-test, post-test and retention test were administered during the experiment. The data collected from the pre-, post- and retention tests were analysed and the two proposed hypotheses were tested using the t-test at  $\alpha = 0.05$ . The findings of the post-test revealed that the PBL learners performed better than those taught using the traditional instruction approach. However, the difference in the post-test's mean scores between the PBL and the traditional instruction group was not statistically significant. Analyses of data collected by the retention test results, with the  $df = 76$  and  $\alpha = 0.05$ , showed the probability value,  $p = 0.00$  and  $t$  (statistics) of 3.93. Consequently, the retention test results showed a significant difference in the mean retention scores of the learners treated using PBL and those taught using traditional instruction approach. The study recommends the use of instructional methods that optimize the cognitive memory load and raise learners' interest, understanding and knowledge retention such as PBL. Hence, Ministry of Education, Arts and Culture should organise professional development activities for in-service teachers on instruction approaches of this kind.

## LIST OF CONFERENCE PROCEEDINGS

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## **LIST OF ACRONYMS**

**PBL:** Problem-Based Learning

**CLT:** Cognitive Load Theory

**WM:** Working Memory

**SDL:** Self-Directed Learning

**UREC:** UNAM Research Ethics Committee

**UNAM:** University of Namibia

**MoEAC:** Ministry of Education, Arts and Culture

## **CHAPTER 1: INTRODUCTION**

### **1.1 Background of the study**

Mathematics is essential in our everyday life; it enables learners to think logically and abstractly (Ministry of Education, 2010). Numeracy skills are used to solve their problems, such as in science, technology and commerce, thus Mathematics is made compulsory for senior secondary school learners in Namibia and a pre-requisite in almost all quantitative disciplines. There is, however, a low retention due to lack of interest, low retention as well as poor performance, as most learners have developed dislike for Mathematics (Kurumeh, Onah & Mohammed, 2012).

The Ministry of Education (2010) links the poor performance in Mathematics to the instructional approaches adopted by teachers. Furthermore, the Ministry of Education (2010) discourages learning through rote memorisation that leads to forgetting and encourages methods that ensure retention and understanding by presenting knowledge in the manner which builds on prior knowledge, experiences and reflects reality. Basically, teachers should acknowledge that learners bring along some Mathematics ideas or concepts to class and are able to construct new concepts and skills independently, however, the instructional approaches used may hinder or promote understanding and knowledge retention. Moreover, learners would find interest in learning concepts that they can apply in real-life, and this is an assumption of Problem-Based Learning (PBL).

Tarmizi and Bayat (2012) identify PBL as an effective teaching strategy that can enhance retention of numeracy skills. The PBL teaching approach, in comparison with traditional instruction method, minimises learners' misunderstandings and misconceptions of Mathematics (Kazemi & Ghoraiishi, 2012), hence improves retention.

This study assessed the effect that PBL, as a teaching and learning approach, has on senior secondary school learners' retention of mathematics (specifically algebraic) knowledge and skills.

## **1.2 Statement of the problem**

For four years as a Mathematics teacher, the researcher consistently observed that most learners could not recall much of what was learnt from the previous grades, making it difficult for the current grade teacher to teach new concepts. The researcher further observed that learners could not relate Algebra to everyday life situations and hence find it difficult to understand algebraic problems.

Despite the importance of Mathematics in quantitative disciplines, it appears that effective teaching and learning strategies are not being employed, resulting in lower Mathematics knowledge retention as well as high failure rates in the subject. However, effective teaching and learning methods that encourage learner's active participation, such as Problem Based Learning (PBL) and other learners-centred approach to learning may enhance knowledge retention and higher performance rates. Therefore, the study assessed the effects of PBL approach on senior secondary school learners' retention of algebraic knowledge and skills.

## **1.3 Research Objective and Hypothesis**

The objective of this study was to determine the effects of PBL on the Grade 11 learners' retention of algebraic knowledge and skills. The study was guided by the following hypotheses that were tested at  $\alpha = 0.05$ .

**The hypotheses of the study were as follows:**

Null hypothesis ( $H_0$ ): There is no significant difference in the mean retention scores of learners taught algebra using Problem-Based Learning (PBL) and those taught using traditional instruction approach.

Alternative hypothesis ( $H_1$ ): There is significant difference in the mean retention scores of learners taught algebra using Problem-Based Learning (PBL) and those taught using traditional instruction approach.

**1.4 Significance of the study**

The findings of this study aim to help curriculum planners and senior secondary school teachers in Ohangwena region and Namibia at large, in designing and using instructional methods that reduce the memory load as well as raise learners' interest and knowledge retention such as PBL, an approach that encourages the retention of Algebraic knowledge and thus improve the learners' performance in Mathematics. Additionally, the results may assist Mathematics teachers in understanding PBL approach to learning. Hence, the study made proper recommendations for Mathematics classroom practices.

The findings of this study also added to the literature on the topic and thus serve as a useful source for other researchers who might want to investigate the effects of PBL approach on the retention in other branches of Mathematics or subjects or in other regions.

**1.5 Limitations of the study**

The study employed a quasi-experimental non-equivalent comparison-group design, in which threats to internal validity may emerge due to the fact that the participants were already assigned to their intact classes and randomization was almost impossible.

Availability of free time to participants was also a challenge, especially during the afternoon study sessions, the time the tests were administered, since learners also had other subjects to attend to, such as those being offered on higher level. This resulted in absenteeism during the administration of the test, especially for the control group's learners and this led to only 36 learners partaking in the control group and 42 for the experimental group. Furthermore, learners in the experimental group were supposed to research further on the problems given on the tutorial sheet prior to each lesson, however due to other commitments they could not always solve all the given problems.

Another limitation was a lack of resources as the PBL group required different resources for the learners to research on their problems; however, the computer laboratory and library were not well equipped to cater for this. Hence, learners were encouraged to make use of their textbooks and hand-outs downloaded by the researcher, as well as the little resources they could find in the school library.

## **1.6 Delimitations of study**

The study strictly assessed the effects of PBL on the retention of Algebraic knowledge and skills by Grade 11 Ordinary Level Mathematics learners at one school, whereby the study compared the retention rate of PBL and traditional instruction approach. The study was conducted in one school only, hence the results of the study cannot be generalised to other secondary schools in the region or the entire country since it only focused on one school in Ohangwena. Consequently, the findings were only generalised to the Grade 11 learners at this particular school.

## 1.7 Definitions of terms

**Knowledge retention** is the “recalling or retrieving of pieces of information or knowledge or processes or skills” that is learnt sometimes in the past (Narli, 2011, p. 37). In this study, retention is referred to the ability to remember what was learnt sometimes back, for instance in previous chapters or grades.

**Working Memory**, also known as the short-term memory, is the “memory of immediate consciousness” with a limited capacity and store information for a short period (Schunk, 2012, p. 183). In this regard, the working memory is that part of the memory where information is instantly stored before it is stored permanently.

**Long-term memory** is the memory system which permanently stores a huge amount of information (De Jong, 2009). In this study, it is regarded as part of the memory responsible for future recalls.

**Problem-Based Learning (PBL)** is an “engaging instructional strategy in which students are given triggers or realistic, simulated real life problems that are ill-structured, vague or ambiguous before they experience any training in a specific content area” (Tarmizi, Tarmizi, Lojinin, & Mokhtar, 2010, p. 4684). In this study, PBL is considered as a teaching and learning approach that allows for active yet collaborative learning opportunities that are centred on solving ill-structured real-life algebraic problems through research or discovery.

**Ill-structured real-life problems** are defined as complex and poorly defined problems with either many possible solutions or many solution’s paths in which learners are required to define the problem, search and select information and skills needed to solve these problems (Hong & Kim, 2016). In this study, ill-structured problems are referred to problems without clear solution

strategies given to learners, whereby the learners through discovery of information or solution strategies, are required to determine the most appropriate solution.

**Traditional instruction** refers to a “teaching strategy in which the teacher introduces the topic or concept to be learnt, then demonstrates using worked examples on the chalkboard and later asks learners to practice in applying the principles and rules learnt by answering similar questions as shown in the worked example” (Tarmizi, Tarmizi, Lojinin, & Mokhtar, 2010, p. 4684). In this study, traditional instruction or teaching approach is referred to as the normal teaching approach that teachers are using, with a balance between teacher-centred and learner-centred strategy, in which the teacher provides learners with rules, principles as well as worked examples, and learners individually, or in groups, apply such rules and principles to other related problems without extra efforts.

**Algebra** is the branch of Mathematics which deals with variables (letters) and numbers connected by arithmetical operations (D’Emiljo, 2012). Similarly in this study, Algebra is also about working with variables and numbers connected by arithmetical operations.

## **1.8 Summary**

This chapter gave the background information of the study about the effects of problem-based learning on Grade 11 learners’ retention of algebraic knowledge at one secondary school in Ohangwena region, Namibia. The effects of problem-based learning on learners’ understanding have been studied world-wide but none has been done in Namibia. Therefore, the study aimed at testing whether there was a significant difference between the retention of algebraic knowledge by the learners learning through PBL and those using the traditional teaching approach.

## **CHAPTER 2: THEORETICAL FRAMEWORK AND LITERATURE REVIEW**

This chapter presents the theoretical framework and a review of the literature related to the study. It mainly focuses on the Cognitive Load Theory (CLT), Mathematics knowledge retention and the teaching and learning approach to Mathematics, Problem-Based Learning (PBL) in particular. The researcher did not find any local study on Problem-Based Learning, hence, international studies on PBL, retentions and CLT are reviewed.

### **2.1 Theoretical framework**

This study is guided by John Sweller's theory, known as the Cognitive Load Theory (CLT). Sweller (1994) suggests that when considering cognition; schema acquisition and automation are the core tools of learning. That is, learning occurs when knowledge and skills (schemas) are acquired and then processed and stored on to the long-term memory. Nevertheless, learners may not learn when they experience the cognitive load extremes (Sweller, 1994; Paas, Renkl & Sweller, 2004). Cognitive load extremes refer to "either excessively low load (underload) or excessively high memory load (overload)" (Paas et al., 2004), and this means that learning cannot take place when a learner experiences an underload or an overload on the working memory (WM).

The cognitive load theory theorizes on "managing the working memory (WM) loads in order to facilitate the changes in long term memory associated with schema construction and automation" (Paas et al., 2004, p. 2). De Jong (2009) indicates that unlike the long term memory, the capacity of the working memory is little and learning may not occur if the item to be learnt requires a capacity larger than that of the WM, unless instructional approaches that optimize the load of the WM are used and the cognitive load extremes are avoided. Consequently, the Cognitive Load Theory focuses on instructional control of these extreme

cognitive loads and thus enables effective learning by managing the cognitive load (Paas et al., 2004).

The Cognitive Load Theory (CLT) distinguishes between three types of cognitive load; the intrinsic, the extraneous and the germane cognitive load to name them. Intrinsic cognitive load is the load caused by the amount of information elements and their interactivity (Paas et al., 2004 & De Jong, 2009). Basically, it is the load imposed on the working memory by the level of complexity of the subject-matter. De Jong (2009) explained that subject matters with several interactive elements tend to be more difficult than those with lesser number of interactive elements (single, simple elements).

Low interactive content can be learnt separately and yet meaningful cognition will occur while in the high interactive ones, individual items may make no or little sense when learnt in isolation. In Mathematics for instance, one may learn the laws of indices separately but will find it difficult to apply these laws when simplifying, multiplying, expanding or dividing algebraic expressions. Hence, indices should be taught under algebra for the learners to link the concepts involved in the two topics. Additionally, some geometry problems (texts) may make little or no sense independent of sketches (visual-aids). Individual items in the high interactive content cannot be learnt independently, since learning such items may require a memory capacity that exceeds the capacity of the working memory and as a result learning will be hampered.

De Jong (2009) indicated that the intrinsic load does not only result from the difficulty of the subject matter but from the learner's prior knowledge as well. There is little load when learning by building on to the prior knowledge as compared to when learning an unfamiliar material. De Jong further emphasized on the premises that intrinsic load emerges from the difficulty of the

content and that instructional interventions cannot reduce or rather prevent intrinsic cognitive load. Thus, teaching methods do not directly contribute to this load.

Some works reviewed by De Jong (2009) revealed that sequencing the content either in the simple-complex or whole-part order, so that the learners do not experience the full “complexity” of content, may reduce intrinsic cognitive load (p. 107). However, both simple-complex and whole-part sequencing begin with few elements and gradually increase the complexity and thus ease the load on the working memory by integrating the elements at later stages of learning.

Despite the above stance indicating the possibility of altering intrinsic load by sequencing, sequencing still starts with learning elements in isolation and thus raises the load in early stages of learning. Hence, this study did not focus on reducing or raising intrinsic load but focused on the loads which can be adjusted by instructional approaches, the extraneous and the germane cognitive load to mention them.

The extraneous load on the other hand, is the load caused by mental activities that do not lead to learning (schema construction) (Paas et al., 2004), which as a result is imposed by the teaching materials used in the presentation of the subject matters (De Jong, 2009). This load is not essential to learning; however, since it emerges from instruction it can be altered with different instructional interventions. De Jong (2009) specifies that extraneous load results from split-attention effect, solving problems without schema-based knowledge, as well as instructional designs that use only one of the subsystems (visual or auditory) of the working memory.

De Jong (2009) refers split-attention effect “to the separate presentation of domain elements that require simultaneous processing” (p. 108). That is, dividing attention in order to do multiple things (elements), which are essential for the understanding of the material, at once. For instance, given a text and a diagram separately to solve an angle or length of a side, or simply

given the text and no diagram yet you need it. This leads to keeping one element in the working memory while searching for another element to relate to and thus impose a heavy load on the memory. However, the integration of domain elements (e.g. texts on diagrams) reduces cognitive load and facilitates learning.

The extraneous load may also emerge when learners have to solve problems in the absence of schema-based knowledge. Consequently, many elements have to be kept in the working memory and this exerts a heavy load on this memory. Concurrently, Paas et al. (2004) and De Jong (2009) suggest that working on goal-free problems, what proponents of Problem-Based Learning (PBL) call ill-structured problems, is a more effective way of acquiring difficult cognitive skills than traditional problem solving as it reduces extraneous load emerging from learning without schema knowledge.

De Jong also indicates that using only one of the subsystems of the working memory (WM) also leads to extraneous load. To be precise, when both the visual and auditory subsystems are used, little load is exerted on working memory. In contrast to the intrinsic, the extraneous load can be directly changed by instructional interventions (De Jong, 2009). Moreover, both intrinsic and extraneous loads are affected by the learners' prior knowledge on that particular contents.

Most of the work reviewed on cognitive load in this study focused on managing extraneous load. Likewise, this study focuses much on reducing the extraneous load using the Problem-Based Learning approach as an instructional intervention. De Jong (2009) on the other hand, indicated that some instructional approaches that may reduce extraneous load may simultaneously lower the germane load.

Germane load is the load caused by information and activities that contribute to schema construction and automation (Paas et al., 2004). It is the load imposed by processes that contribute to learning skills such as interpreting, demonstrating, classifying, inferring,

differentiating, and organizing (De Jong, 2009). Paas et al. (2004) indicate that a large germane load is necessary when learning. That is, the content must be presented through a medium that increases the germane load but minimizes the extraneous load.

For as long as the cognitive load is optimum and manageable then learning will still occur. Learners should therefore not waste time and resources on processes that do not contribute to schema construction and automation. Accordingly, learning only occurs if the sum of these loads does not exceed the available memory capacity of the working memory (Paas et al., 2004). Moreover, knowledge and high cognitive skills can be retained by using goal-free problems, the primary principle of problem-based learning, which is the central idea of this study.

## **2.2 Retention of learning**

Retention of learning (also referred to as knowledge retention) is the ability to recall from the memory information or knowledge or processes of skills that were learnt sometimes in the past (Narli, 2011). Based on Tileston (2000) retention implies teaching or learning for long term memory; however also indicated that every teacher had experienced a case whereby learners were unable to recall what was taught some weeks after instruction. This means that learners often forget what they learn and as a result it is not transferred to the permanent, long term memory. This could be a result of failure to optimize and manage the cognitive load on the working memory.

Learners may permanently retain information that they can remember even 12 weeks after instruction (Slavin, 2003). Therefore, Mathematics teachers should ensure that learners retain the content learnt for at least 12 weeks, so it can be transferred to the long-term memory. Furthermore, Tileston (2000) and Narli (2011) argue that knowledge retention depends on the extent to which connection is made between prior knowledge and new materials. They

emphasize that prior knowledge has an effect on new learning, and when a link is formed between prior and new learning, information transfer is made possible and easy.

Narli (2011) as well as Kurumeh, Onah and Mohammed (2012) also linked knowledge retention to the way the content is delivered to the learners. According these authors, traditional teaching such as rote learning, memorization and lecturing may lead to lack of interest, low knowledge retention and poor performance in Mathematics. Narli however indicates the existence of instructional methods which improve retention and recommends those that favor active learning and engage learners in inquiry and discovery.

### **2.3 Retention versus forgetting**

Generally, any knowledge learnt is stored in the sensory register for a short period of time then it is moved to the working (short-term) memory, where it is stored for a further short period of time before it is sent to the long-term memory, however, only when it is stored in the long term memory can future recalls be possible (Mwamwenda, 2004). From the Cognitive Learning Theory perspective, knowledge transfer is only possible when the load of the working memory is manageable, otherwise, learning will be hampered. Additionally, Slavin (2003) notes that retention drops quickly in the first few weeks of learning, but then levels off and whatever learners retained for at least 12 weeks after learning will be stored permanently.

Narli (2011) proposes that knowledge retention depends on the extent to which connection is made between the existing knowledge and new materials. Meaningfulness, explanation and organisation of the subject matter heighten the retrieval of knowledge (Schunk, 2012). Algebra for instance may be abstract to learners if meaningful elaboration and real-life applications are not offered. Lack of relevance may also result in load extremes and interfere with future knowledge retrieval. On the other hand, the knowledge that is not retained is forgotten.

Forgetting is described as the inability to recall information from the memory (Schunk, 2012). Researchers on the memory such as Arthur, Bennett, Stanush and McNelly (1998); Slavin (2003); Mwamwenda (2004); Narli (2011) and Schunk (2012) oppose the belief that information can completely disappear from the memory. They, instead link forgetting to the inactivity of retrieval cues and/or the interferences resulting from new learning. Mwamwenda (2004) for instance, indicates that forgetting happens only if “the retrieval of information is not activated by the cues” required to identify such information (p. 212). In other words, forgetting does not mean what was learnt disappears completely from the memory. Instead, it implies that cues to recall what was learnt are inactive and once they are active, information will be remembered again.

Slavin (2003) also supports Mwamwenda, that forgetting results from losing the ability to remember the information from long term memory. However, Slavin also noted that knowledge may only be transferred to long-term memory if the load extremes of the working memory are controlled. Likewise Tileston (2000), Slavin (2003) and Mwamwenda (2004) further indicate that when new learning occurs, a search of past knowledge linked to the new learning takes place in the long-term memory and if the link is found then retention occurs, otherwise, if the new learning interferes with the past knowledge then past knowledge may be forgotten (retroactive interference) and if the past knowledge interferes with the new learning then new knowledge may be forgotten (proactive inhibition or interference). Interference means the prior and new knowledge are in conflict or competition and if one is inhibited then it may be forgotten (Mwamwenda, 2004). For instance, a learner who got used to solving simultaneous algebraic equations by the substitution method may find it hard to learn to solve by elimination. That is, the old method inhibits learning of the new one and therefore the new one may be forgotten.

Arthur, Bennett, Stanush and McNelly (1998) as well as Narli (2011) argue that knowledge and skills decay occur as a result of extended periods of non-use. This simply means that people tend to forget knowledge that they have not used for some time and therefore Mathematics teachers must ensure that this does not happen if retention has to be ensured.

Based on Guthrie's theory of learning, forgetting involves new learning and it results from interference in which an alternative response is made to an old stimulus (Schunk, 2012). That is, Guthrie does not explain forgetting as a result of inactive cues but of interferences of past learning with new information and consequently wrong or different responses may be elicited. Therefore, Problem Based Learning may help in reducing excessive forgetting, whereby problems should be presented in an ill-structured and goal free manner to shorten periods of non-use and combat interferences.

## **2.4 Problem Based Learning (PBL) approach**

### **2.4.1 The overview of Problem Based Learning**

The importance of Mathematics knowledge to human life cannot be ignored, since every one of us needs some kind of Mathematics in life. The significance of Mathematics to life is evident worldwide, as Mathematics allows people to investigate, model and interpret numerical and spatial relationships and this is also why Mathematics was made compulsory in Namibian schools (Ministry of Education, 2008).

Studies conducted on Mathematics performance and retention rate found a link between poor performance and inappropriate teaching strategies as well as between low retention rate and inappropriate teaching strategies (Kurumeh, Onah & Mohammed, 2012 and Kurumeh, Obarakpo, Odoh & Ikyereve, 2016). On the same note, Gorghiu, Draghinescu, Cristea, Petrescu and Gorghiu (2015) criticised the traditional teaching due to the fact that it passively

engages learners in problem solving and focuses mostly on the provision of already constructed knowledge, whereby the learners memorise and reproduce this knowledge in assessment activities. This indicates that the traditional teaching results in learning and immediate recall. However, Gorghiu et al. (2015) propose what they also called a “discovery apprenticeship”; a problem-based learning approach (p. 1866).

Mari (2012) indicates that manipulating appropriate teaching strategies may increase the retention of knowledge in Mathematics and Science. Bearing this in mind, new effective teaching and learning strategies have to be designed and existing ones deserve recommendations. Additionally, Curriculum planners and developers worldwide advocate for a paradigm shift, from the teacher-centred to the learning-centred teaching and learning strategies. Tarmizi and Bayat (2012) identify Problem Based Learning (PBL) as an effective teaching and learning strategy that can enhance retention of numeracy skills. Hence this study focused on testing whether there is a significant difference between the retention scores of learners of learners learning through PBL and those learning through the traditional teaching methods.

Abdullah, Tarmizi and Abu (2010) discovered that learners learning Mathematics through the traditional, teacher-centred, teaching are preoccupied by exercises, algorithms and formula that need to be mastered nevertheless are of little use in other and unfamiliar situations such as solving real-life Mathematics problems. Thus, finishing the Mathematics syllabus, mastering the questions and their solutions which may result in passing the examinations are the core activities of the traditional Mathematics classroom environment. Problem based learning environment, which is learner-centred on the other hand, affords the learners with the skills to accommodate and adjust procedures to fit unfamiliar situations.

Problem-based learning (PBL) is an engaging teaching and learning approach in which learners are given “ill-structured, vague, or ambiguous” real-life problems to solve before they are provided with any sort of teaching (Tarmizi, Tarmizi, Lojinin & Mokhtar, 2010, p. 4684). Learners learning through PBL have to solve problems through research or discovery. Furthermore, PBL is a teaching and learning strategy in which learners learn through solving problems and reflecting on their experiences (Kazemi & Ghoraishi, 2012). The difference between PBL and the Traditional teaching methods is that PBL focuses on engaging learners actively in the exploration of real-life problems rather than imposing already made knowledge on learners, a traditional teaching perspective (Carriger, 2016).

The PBL learners first do self-directed and self-regulating exploration on the ill-structured problems individually before regrouping in their small groups to discuss and refine the learnt knowledge (Wood, 2003). The problems are ill-structured since learners have insufficient information from teachers to generate the solution, and are therefore required to seek the information and knowledge they need to acquire and apply in order to solve the problems (Kazemi & Ghoraishi, 2012 and Tarmizi & Bayat, 2012).

Ill-structured problems allow “free inquiry” (Carriger, 2016, p. 93). Ill-structured problems allow free inquiry because solving these problems begin with a question followed by investigating solutions, creating new knowledge as knowledge is gathered and understood, discussing discoveries and experiences, and reflecting on newly-discovered knowledge and these motivate learners to do inquiry. Kazemi and Ghoraishi (2012) further stressed that instead of generating a single answer, learners do research, discover solutions and make conclusions. PBL focuses on problem-solving, however learners seek for the knowledge required to solve the problems through research (Ribeiro, 2011).

Problem-Based Learning (PBL) has a generation effect. That is, learners retain the knowledge that they have generated better than knowledge that have been given by others for instance by teachers (Foos, Mora & Tkacz, 1994). Concurrently, Problem Based Learning was devised with a view based on constructivism because it involves learners in real world situations as well as interaction and collaboration with other learners. These are also the main features of constructivism (Ozer, 2004). PBL is centred on the assumption that learning is a process of constructing new knowledge rather than a reception of pre-given knowledge (Ribeiro, 2011). Thus, viewed from a PBL perspective, learning does not imply imposing information on others; but it is the discovery of new knowledge by the learners themselves.

Kazemi and Ghoraishi (2012) identified the following six key characteristics of Problem-Based Learning: the use of real-world problems; encouragement of learners' active participation; integration of diverse view points; encouragement of self-directed learning; encouragement of team collaboration; and enhancement of education quality.

The Problem Based Learning process entails the presentation of the problem to the learners in small groups, gathering and organization of individual ideas, evaluation of these ideas, defining the nature of the problem as well as solving it with the gathered and recreated knowledge; followed by the discussions of the problem and identification of further learning; at the same time, allocating different roles to group members as well as proposing an informed solution (Ribeiro, 2011).

#### **2.4.2 Problem Based Learning versus Constructivist learning approach**

The two learning approaches coincide in many ways. However, they also do have few differences. Both the Problem-Based Learning (PBL) and the constructivist learning approach are based upon the constructivist learning theory, constructivism (Kemp, 2001).

Constructivism contends that learners construct most of the knowledge they learn and understand (Schunk, 2012). This is, learning centred on learners and thus knowledge should be formed by learners through enquiry and hence teachers should not impose the already made knowledge on learners. Likewise constructivist learning approach, PBL also places an emphasis on knowledge construction by the learners themselves. PBL also overlap with the constructivist learning approach in the sense that they both involve collaboration and self-directed learning; teachers are facilitators; learners are researchers and learning is inquiry based; and therefore learners develop critical thinking, problem solving and collaborative skills as they identify problems, formulate hypotheses, seek for information, perform experiments, formulate solutions and make decision on the most suitable solutions (Kemp, 2011). Furthermore, both these methods require purposeful collaboration between learners within their small groups and the entire class.

These two approaches (PBL and constructivist approach) however differ in the way that PBL learners are presented with an ill-structured problem while the constructivist learners decide on the problem they want to investigate (Kemp, 2011). Additionally, the PBL learners are given an ill-structured problem that they should define and solve while constructivist learners are given a topic and they decide on what to learn and the way they want to learn.

#### **2.4.3 The teachers and learners' role in the Problem Based Learning classroom environment**

The Problem Based Learning (PBL) strategy requires learners to take responsibility of their own learning, by solving problems and reflecting on their experiences (Hmelo-Silver, 2004). They do so by actively engaging in relevant, real-life problems. Consequently, the role of the PBL learners is to actively engage in the learning process and construct own understanding under the guidance of the teacher (Kazemi & Ghorraishi, 2012). According to Abdullah, Tarmizi

and Abu (2010), PBL learners are active problem solvers, contributors and participants in group discussions. In their groups, they have to work together as a team, share the available information, Mathematical resources and notes in order to solve problems. Furthermore, PBL centres at team work, whereby learners first discover information they need in order to solve problems in small cooperative learning groups (Hmelo-Silver, 2004). Thus prior learning also has a role to play in PBL and discovery is the mean to acquire it.

Since the learners construct their own understanding, the PBL teacher does not impose knowledge on learners. Wood (2003), Hmelo-Silver (2004) and Carriger (2016) explain that the PBL teacher's duty is to facilitate learning rather than to provide knowledge. Hence, the teacher presents the ill-structured problems to learners who formulate and analyse it as well as construct the conjectures about the solutions to the problems. Likewise the constructivist teacher, the PBL teacher is a facilitator. Furthermore, the teacher is responsible for making sure that the group accomplishes the set learning objectives as well as encouraging active participation by all group members (Wood, 2003).

Based on Kazemi and Ghoraiishi (2012), the facilitator in a PBL classroom is “an expert learner, able to model good strategies for learning and thinking, rather than providing expertise in specific content” (p. 3853). Similarly, Abdullah et al. (2010) as well as Kazemi and Ghoraiishi (2012) in their studies, described the PBL teachers as facilitators of learning, who provide guidance to learners and their involvement fade away as learners take responsibility of their own learning process. Their description concurs with that of Hmelo-Silver (2004) that the PBL facilitator scaffolds learners' learning by modelling and coaching. However, these scaffoldings decline as learners become more knowledgeable in the concepts. On the same hand, the monitoring process remains the facilitator's duty even though scaffolding fades away through experience.

Another similarity that both PBL and constructivist approach have is the role of the teacher and learners. Whereby, in both these two approaches, the teacher is responsible for providing high skilled questions and creating a collaborative, problem-solving environment where learners are encouraged to make discoveries and to construct knowledge from these discoveries, by generating and testing hypotheses as well as looking for generalizations (Fears, 2008).

#### **2.4.4 Advantages of Problem Based Learning**

According to Wood (2003), Problem Based Learning (PBL) brings about knowledge acquisition and retention, good communication skills, teamwork, problem solving, active and self-directed learning, information sharing, and acknowledgement of individual differences and views. It benefits the learners both academically and socially by elevating the learners' life skills. At the same time, PBL raises learners' motivation and interest, fosters deep learning and real-life applications of knowledge, offers learners an opportunity to activate prior knowledge to construct new schema (Wood, 2003).

#### **2.4.5 The Challenges of Problem Based Learning**

Implementing Problem Based Learning (PBL) comes with challenges. These challenges need to be tackled. De Simone (2014) identified resource-intensiveness and implementation dip as major challenges to problem-based learning. These challenges are discussed in details in the sub-sections that follow.

##### **2.4.5.1 Resource-intensiveness**

Wood (2003) associates Problem Based Learning with “implications for staffing and learning resources” and demands a different timetabling approach, workload, and assessment (p. 2). Additionally, De Simone (2014) stresses that in the implementation of PBL; changes in the planning, curriculum and assessment are required. Consequently, there is need for extra

resources especially when PBL is to be offered to big groups. Furthermore, the set-up and maintenance costs, technical know-how, access to library and online PBL aids need to be considered (De Simone, 2014).

#### **2.4.5.2 Implementation dip**

Implementation dip is “a drop in performance and confidence as users (teacher in this case) encounter an innovation that requires new skills and understanding” (De Simone, 2014, p. 22). It is the incompetence that results from the teachers’ inability to cope and adjust to the newly implemented method. Despite that curriculum planners and designers as well as education policymakers are advocating for a learner centred approach to teaching and learning; most curriculum implementers (teachers) still opt to use traditional teacher-centred approaches. This could be resulting from the implementation dip imposed on the teachers by learner-centred approaches. That is, most often implementers cannot cope with the pressure and demand of the new approach.

De Simone (2014) stressed that once a new approach is introduced, PBL for instance, implementers would experience the need for more direction (the technical capacity) on how to go about the new method, the difficulties understanding their roles and roles of other stakeholders, difficulties working with others as well as the difficulties in understanding expectations. Hence, many teachers may struggle to keep up with the methods different from their usual method as a result of their inability to cope and adapt to the changes and demands in the new approach.

Once a new method or innovation is introduced, it may take longer for it to gain recognition and only after it showcase good results that it may be considered successful. However, De Simone (2014) suggests that for PBL facilitators to succeed in implementing the approach and

deal with the implementation dip, they must adapt to many roles and learn some new roles as well. This may not happen in an instant of time and may require professional development.

#### **2.4.6 Effects of Problem Based Learning (PBL) on the learning of Mathematics**

Research on Problem Based Learning (PBL) is not recent, it dates back from the 1960s (Hmelo-Silver, 2004; Kazemi & Ghoraishi, 2012). PBL has been applied to different fields, yet yielding positive effects on academic performances. For instance, in the medical school program at McMaster University where its first application took place back in 1970s (Oğuz-Ünver & Arabacıoğlu, 2011 and Tarmizi & Bayat, 2012), high school economics (Mergendoller, Maxwell & Bellisimo, 2001), high school Mathematics, statistics in particular (Abdullah et al., 2010 and Tarmizi & Bayat, 2012), university Mathematics (Kazemi & Ghoraishi, 2010), in Sciences (Gorghiu et al. 2015) as well as education management (Carriger, 2016). However, its positive effect on both performance and interest towards learning is evident (Üzel & Özdemir, 2012; Kazemi & Ghoraishi, 2012; Abdullah et al., 2010 and Carriger, 2016).

Even though it was first applied in medical schools, PBL has been studied and researched on in other fields including education, Mathematics education in particular. For instance, Abdullah et al. (2010) concluded that Problem Based Learning (PBL) is effective and learners taught using this method consistently display better Mathematics communication skills and worked together well compared to those taught using traditional teaching methods. In their study, Abdullah et al. (2010) reported that learners agreed that the PBL instructional strategy was a more effective approach in explaining difficult mathematical concepts and led them to understand the content better. However, they found no significant differences as both the PBL and Traditional teaching groups displayed positive interest and perception towards group work and valued the importance of helping others and working in teams. Additionally, Abdullah et al. (2010) posited that stimulating and engaging problems bring about a deep understanding and

skill development as compared to traditional instruction. Carriger (2016) also indicates that PBL is superior to traditional lecture. He, however, suggests that a hybrid of these two teaching methods may supplementarily yield the best learning outcomes.

There is consistence on the results of research conducted on the effects that PBL has on the learning of mathematics. Üzel and Özdemir (2012) investigated the achievements and attitudes of prospective teachers towards mathematics in problem-based e-learning and found a significant deference between the results of learners learning through Problem Based Learning (PBL) and those who were offered the traditional instruction approach to teaching and learning. In their study, they reported that the studied prospective teachers showed positive attitude towards mathematics when taught through PBL compared to those taught through traditional instruction approaches. Furthermore, their findings coincided with that of Gunhan (2006) in Üzel and Özdemir (2012) who showed that PBL approach improved learners' positive attitude towards mathematics and increased their level of knowledge.

Similarly, when they investigated the effects of a problem-based learning (PBL) approach on attitude, misconceptions and mathematics performance among university students; Kazemi and Ghoraishi (2012) found that the PBL method positively affects learners' performance and attitude toward Mathematics. Kazemi and Ghoraishi (2012) also conclude that the PBL teaching approach, in comparison with traditional methods, helps in minimising learners' misunderstandings and misconceptions of the Mathematics. Concurrently, Tarmizi and Bayat (2012) also indicate the positive influence that PBL has on the performance and teamwork skills.

In their study on the effects of PBL on the learning outcomes of an educational statistics course, Tarmizi and Bayat (2012) found a significant difference in the performance of their participants. They stated that the PBL group performed better than the traditional (conventional) group. Furthermore, Tarmizi and Bayat (2012) explained that the PBL approach afforded learners an

opportunity to work collaboratively as a team, discovering knowledge and solving real-life problems together. Their results coincide with that of Tarmizi et al. (2010). Both the former and the latter found that the PBL group out-performed the Traditional Teaching group in Mathematics tests written. Upon data analysis, the latter further found that the majority of PBL learners (62.3%) scored A-B grades against 45.9 percent of the traditional group. Hence, they concluded that the PBL approach is effective and its effects on learning are significant.

Although not the focus of this study, worth-mentioning is what Tarmizi et al. (2010) also found about how most learners prefer learning Mathematics in groups rather than individually. This is another valid reason as to why the implementation of PBL in the learning of Mathematics should be recommended. Additionally, Tarmizi et al. (2010) indicated that learners perceive PBL as an approach that enables people to discuss and clarify misconceptions as well as strengthen the development of skills transfer. Accordingly, PBL may ensure retention. Moreover, learners studied by Tarmizi et al. (2010) also expressed that PBL is an approach that offers them opportunities to do self-study and knowledge exploration, supplemented by the ability to construct own understanding, apply the learnt Mathematics concepts to new situations, and accept and appreciate different perspectives within groups.

In their study, Oğuz-Ünver and Arabacıoğlu (2011) also could not trace any negative effects of PBL and indicated that this approach allows learners to develop effective problem-solving, self-directed and lifelong learning skills. Similarly, De Simone (2014) reflected Oğuz-Ünver and Arabacıoğlu (2011) that fostering self-directed learning (SDL) and lifelong learning skills among others are key goals of PBL. Furthermore, Hmelo-Silver (2004) and De Simone (2014) are in agreement with the literature above, indicating that PBL also assist learners to construct knowledge; and acquire effective problem-solving, self-directed learning and effective collaboration skills in addition to helping them become motivated intrinsically. As a result, this approach also increases learners' interest in learning.

Hmelo-Silver, Duncan and Chinn (2007) also added to the literature of the effects of Problem Based Learning (PBL). Hmelo-Silver et al. indicated that the Problem Based Learning (PBL) learners are able to transfer the learnt knowledge to new situations, an ability that their traditionally-taught counterparts find hard to solve. PBL learners also develop a deep understanding of the knowledge they construct. However, PBL learners are more vulnerable to making errors compared to learners learning through traditional approaches (Hmelo-Silver et al., 2007). Despite their likelihood of making errors, Hmelo-Silver (2004) suggests that errors are essential when applying new knowledge; however, PBL learners stand a better chance to correct their misconceptions when correct feedback is provided. Based on Hmelo-Silver (2004) PBL learners retain much knowledge and for longer time than traditional instruction learners.

In short, the reviewed literature indicates that Problem Based Learning (PBL) is effective as it brings about better communication skills and team work among the learners, good performances as well as positive attitudes towards Mathematics. Additionally, PBL affords learners an opportunity to do self-study, knowledge exploration, construction of own understanding besides developing effective problem-solving skills as well as self-directed and life-long learning skills. Furthermore, the reviewed literature also indicates that PBL helps in minimising learners' misconceptions. However, none of the reviewed literature or study investigates the effects that PBL has on knowledge retention and transfer of Mathematics skills to long-term memory. Hence, this study intends on determining the effects of PBL on the Grade 11 learners' retention of algebraic knowledge and skills.

#### **2.4.7 Retention from both the Problem Based Learning (PBL) and Cognitive Load Theory (CLT) perspectives**

In a study carried out in Nigeria; Kurumeh et al. (2012) argue that the low performance rate in Mathematics among Nigerian learners may be due to poor retention rate and lack of interest in

the subject. They further criticise poor teaching methods such as rote learning and traditional instruction saying that these methods are some of the factors, among others, that contribute to low retention and achievement rates. However, Tarmizi and Bayat (2012) stress that Problem-Based Learning (PBL) is an “instructional strategy that may effectively increase learners’ motivation and knowledge retention” due to the fact that PBL actively focuses on the utilisation of critical thinking skills in problem solving (p. 346). In addition, PBL, in contrast with the traditional convectional teaching, affords learners chances of engaging actively in the learning process and in exploration of solutions to problems as well as developing critical thinking and making relevant connections between the content and other situations.

Managing the cognitive load of the working memory (WM) through appropriate sequencing of the content as well as using instructional interventions that do not impose too much load on the WM is the core aim of this study and an effort to ensure Mathematics knowledge retention among the Grade 11 learners. Additionally, PBL is reviewed as an instructional intervention that may ensure a manageable load and thus perhaps enhance knowledge retention.

Most of the literature reviewed centred at the effects that PBL has on performance but paid no or little attention to the effects it has on knowledge and skill retention. Hence, it is an obligation for this study to look into the matter.

## **2.5 . Summary**

This study is guided by John Sweller theory, the Cognitive Load Theory (CLT). CLT explains that learning does not occur if learners experience any kind of cognitive load extremes (underload or overload). The capacity of the working memory (WM) is limited and learning may not occur if the item to be learnt requires a capacity larger than that of the WM. However, learning may occur if instruction approaches, such as Problem-Based learning (PBL), that optimize the load of the WM are used and reduce the cognitive load extremes. Traditional

teaching such as rote learning, memorization and lecturing is criticised as it leads to lack of interest, low knowledge retention and poor performance in Mathematics. PBL on the other hand, is favoured by many such as Oğuz-Ünver and Arabacıoğlu (2011); Tarmizi et al. (2010); Tarmizi and Bayat (2012) as well as Kazemi and Ghoraishi (2010), since it actively engages learners in real-life problems.

In PBL, learners do self-directed and self-regulated exploration of the problem before regrouping in their small groups to discuss and refine the learnt knowledge. Even though it was first applied in the medical program at McMaster University in the 1970s, it has been studied and researched in other fields including education, Mathematics education in particular. However, nothing was done on the effects that PBL has on the retention of Mathematics knowledge locally. Hence, it is the responsibility of this study to investigate that. The following chapter gives the methods by which data was collected.

## CHAPTER 3: METHODOLOGY

This chapter discusses the methodology used in the study. It discusses the research design and how the research participants were selected. It further explains the instruments and methods used in collecting the data from the sample, as well as how the data was analysed. Finally, it discusses the research ethics that were adhered to, to ensure integrity.

### 3.1 Research design

Given the nature of the study, a quantitative research paradigm, a non-equivalent comparison group quasi-experimental research design in particular, was used. A quantitative research paradigm is a “systematic and objective” process which is characterised by the collection of numerical data from the selected sample to generalise the findings on the population being studied (Johnson & Christensen, 2012, p. 34). Johnson and Christensen (2012) define the quasi-experimental research design as “an experimental research design that does not provide for a full control of potential confounding variables primarily because it does not randomly assign participants to comparison groups” (p. 319).

The study adopted a non-equivalent comparison-group quasi-experimental research design because during normal school hours, randomization of learners was not possible due to the fact that learners were already assigned to different classes and hence such intact classes were used. Non-equivalent comparison-group quasi-experimental research design is a “design consisting of a treatment group and a non-equivalent untreated comparison group, both of which are administered for pre-test and post-test measures” (Johnson & Christensen, 2012, p. 321). In this study, the experimental group was offered Problem-Based Learning while the control group was treated with traditional teaching approach.

### **3.2 Population**

A population is the whole group to which a researcher intends to generalise his or her sample results (Johnson & Christensen, 2012). The population consisted of all Grade 11 Mathematics Ordinary Level learners at one secondary school in Ohangwena region. The school has twelve (12) Grade 11 Ordinary-level classes. Since Mathematics is compulsory to all Grade 11 learners, the population consisted of all the 480 Mathematics Ordinary level learners.

### **3.3 Sample and sampling procedures**

Two (2) out of twelve (12) intact classes were sampled using simple random sampling. The name of each of the 12 intact classes (A to L) was written on a separate piece of paper and these pieces of paper were placed and reshuffled in a box. Two classes were picked at random. Furthermore, one (1) of the two classes selected was assigned as the experimental and another as the control group using probability sampling. Additionally, all learners from the 2 classes participated. As a result, a total of 72 learners took part in the study.

### **3.4 Research instruments**

Fifteen lesson plans on the topic of Algebra were used for each group. Mathematics Tests set by the researcher were used to test for the learning objectives of Algebra. These tests were used as pre-, post- and retention test in the experiment. The pre-test which serves as a diagnostic test assessed what learners retained from Grade 10 Mathematics. While the post-test and retention test assessed what the learners learnt and retained from the intervention offered. All the three tests were marked by the researcher, whereby the researcher also scored part marks for correct methods for a step in the solution. Additionally, the post- and retention test were the same test, however questions were reshuffled to minimise recall of answers to questions previously seen and for the test to appear different.

### **3.5 Pilot study**

The pre-test and post-test were piloted with two (2) Grade 11 classes, other than the one sampled. This was done to determine whether the instrument would elicit the intended results. Due to time, only few pre- and post-test questions were administered. However, during the intervention provided in the pilot, it was found that the PBL group required more time than the traditional instruction group. They needed time to go research before regrouping to discuss their findings. To afford the experimental group the time they needed, tutorial sheets were prepared and distributed to the PBL learners a day prior to each lesson so that learners could do research during their free time. This was done to give these learners an opportunity to do their discovery and discuss the problems given in the tutorial sheets before regrouping to present their solutions to the rest of the class. However, each lesson duration remained forty (40) minutes for both groups. But no changes were made to the research instruments.

### **3.6 Data collection procedures**

Upon permission granted by different authorities; University of Namibia, the permanent secretary of the Ministry of Education, Arts and Culture, the Director of Education of Ohangwena region, the principal of the selected school and participants respectively; the experiment commenced. A pre-test (see appendix H) was administered to both groups to identify the learners' performances on prior knowledge, followed by an intervention, whereby the researcher taught the topic Algebra to both groups. Both the groups covered all the learning objectives included in the topic of Algebra at Namibia Senior Secondary Certificate (NSSC) ordinary level.

The experimental group was taught using Problem Based Learning (PBL) approach while the control group using the traditional instruction for three weeks. A total of fifteen (15) lessons, 40 minutes each were taught to each group on the concepts of algebraic expressions and equations. Three afternoon studies were also used for the administration of the tests, due to the fact that the tests administered required an hour or more, a period of time the class groups would not have during normal school time.

For the PBL group, a tutorial sheet prepared by the researcher on Algebra was first given to the learners, who then through discoveries solved it individually before they brought their constructions or solutions together for discussions by their self-directed groups of 3-5 participants. Secondly, these active group members discussed their constructions by writing self-explanations at each step of problem-solving. As a facilitator, the teacher then walked around the class to give guidance and pose questions to clarify problems for understanding. Each group was then afforded an opportunity to present their constructions to the rest of the class and finally the teacher provided feedback. On the other hand, the control group was offered the traditional teaching, in which the teacher solved some examples and the learners applied what they learned from examples to other related problems. Both groups then sat for a post-test (see appendix I). Finally, after the three weeks' time lapsed, the learners in both groups sat for the retention test (see appendix J).

### **3.7 Data analysis procedures**

The data collected were analysed by comparing the experimental and control groups' means of the pre-test and those of the post- and retention-test scores using the t-test in Microsoft Excel to test the null and alternative hypotheses at  $\alpha = 0.05$ .

### **3.8 Ethical considerations**

The researcher obtained an ethical clearance for conducting the study from the UNAM Research Ethics Committee (UREC), then permission to conduct the study in the selected secondary school was sought from the Ministry of Education, Arts and Culture, as well as the Ohangwena Regional Directorate of Education and the school principal of the selected secondary school. After the ethical clearance was granted by UREC, the informed consent was obtained from participants.

The data collected is and will be treated with utmost confidentiality as it will not be used for any purpose other than the study at hand. Participants were assigned pseudonyms for anonymity purposes and their identities will not be revealed under any circumstance, either on the data collection instruments or in the final thesis report as well as other publications resulting from this work. Hard copy documents and computer hardware containing raw data are and will be stored in a locked cupboard for five (5) years, before disposed by shredding hard copy documents and formatting computer hardware that contain soft copy documents with raw data. Furthermore, findings of the study are communicated by handing out one copy of the study to all stakeholders involved; Ministry of Education, Arts and Culture, UNAM, Ohangwena Education Directorate, and the school principal of the selected school to mention them.

### **3.9 Summary**

The study employed a non-equivalent comparison-group quasi-experiment quantitative research design. The study was carried out over a total six (6) weeks. Two out of 12 class groups were sampled using simple random sampling, of which one of the two sampled classes was further assigned as experimental and the other a control group using probability sampling. Experimental and control groups were treated using PBL and the traditional instruction

approach respectively. 78 learners out of 480 of the population participated. Two Grade 11 classes, other than the ones sampled, were used in the pilot study. Three tests were administered during the experiment. The data collected from the post and retention tests were analysed using a t-test to test the hypotheses.

## **CHAPTER 4: DATA ANALYSIS, INTERPRETATIONS AND DISCUSSIONS**

This chapter presents and discusses the data collected in this study. The study sought to determine the effects of Problem-Based Learning (PBL) on Grade 11 learners' understanding and retention of algebraic knowledge at one secondary school in Ohangwena region. The current chapter discusses the findings on how much learners retained from the previous grade, which is Grade 10, as well as what they retained from the intervention, which is the teaching through a PBL approach.

### **4.1. Biographical information of the participants**

#### **4.1.1. Gender**

A total of 78 Grade 11 learners took part in the study, of which 42 learners were from the PBL group and the other 36 learners were from traditional instruction group. 18 out of the 42 PBL learners were male and 24 were female. Moreover, 14 out of the 36 traditional instruction were male and 22 were female.

#### **4.1.2. Age**

The ages of the participants are summarized in the following table.

**Table 1: Ages of participants**

<b>Ages/years</b>	<b>Frequency</b>	
	<b>PBL</b>	<b>Traditional Instruction</b>
<b>17</b>	32	24
<b>18</b>	4	6
<b>19</b>	4	5
<b>20</b>	2	1
<b>Total</b>	<b>42</b>	<b>36</b>

Table 1 shows that the learners were aged between 17 and 20, of which learners who were 17 years were the most frequent, with 32 and 24 learners for the PBL and traditional instruction group respectively.

#### 4.2. Retention from previous grades as assessed by pre-test.

The learners were pre-tested to identify and bridge the gap in their previous grade algebraic knowledge and skills through interventions. A t-test could not be run on the pre-test since the objective of this study was to strictly assess the effects of Problem Based Learning (PBL) on the Grade 11 learners' retention of algebraic knowledge and skills. Nevertheless, the pre (diagnostic)-test assessed learners on Grade 10 algebraic knowledge and in spite of learners coming from different Grade 10s, whereby the teaching methods used are not known; the scores' means of both the groups on the pre-test were relatively close to one another. This can be seen in Table 1 below.

**Table 2: Group mean summary of learners' performance from the diagnostic pre-test on algebra**

Teaching and learning methods	Particints	Mean
PBL	42	61.14
Traditional instruction approach	36	62.97

On the comparison of groups' means of the pre (diagnostic) -test administered, there was a slight difference in the means. However, the mean of the control group (62.97 %) and experimental group (61.14 %) were relatively close to one another, with the control groups' mean being a little bit greater than that of the experimental group with 1.83 percent. The diagnostic test assessed what the participants retained from Grade 10 and consequently found out that on average the participants, in both groups, fairly answered the items administered in the pre-test. Despite the fact that the participants might have been enrolled in Grade 10 from different schools before meeting in Grade 11 at the selected school, they still possess adequate prior knowledge on average, resulting in an average of within a  $\pm 1$  of each other. This pre-test was administered as a surprise test to minimise the effect that studying may have on the participants' performances.

### 4.3. The effects of PBL on Grade 11 learners’ retention of algebraic knowledge at one secondary school in Ohangwena region

After the administration of the pre-test, intervention took place in the form of teaching the experimental and control groups using PBL and traditional instruction respectively. Upon completion of the intervention, both the groups sat for the same post-test. The table below show the results’ summary for the post-test as it was analysed using the t-test.

**Table 3: t-test results summary for the post test**

t-Test: Two-Sample Assuming Unequal Variances		
	<i>PBL</i>	<i>Traditional teaching</i>
Mean	42.83	34.50
Variance	498.53	268.49
Observations	42.00	36.00
Hypothesized Mean Difference	0.00	
df	76.00	
t Stat	1.90	
P(T<=t) one-tail	0.03	
t Critical one-tail	1.67	
P(T<=t) two-tail	0.06	
t Critical two-tail	1.99	

T-test for independent samples was used to analyse data in this study. Based on their book, Johnson and Christensen (2012) argued that a t-test for independent samples should be used when dealing with quantitative dependent variable (e.g. retention in this case) and a “dichotomous” independent variable (e.g. teaching method, which is Problem Based Learning (PBL) and tradition instruction in this study) (p. 503).

Table 2 above indicates that participants in the PBL (mean score = 42.83 %) class performed and retained much on average in comparison with the traditionally taught counterpart whose mean score is 34.50%. From the results on the post-test above, the researcher compared the

groups' means to determine the teaching method (PBL or Traditional instruction approach) which enhances retention immediately after intervention.

The null hypothesis states that, there is no significant difference in the mean retention scores of learners taught algebra using PBL and those taught using traditional instruction approach. However, Johnson and Christensen (2012) in rule 1 of testing the hypothesis, indicates that "if the probability (p) value is less than or equal to the significance level", the null hypothesis should be rejected and hence concludes that the finding is statistically significant (p. 502). For this study, the degree of freedom,  $df = 76$ , the sample sizes were  $n_1 = 42$  and  $n_2 = 36$  for PBL and the traditional instruction respectively, and the probability value (p) for two tails obtained was equal to 0.06. The probability value ( $p = 0.06$ ) found is greater than the significance level ( $\alpha = 0.05$ ). Hence, the study fails to reject the null hypothesis based on the post-test results.

Johnson and Christensen (2012) also state that one should reject the null hypothesis, when the value of the t (statistics) obtained is larger in one of the two tails of the t distribution. That is, when  $t(\text{statistics}) > 1.99$ . However, for this study,  $t = 1.91$  which is less than 1.99 and this further fails to reject the null hypothesis. Therefore, the researcher concludes that there is in fact no significant difference in the mean scores of learners taught Algebra using PBL and those taught using traditional instruction approach based on the post-test scores. This simply means that both the two methods are all have an immediate effect on learning. That is, if assessed immediately after instruction took place, there may not be much of the differences between the mean scores of learners taught using problem-based learning and those who are taught using the traditional instruction. Narli (2011) revealed the same finding that both the traditional and alternative instructions have similar immediate effects after interventions. In this study the PBL group has performed slightly better than the traditional learners on comparison, however the difference in the mean scores was not statistically significant.

A time of at least three weeks was allowed to elapse before learners could sit for a retention test. This retention test was the same as the post-test, however the questions were reshuffled.

The following table shows the results' summary for the retention test as it was analysed using the t-test. The table gives the analysis of data collected from the retention test of the two groups, which is used to compare the retention of learning by the learners taught using PBL and traditional instruction approaches respectively.

**Table 4: t-test results summary of the retention test**

t-Test: Two-Sample Assuming Unequal Variances		
	<i>PBL</i>	<i>traditional instruction</i>
Mean	55.33	39.64
Variance	369.45	257.32
Observations	42.00	36.00
Hypothesized Mean Difference	0.00	
df	76.00	
t Stat	3.93	
P(T<=t) one-tail	0.00	
t Critical one-tail	1.67	
P(T<=t) two-tail	0.00	
t Critical two-tail	1.99	

The retention test result scores were analysed using t-test for two sample independent variable assuming unequal variances too. With the degree of freedom,  $df = 76$ , the probability value for the two tails ( $p = 0.00$ ) is less than the significance level ( $\alpha = 0.05$ ). Furthermore, the t statistics = 3.93 which is greater than 1.99, hence the study rejects the null hypothesis based on the retention test results. Thus, there is of course a significant difference between the retention of Mathematics (Algebra) knowledge between learners learning through Problem Based Learning (PBL) and those that are learning through the traditional method of teaching.

The PBL group mean obtained is 55.33 % against the 39.64 % of their traditional counterpart. Consequently, the study revealed that learners learning through PBL retained much from their lessons than those who are learning through the traditional method. Therefore, PBL approach enhances retention of Mathematics knowledge among learners, algebraic knowledge to be more specific. This concurs with Narli (2011) that alternative teaching does of course ensure retention compared to traditional instruction. Retention of knowledge and skills is made possible when the cognitive load of the working memory is optimized. This implies that the PBL approach reduces the cognitive load of the working memory, offering an opportunity of transferring the knowledge and skills into long-term memory so that it could be stored permanently.

#### **4.4. Summary**

The group means of the pre-test showed that the two groups were relatively close in terms of prior knowledge of Algebra. The group means were 61.14% and 62.97% for PBL and the traditional instruction respectively. Hence, on average the two groups are relatively comparable, despite the fact that the groups consisted of learners who have attended Grade 10s from different schools. On the post-test, an immediate effect of PBL on retention was observed as the experimental group performed better on average than the control group. However, based on data analysis, the researcher found that the difference between the post-test's mean scores of learners taught using PBL and those of traditional instruction approach was not significant.

According to the analysis of the retention test results, the PBL group still performed better on average than its traditional counterpart. However, unlike for the post-test results, the researcher found a significant difference between the mean retention scores of the PBL and that of traditional instruction group. Hence, the study rejects the null hypothesis (no significant differences exist in the mean retention score of learners taught Algebra through the PBL method and those taught through the traditional instruction approach) and thus accepts the alternative

hypothesis, which states that there is a significant differences in the mean retention score of learners taught Algebra through the PBL method and those taught through the traditional instruction approach.

## **CHAPTER 5: CONCLUSION, RECOMMENDATIONS AND SUMMARY**

This chapter presents the conclusion, recommendations as well as summary of this study.

### **5.1 Conclusion**

In this study, both the experimental (PBL) and control (traditional instruction) group were relatively close in terms of prior knowledge, giving an average of 61.14% and 62.97% for the experimental and control group respectively. Based on the data analysis of the retention test results, the probability value,  $p = 0.00$ ; the  $t$  (statistics) obtained,  $t = 3.93$  and the significance level used,  $\alpha = 0.05$ . Since  $p < \alpha$  and  $t > 1.99$ , the study rejected the null hypothesis and accepted the alternative hypothesis. As a result, the study concludes that there is a significant difference in the retention of algebraic knowledge and skills between learners learning through PBL and the traditional instruction approach. That is, PBL learners understood and retained more algebraic knowledge and skills than their traditional fellows. Hence, PBL as a teaching and learning approach increases retention of Algebraic knowledge and skills.

The above conclusion enables a further conclusion that PBL as well as other instructional approaches that focus on working on ill-structured problems reduce the cognitive load of the working memory, hence ensuring retention and understanding as argued in the literature (Paas, Renkl & Sweller, 2004; De Jong, 2009 and Tarmizi & Bayat, 2012). On the contrary, when the content is learnt in isolation, that is one at time, which is a base to the traditional teaching; there is little connection from one item to another and thus imposing a cognitive load on the working memory (De Jong, 2009). Furthermore, this study proved that Problem Based Learning is better than teacher-centred methods because it yields better results.

## **5.2 Recommendations**

Based on the findings of this study and the conclusions made thereof, the following recommendations are made:

1. The study recommends the use of instructional methods that reduce the memory load and raise learners' interest and knowledge retention such as Problem Based Learning (PBL). Hence, teachers should focus on and exercise the use of teaching and learning methods that increase learners' interest and retention of knowledge.
2. The Ministry of Education, Arts and Culture (MoEAC) should organise professional development activities for in-service teachers on instructional approaches of this kind. MoEAC through its Mathematics Advisory Teachers should prepare and encourage the teachers on the use of PBL through workshops and short in-service training courses.
3. Since PBL learners learn through collaborative discovery which requires different books and other learning aids, MoEAC should equip school libraries to make information available.
4. Researchers on Mathematics education should do further research on instructional approaches of this kind and assess their impact on prospective teachers' practices in the field after graduation.
5. It is recommended further that research on the effects of PBL, and other teaching and learning methods at senior secondary school level, be carried out with a large sample at a national level.

### **5.3 Summary**

This study assessed the effects of Problem-Based Learning (PBL) on Grade 11 learners' retention of algebraic knowledge at one secondary school in Ohangwena region, Namibia. The study employed a quantitative research approach, a non-equivalent comparison-group quasi-experiment research design in particular. The study was guided by John Sweller theory, the Cognitive Load Theory and was carried out in the span of six weeks. Two out of twelve class groups at the school were sampled using simple random sampling, of which the two sampled classes were further assigned one as an experimental and the other a control group, using probability sampling. Both the experimental and control group were treated using PBL and the traditional instruction respectively.

Both groups were taught by the researcher, using the PBL and traditional instruction approach for the experimental and the traditional instruction for the control group respectively. The PBL learners were first given group work with ill-structured problems before they were taught. They solved the problem through discoveries, based on the explanations and examples given in their textbook, at the same time writing their self-explanation at each step of problem solving. The learners then presented their solutions as a group. This was followed by the presentation of the lesson as reinforcement of the concept learnt. The control group on the other hand, was taught using the traditional teaching approach, in which the teacher provides learners with rules, principles as well as worked examples and learners applied such rules and principles to other related problems.

The population was all the 480 learners in the twelve Grade 11 class groups at the selected school. All the 78 learners from the two sampled classes took part in the study. Three Mathematics tests on algebraic knowledge, namely the pre-test, post-test and retention test set by the researcher were administered during the experiment.

The pre-test was a diagnostic test and assessed the algebraic knowledge and skills that learners learnt from Grade 10. Moreover, the post and retention test contained the same content and assessed the algebraic knowledge learnt during the experiment. Despite the post and retention test being the same, questions in the retention test were reshuffled to change the order of questions and minimise the chances of learners recalling items in the order they have seen it previously.

The data collected from the pre, post and retention tests were analysed and the two proposed hypotheses were tested using the t-test at  $\alpha = 0.05$ . Furthermore, t-test in Microsoft Excel was used to test the null and alternative hypotheses of the study. The analysis of the post-test revealed that the difference in the mean scores of the two groups was not significant. However, analyses of data collected by the retention test results indicated that of course PBL has post effects on the retention of algebraic knowledge. Analyses of data collected by the retention test results, with the degree of freedom,  $df = 76$  and  $\alpha = 0.05$ , showed the probability value,  $p = 0.00$  and  $t$  (statistics) of 3.93. Since  $p$  value ( $0.00$ )  $< \alpha$  ( $0.05$ ) and  $t$  ( $3.93$ )  $> 1.99$  then the study rejects the null hypothesis. Hence, the retention test results showed a significant difference in the mean retention scores of the learners treated using PBL and those taught using the traditional instruction approach. Therefore, it was concluded that PBL learners retained much more knowledge than their traditional counterparts.

Based on the findings, the study recommended the following: the use of instructional methods that reduce the memory load as well as raise learners' interest and knowledge retention such as Problem Based Learning (PBL); the Ministry of Education, Arts and Culture to organise professional development workshops on instructional approaches of this kind as well as equipping school libraries. Finally, researchers in Mathematics education should assess the impact of PBL on prospective teachers' practices in the field after graduation.

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## APPENDICES

### APPENDIX A: Ethical clearance certificate from UNAM



#### ETHICAL CLEARANCE CERTIFICATE

Ethical Clearance Reference Number: FOE/315/2017      Date: 10 October, 2017

This Ethical Clearance Certificate is issued by the University of Namibia Research Ethics Committee (UREC) in accordance with the University of Namibia's Research Ethics Policy and Guidelines. Ethical approval is given in respect of undertakings contained in the Research Project outlined below. This Certificate is issued on the recommendations of the ethical evaluation done by the Faculty/Centre/Campus Research & Publications Committee sitting with the Postgraduate Studies Committee.

**Title of Project:** The Effects of Problem-Based Learning On Grade 11 Learners' Retention of Algebraic Knowledge at One Secondary School in Ohangwena Region, Namibia

**Student Name:** Tuhafeni Ilenikwaame Kaufilua

**Student Number:** 201067188

**Faculty:** Faculty of Education

**Supervisor(s):** H Miranda (Main) N Gaoseb (Co)

Take note of the following:

- (a) Any significant changes in the conditions or undertakings outlined in the approved Proposal must be communicated to the UREC. An application to make amendments may be necessary.
- (b) Any breaches of ethical undertakings or practices that have an impact on ethical conduct of the research must be reported to the UREC.
- (c) The Principal Researcher must report issues of ethical compliance to the UREC (through the Chairperson of the Faculty/Centre/Campus Research & Publications Committee) at the end of the Project or as may be requested by UREC.
- (d) The UREC retains the right to:
  - (i) Withdraw or amend this Ethical Clearance if any unethical practices (as outlined in the Research Ethics Policy) have been detected or suspected,
  - (ii) Request for an ethical compliance report at any point during the course of the research.

UREC wishes you the best in your research.

Prof. P. Odonkor: UREC Chairperson

A handwritten signature in black ink, appearing to be "P. Odonkor", written over a horizontal line.

Ms. P. Claassen: UREC Secretary

A handwritten signature in black ink, appearing to be "Paola Claassen", written over a horizontal line.

**APPENDIX B: permission from permanent secretary**

P/BAG 88008

Eenhana

Ohangwena region

13<sup>th</sup> October 2017

To: Ms S Steenkamp

The Permanent Secretary

Ministry of Education, Arts and Culture

Dear Ms Steenkamp

**Re: Request for permission to conduct research at one Secondary School in Ohangwena Region on the topic: The Effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge.**

I am a part-time student for a Master's degree in Mathematics Education at the University of Namibia. In order to complete my course, I am required to conduct a research on the topic stated above. The purpose of my research is to determine the effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge. Moreover, the study will make proper recommendations for mathematics classroom practices.

Since 2012 Mathematics was made compulsory to all Namibian learners in public schools due to the fact that Mathematics is relevant to all people, yet learners perform poorly and retain little from grade to grades or topic to topic. Additionally, literature reviewed links poor performance, low retention and low interest in the subject with the poor teaching and learning strategies.

Literature also states that the learners' Mathematics interest, performance and retention maybe improved by making the content relevant to learners' life through the use of appropriate method. Hence, I chose to investigate in the matter using Problem Based Learning (PBL).

I kindly therefore request for permission from your good office to allow me to use the selected school in Ohangwena region as my research site. I will need to conduct the study with two classes doing Mathematics ordinary level. The data will be collected using a pre-test, post-test and retention test. The lessons will be based on Algebra at ordinary level of Mathematics. I planned to complete the data collection process at the end of November 2017. The school and participants will be assured of confidentiality and anonymity both during the experiment and in the final research report. The normal class teaching time slots and intact classes will be used and will not be changed in any ways.

This research study has been cleared by the UNAM Research Ethics Committee and the ethical clearance certificate is attached. Should there be any queries about this request, please contact me at +264 81 494 3704 or my Main Supervisor Dr Miranda at +264 81 322 2449.

I look forward to hear from your good office.

Yours Sincerely,

Tuhafeni I.M. Kaufilua

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**APPENDIX C: Approval from the Permanent Secretary of the Ministry of Education,**

**Arts and Culture**



REPUBLIC OF NAMIBIA

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**MINISTRY OF EDUCATION, ARTS AND CULTURE**

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Tel: +264 61 -2933200/86  
Fax: +264 61- 2933922  
Enquiries: C. Muchila/G Munene  
Email: Cavin.Muchila@moe.gov.na/gm12munene@yahoo.co.uk

Luther Street, Govt. Office Park  
Private Bag 13186  
Windhoek  
Namibia

File no: 11/1/1

Mr. Tuhafeni I. M. Kaufulua  
Private Bag 88008  
Ohangwena  
Cell: +264 814943704

Dear Mr. Kaufulua

**SUBJECT: PERMISSION TO CONDUCT RESEARCH IN OHANGWENA REGION**

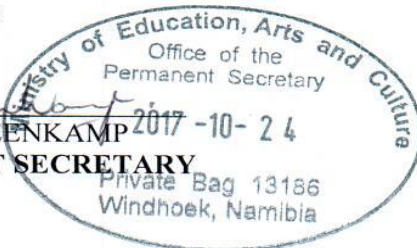
Kindly be informed that permission to conduct research for your Master's Degree in "*The effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge in Ohangwena region*" is herewith granted. You are further requested to present the letter of approval to the Regional Director to ensure that research ethics are adhered to and disruption of curriculum delivery is avoided.

Furthermore, we humbly request you to share your research findings with the ministry. You may contact Mr C. Muchila/ Mr. G. Munene at the Directorate: Programmes and Quality Assurance (PQA) for provision of summary of your research findings.

I wish you the best in conducting your research and I look forward to hearing from you soon.

Sincerely yours

  
SANET L. STEENKAMP  
PERMANENT SECRETARY



24.10.17  
Date

*All official correspondences must be addressed to the Permanent Secretary*

**APPENDIX D: letter to the director of Ohangwena region**

P/BAG 88008

Eenhana

Ohangwena region

13<sup>th</sup> October 2017

To: Mr. Hamatwi

The Director of Ohangwena Education region

Ministry of Education, Arts and Culture

Dear Mr. Hamatwi

**Re: Request for permission to conduct research at one Secondary School in Ohangwena Region on the topic: The Effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge.**

I am a part-time student for a Master's degree in Mathematics Education at the University of Namibia. In order to complete my course, I am required to conduct a research on the topic stated above. The purpose of my research is to determine the effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge. Moreover, the study will make proper recommendations for mathematics classroom practices.

Since 2012 Mathematics was made compulsory to all Namibian learners in public schools due to the fact that Mathematics is relevant to all people, yet learners perform poorly and retain little from grade to grades or topic to topic. Additionally, literature reviewed links poor performance, low retention and low interest in the subject with the poor teaching and learning strategies.

Literature also states that the learners' Mathematics interest, performance and retention maybe improved by making the content relevant to learners' life through the use of appropriate method. Hence, I chose to investigate in the matter using Problem Based Learning (PBL).

I kindly therefore request for permission from your good office to allow me to use the selected school in Ohangwena region as my research site. I will need to conduct the study with two classes doing Mathematics ordinary level. The data will be collected using a pre-test, post-test and retention test. The lessons will be based on Algebra at ordinary level of Mathematics. I planned to complete the data collection process at the end of November 2017. The school and participants will be assured of confidentiality and anonymity both during the experiment and in the final research report. The normal class teaching time slots and intact classes will be used and will not be changed in any ways.

This research study has been cleared by the UNAM Research Ethics Committee and the ethical clearance certificate is attached. Should there be any queries about this request, please contact me at +264 81 494 3704 or my Main Supervisor Dr Miranda at +264 81 322 2449.

I look forward to hear from your good office.

Yours Sincerely,

Tuhafeni I.M. Kaufilua

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**APPENDIX E: letter to the school principal**

P/Bag 88008

Eenhana

Ohangwena region

23<sup>rd</sup> July 2017

The Principal

The Selected Secondary School

P/Bag 88008

Eenhana

Dear Mr Principal

**Re: research to be conducted at the Selected Secondary School.**

I am a part-time student for a Master's degree in Mathematics Education at the University of Namibia. In order to complete my course, I am required to conduct a research on the topic stated above. The purpose of my research is to determine the effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge. Moreover, the study will make proper recommendations for mathematics classroom practices.

Since 2012 Mathematics was made compulsory to all Namibian learners in public schools due to the fact that Mathematics is relevant to all people, yet learners perform poorly and retain little from grade to grades or topic to topic. Additionally, literature reviewed links poor performance, low retention and low interest in the subject with the poor teaching and learning strategies. Literature also states that the learners' Mathematics interest, performance and retention maybe

improved by making the content relevant to learners' life through the use of appropriate method. Hence, I chose to investigate in the matter using Problem Based Learning (PBL).

I kindly therefore request for permission from your good office to allow me to use the selected school in Ohangwena region as my research site. I will need to conduct the study with two classes doing Mathematics ordinary level. The data will be collected using a pre-test, post-test and retention test. The lessons will be based on Algebra at ordinary level of Mathematics. I planned to complete the data collection process at the end of June 2018. The school and participants will be assured of confidentiality and anonymity both during the experiment and in the final research report. The normal class teaching time slots and intact classes will be used and will not be changed in any ways.

Should there be any queries about this request, please contact me at +264 81 494 3704 or my Main Supervisor Dr. Miranda at +264 81 322 2449.

I look forward to hear from your good office.

Yours Sincerely,

Tuhafeni I.M. Kaufulua

Masters' Student

University of Namibia

## **APPENDIX F: Consent form for parents**

P/Bag 88008

Eenhana

26<sup>th</sup> July 2017

Dear Parent/Guardian

Re: INFORMATION LETTER AND CONSENT FORM FOR RESEARCH STUDY TO BE CONDUCTED

### **Title of Study**

The Effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge at One Secondary School in Ohangwena Region, Namibia

Principal Investigator: Tuhafeni Kaufulua

University of Namibia

Master of Education

(Mathematics Education)

[ktimson.kaufilua@gmail.com](mailto:ktimson.kaufilua@gmail.com)

0814943704

### **Background**

Your child, \_\_\_\_\_ is being invited to take part in a research study. Before you decide to allow your child participate in this study, it is important that you understand why the research is being done and what it will involve. Please take the time to read

the following information carefully. For more information you may ask me on the contact details provided.

### **Purpose and Benefit**

The purpose of my research is to determine the effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge. The study will be conducted toward the attaining of the Masters of Education (Mathematics Education). Furthermore, the study will make proper recommendations for mathematics classroom practices. Moreover, the learner active participation in this study will be appreciated.

### **Study Procedure**

The learners will be taught using Problem-based learning approach. The expected time commitment for this study is 3 weeks, after which they may be expected to write tests.

### **Risks**

There will be no risks or harm in participating in this study. If you find anything upsetting in the study, do not hesitate to bring it to my notice. In case of unforeseeable risks, efforts will be made to minimize such risks.

### **Confidentiality and anonymity**

Confidentiality will be maintained throughout the study, since the study will only be used for academic, professional presentations as well as publications purposes. Data collected by this study anonymous through the use of pseudonyms. The name of your child will not be reported, either on the data collection instrument, research reports or any final publication.

Participants involved in this study will not be identified and their anonymity will be maintained.

### **Costs to Subject and Compensation**

There are no costs and no monetary compensation to you for your child participation in this study.

**Consent**

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions. I understand that my child's participation is voluntary and that s/he is free to withdraw at any time, without giving a reason and at no harm or cost. I understand that I will be given a copy of this consent form. I voluntarily agree on behalf of my child (a minor) to take part in this study.

Signature \_\_\_\_\_ Date \_\_\_\_\_

**APPENDIX G: Consent form for learners**

P/Bag 88008

Eenhana

26<sup>th</sup> July 2017

Dear learners

Re: INFORMATION LETTER AND CONSENT FORM FOR RESEARCH STUDY TO BE CONDUCTED

**Title of Study**

The Effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge at One Secondary School in Ohangwena Region, Namibia

Principal Investigator:       Tuhafeni Kaufulua  
University of Namibia  
Master of Education  
(Mathematics Education)

[ktimson.kaufilua@gmail.com](mailto:ktimson.kaufilua@gmail.com)

0814943704

**Background**

You, ....., being invited to take part in a research study. Before you decide to participate in this study, it is important that you understand why the research is being done and what it will involve. Please take the time to read the following information carefully. For more information you may ask me on the contact details provided.

### **Purpose and Benefit**

The purpose of my research is to determine the effects of Problem-Based Learning on Grade 11 Learners' Retention of Algebraic Knowledge. The study will be conducted toward the attaining of the Masters of Education (Mathematics Education). Furthermore, the study will make proper recommendations for mathematics classroom practices. Moreover, your active participation in this study will be appreciated.

### **Study Procedure**

You will be taught using Problem-based learning approach. The expected time commitment for this study is 3 weeks, after which you may be expected to write tests.

### **Risks**

There will be no risks or harm in participating in this study. If you find anything upsetting in the study, do not hesitate to bring it to my notice. In case of unforeseeable risks, efforts will be made to minimize such risks.

### **Confidentiality and anonymity**

Confidentiality will be maintained throughout the study, since the study will only be used for academic, professional presentations as well as publications purposes. Data collected by this study will be anonymous through the use of pseudonyms. Your name will not be reported, either on the data collection instrument, research reports or any final publication.

### **Costs to Subject and Compensation**

There are no costs and no monetary compensation to you for your participation in this study.

**Consent**

By signing this consent form, I confirm that I have read and understood the information and have had the opportunity to ask questions. I understand that my participation is voluntary and that I am free to withdraw at any time, without giving a reason and at no harm or cost. I understand that I will be given a copy of this consent form. I voluntarily agree to take part in this study.

Signature \_\_\_\_\_ Date \_\_\_\_\_

**APPENDIX H: Pre-Test (Diagnostic) in Algebra**

1. Write down the number of terms in the following algebraic expressions.

a)  $3a - ac + 2d$

..... [1]

b)  $3a - (ac + 2d)$

.....[1]

2. Simplify the following

a)  $2a - 3 - a + 2 = \dots\dots\dots$

[1]

b)  $-b + 2a - 3a = \dots\dots\dots$

[1]

3. Expand and simplify each of the following expressions.

a)  $2t^2(2t - 3st^3)$

[1]

b)  $\frac{1}{2}(6x - 2) - 3(x - 1)$

[2]

c)  $(a-b)(a+b)$

[1]

4. Express each of the following expressions as a single fraction in its simplest form

a)  $\frac{3}{a+3} + \frac{2}{a-1}$

[3]

b)  $\frac{4c}{2d} - \frac{c}{2d}$

[1]

c)  $\frac{2x}{y^3} \div \frac{4x^2}{y}$

[3]

5. A man is  $x$  years old.

a) How old will he be in 8 years from now?

[1]

b) How old was he 10 years ago?

[1]

c) How old will he be in  $x$  year time?

[1]

6. Factorize completely

(a)  $2x^2y + 16xy^3$

[1]

(b)  $xy + y - xz - z$

[2]

7. Given that  $a = 1$  and  $b = -2$ , find:

a)  $a - b$

[1]

b)  $2a^2 + b$

[2]

8. Solve the equations and inequality.

a)  $(3x+1)-(2x-7)=0$

[2]

b)  $6-3x=5x+1$

[1]

c)  $2x \leq 4$

[1]

9. Chosi is  $x$  years old. His sister is 8 years older and his brother is 12 years younger than he is. If their total age is 50 years, how old are they?

[2]

## APPENDIX I: Post-Test in Algebra

1. Simplify the following:

a)  $-2x + 6y + 3x + y$

[2]

b)  $\frac{2x^3 + 4x}{2x}$

[2]

2. Remove the brackets

a)  $(-3ab^2c^3)^2$

[2]

b)  $(x-5)^2$

[2]

3. Find the **quotient** and **remainder** when  $-2x^3 + 3x^2 - 3x + 10$  is divided  $x + 2$

[3]

4. Factorise  $6x^3 - 6x$  completely

[2]

5. Work out as a single fraction

a)  $\frac{5}{s-2} - \frac{2}{s+3}$

[2]

b)  $\frac{5}{s-2} \div \frac{15}{s^2-2s}$

[3]

6. Solve the following equations and inequality

a)  $\frac{2}{y} = \frac{3}{y-2}$

[2]

b)  $6x^3 - 6x = 0$

[2]

c)  $-2x + 4 \leq 8$

[1]

7. The formula for calculating Kinetic Energy is given by the formula,  $KE = \frac{1}{2}mV^2$ .

a) Make  $V$  the subject of the formula

[1]

b) Find  $KE$ , when  $m = 4$  and  $V = 4$

[1]

8. The distance round a rectangular field is 400 m. The length of the field is 26 m longer than the breadth.

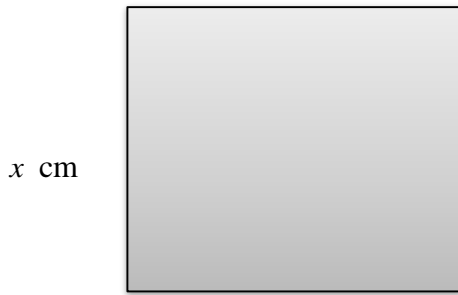
i) Calculate the length and the breadth of the field

[3]

9. In 9 years time a mother will be twice as old as her son. 3 years ago, she was four times as old as her son. Find their present ages.

[2]

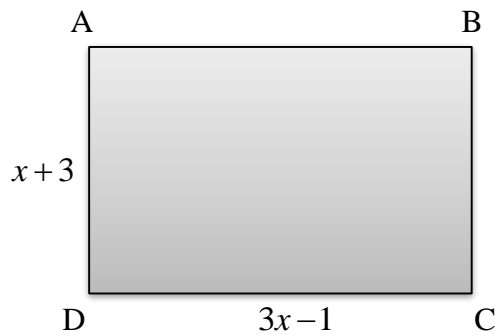
10. (a) The diagram shows a square of side  $x$  cm.



When each side of a square is reduced by 3 cm, the area is reduced by  $39 \text{ cm}^2$ . Find the length of the side,  $x$  cm, of the original square.

Answer (a)  $x = \dots\dots\dots\text{cm}$  [3]

11. **ABCD** is a rectangle in which length **AD** =  $(x + 3)$ cm and **DC** =  $(3x - 1)$ cm.



- a) If the perimeter of the rectangle is 25cm, form an equation in  $x$  and solve it.

[2]

b) **PQRS** is another rectangle in which **PQ** =  $4x$  cm and **QR** =  $x-3$  cm. the rectangle PQRS has an area which is twice the area of **ABCD**. Form an equation in  $x$  and show that it reduces to  $x^2 + 14x - 3 = 0$ . Solve this equation, **giving your answer correct to 1 decimal place.**

[5]

## APPENDIX J: RETENTION TEST IN ALGEBRA

1. Remove the brackets

a)  $(x-5)^2$

[2]

b)  $(-3ab^2c^3)^2$

[2]

2. Factorise  $6x^3 - 6x$  completely

[2]

3. Solve the following equations and inequality

a)  $\frac{2}{y} = \frac{3}{y-2}$

[2]

b)  $-2x + 4 \leq 8$

[1]

c)  $6x^3 - 6x = 0$

[2]

4. Work out as a single fraction

a)  $\frac{5}{s-2} - \frac{2}{s+3}$

[2]

b)  $\frac{5}{s-2} \div \frac{15}{s^2-2s}$

[3]

5. Simplify the following:

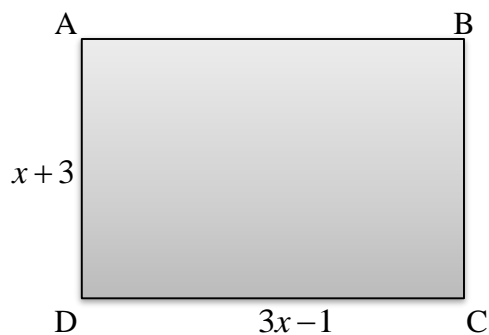
a)  $-2x + 6y + 3x + y$

[2]

b)  $\frac{2x^3 + 4x}{2x}$

[2]

6. **ABCD** is a rectangle in which length **AD** =  $(x + 3)$ cm and **DC** =  $(3x - 1)$ cm.



a) If the perimeter of the rectangle is 25cm, form an equation in  $x$  and solve it.

[2]

b) **PQRS** is another rectangle in which **PQ** =  $4x$  cm and **QR** =  $x - 3$  cm. the rectangle PQRS has an area which is twice the area of **ABCD**. Form an equation in  $x$  and show that it reduces to  $x^2 + 14x - 3 = 0$ . Solve this equation, **giving your answer correct to 1 decimal place.**

[5]

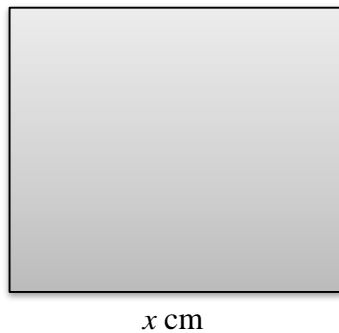
7. Find the **quotient** and **remainder** when  $-2x^3 + 3x^2 - 3x + 10$  is divided  $x + 2$

[3]

8. In 9 years' time a mother will be twice as old as her son. 3 years ago, she was four times as old as her son. Find their present ages.

[2]

9. (a) The diagram shows a square of side  $x$  cm.



When each side of a square is reduced by 3 cm, the area is reduced by  $39 \text{ cm}^2$ . Find the length of the side,  $x$  cm, of the original square.

Answer (a)  $x = \dots\dots\dots$ cm [3]

10. The formula for calculating Kinetic Energy is given by the formula,  $KE = \frac{1}{2}mV^2$ .

a) Make  $V$  the subject of the formula

[1]

b) Find  $KE$ , when  $m = 4$  and  $V = 4$

[1]

11. The distance round a rectangular field is 400 m. The length of the field is 26 m longer than the breadth.

a) Calculate the length and the breadth of the field

[3]

## APPENDIX K: MARKING SCHEMES FOR PRETEST

1. (a) 3 terms

(b) 2 terms

2. (a)  $2a - 3 - a + b = a - 1$

(b)  $-b + 2a - 3a = -b - a$  or  $-a - b$

3. (a)  $2t^2(2t - 3st^3) = 4t^3 - 6st^5$

(b)  $\frac{1}{2}(6x - 2) - 3(x - 1)$

$$= 3x - 1 - 3x + 3$$

$$= 2$$

(c)  $(a - b)(a + b)$

$$= a^2 + ab - ab - b^2$$

$$= a^2 - b^2$$

4. (a)  $\frac{3}{a+3} + \frac{2}{a-1}$

$$= \frac{3(a-1)+2(a+3)}{(a+3)(a-1)}$$

$$= \frac{3a-3+2a+6}{(a+3)(a-1)}$$

$$= \frac{5a+3}{(a+3)(a-1)}$$

(b)  $\frac{4c}{2d} - \frac{c}{d}$

$$= \frac{4c-c}{2d}$$

$$= \frac{3c}{2d}$$

(c)  $\frac{2x}{y^3} \div \frac{4x^2}{y}$

$$= \frac{2x}{y^3} \times \frac{y}{4x^2}$$

$$= \frac{1}{2xy}$$

5. (a)  $x + 8$

(b)  $x - 10 - 10$

$$x + y$$

6. (a)  $2x^2y + 16xy^3 = 2xy(x + 8y)$

(b)  $xy + y - xz - z - z$

$$= y(x + 1) - z(x + 1)$$

$$= (x + 1)(y - z)y - z$$

7.  $a - b = 1 - (-2) = 3$

(b)  $2a^2 + b = 2(1)^2 + (-2) = 2 - 2 = 0$

8. (a)  $(3x + 1) - (2x - 7) = 0$

$$3x + 1 - 2x + 7 = 0 \quad 1 - 2x + 7 = 0$$

$$x + 8 = 0 \quad 8 = 0$$

$$x = -8$$

(b)  $6 - 3x = 5x + 1$

$$6 - 1 = 5x + 3x$$

$$\frac{5}{8} = \frac{8x}{8}$$

$$\therefore x = \frac{5}{8}$$

(c)  $2x \leq 4 \rightarrow x \leq 2$

9. Chosi =  $x$  years

Sister =  $x + 8$  years

Brother =  $x - 12$  years

$$x + x + 8 + x - 12 = 50 \Rightarrow x + x + 8 + x - 12 = 50$$

$$3x - 4 = 50$$

$$\frac{3x}{3} = \frac{54}{3}$$

$$x = 18$$

$\therefore$  chosi = 18 years; sister = 26 years and brother = 6 years ears; sister=26 years  
and brother=6years

## APPENDIX L: MARKING SCHEME OF POST AND/OR RETENTION TEST

1. (a)  $-2x + 6y + 3x + y$

$$= x + 7y$$

(b)  $\frac{2x^3+4x}{2x}$

$$= \frac{2x^3}{2x} + \frac{4x}{2x}$$

$$= x^2 + 2$$

2. (a)  $(-3ab^2c^3)^2$

$$= 9a^2b^4c^2$$

(b)  $(x - 5)^2$

$$= x^2 - 10x + 25$$

3. Quotient =  $x^2 + 7x - 17$  and Remainder = 44

4.  $6x(x^2 - 1)$

$$= 6x(x - 1)(x + 1) + 1$$

5.  $\frac{5}{s-2} - \frac{2}{s+3}$

$$= \frac{5(s+3) - 2(s-2)}{(s-2)(s+3)}$$

$$= \frac{3s+19}{(s-2)(s+3)}$$

(b)  $\frac{5}{s-2} \div \frac{15}{s^2-2s}$

$$= \frac{5}{s-2} \times \frac{s^2-2s}{15}$$

$$= \frac{s}{3}$$

6. (a)  $\frac{2}{y} = \frac{3}{y-2}$

$$3y = 2y - 4 \text{ and } \therefore y = -4$$

(b)  $6x^3 - 6x = 0$

$6x(x - 1)(x + 1) = 0$

$x = 0$  or  $x = 1$  or  $x = -1$  r  $x=1$  or  $x=-1$

$-2x + 4 \leq 8$

$-2x \leq 4$

$x \geq -2$

7. (a)  $v = \sqrt{\left(\frac{2KE}{m}\right)}$

(b)  $KE = 32$

8. (i)  $400 = 2(a + 26) + 2a$

$a = 87m$  and  $\therefore$  breadth = 87m and length = 113mnd  $\therefore$  breadth=87m and length=113m

9.

	Present age	In 9 yrs time	3 yrs ago
Mother	$y$	$y + 9$	$y - 3$
Her son	$x$	$x + 9$	$x - 3$

$y + 9 = 2(x + 9)$ .....eq1

$y - 3 = 4(x - 3)$ .....eq2 solving these simultaneously give

$x = 9$  and  $y = 279$  and  $y=27$

10.  $(x - 3)^2 = x^2 - 39$

$x^2 - x^2 + 6x = 48$

$x = 8$

11. (a)  $25 = 2(x + 3) + 2(3x - 1)$

$25 = 2x + 6 + 6x - 2x-2$

$$x = \frac{21}{8}$$

(b)  $x^2 + 14x - 3 = 0$

$a = 1, b = 14$  and  $c = -3$  and  $c=-3$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = -14.2$  or  $x = 0.2$ , or  $x=0.2$ , distance cannot be negative and therefore  $x = 0.2$

## APPENDIX M: LESSON PLAN FOR PBL GROUP

### LESSON PLAN

**SUBJECT:** MATHEMATICS

**TIME:** 40 MINS

**TOPIC:** ALGEBRA

**SUBTOPIC:** PROBLEMS SOLVED BY QUADRATIC EQUATION

#### LEARNERS' PRIOR KNOWLEDGE:

- Learners are expected to know the formula used to calculate the area of shapes such as triangle, rectangle etc.
- Learners are expected to know how to solve quadratic equations given in factorized form, e.g.  $(x - 2)(x + 3) = 0$

#### BASIC COMPETENCIES:

Learners should be able to:

- solve quadratic equations by factorisation, by completing the square and by use of the formula

#### LEARNING AND TEACHING RESOURCES TO USED:

- $Y=mx+c$  Mathematics textbook
- Namcol Mathematics textbook module 1
- Any Mathematics textbook found in the school library
- Flip chart from each group
- Chalkboard

## INTRODUCTION TO THE LESSON:

- Quadratic equations and their solutions can also be applied in other branches of Mathematics such as in Mensuration and Geometry. Yet the resulting equation are solved using methods such as factorization, completing the square or quadratic formula etc..

## PRESENTATION OF SUBJECT CONTENT AND LEARNING TASKS

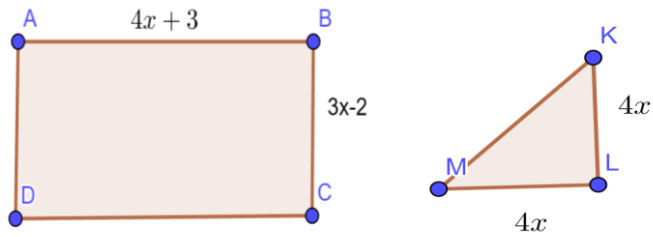
Teacher's activities	Learners activities
<ul style="list-style-type: none"><li>• teacher motivate the learners for objectives</li><li>• Teacher re-pose the questions or problems given in the previous lesson and allow the learners (<b>in their group</b>) to discuss their solutions</li><li>• Teacher facilitate the learning process, going from group to group, simplifying the problems where necessary, guiding and coaching learners where they need help and asking the learners for clarity purposes.</li><li>• Affords each group an opportunity to send a group representative to go present their solutions using the chalkboard and flip charts.</li><li>• Affords the rest of the groups an opportunity to ask questions, criticise and offer alternative solution</li><li>• Re-enforce by summarising the lesson and providing feedback</li><li>• Give assessment activity and problems for the next lesson</li></ul>	<ul style="list-style-type: none"><li>• Prior to the lesson, learners carry out independent research on the given problem before regrouping to discuss and combine their discoveries</li><li>• Ask guidance and assistance from the teacher</li><li>• A group choose a representative, who present the groups finding (solution) to the class.</li><li>• The rest of the learners ask questions and the group members of the group representing respond to these questions</li><li>• Learners also take notes once the teacher provide feedback.</li></ul>

## CONSOLIDATION OF THE LESSON:

- In case where the solutions describe dimensions that can not be negative e.g length, the solution that makes these dimensions negative is considered invalid.

**ASSESSMENT/HOMEWORK/TASKS/EXERCISE:**

The diagram shows rectangle ABCD and triangle KLM.



Given that the area of the two shapes are equal, find the length of AB, BC and KL correct to 2 d.p.? [7]

**Solution:**

$$\text{Area}(\text{rectangle } ABCD) = \text{Area}(\text{triangle } KLM)$$

$$lb = \frac{1}{2}(b)(h)$$

$$(4x + 3)(3x - 2) = \frac{1}{2}(4x)(4x)$$

$$12x^2 + x - 6 = 8x^2 \quad [1 \text{ mark}]$$

$$\text{Simplifies to } 4x^2 + x - 6 = 0 \quad [1 \text{ mark}]$$

Since the above equation cannot factorise, then solve using completing the square or quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 4, b = 1 \text{ and } c = -6$$

$$\therefore x = 1.11 \text{ or } x = -1.36$$

$x = -1.36$  (invalid since length cannot be negative and therefore  $x = 1.11$  is the only solution)

[2 marks]

$$\text{Length } AB = (4x + 3) = 4(1.11) + 3 = 7.44$$

$$BC = (3x - 2) = 3(1.11) - 2 = 1.33$$

$$KL = 4x = 4(1.11) = 4.44 \quad [3 \text{ marks}]$$