

Effective teaching practices from the perspective of Kilpatrick, Swafford and Findell's (2001) model: A video-based case study analysis of the teaching of geometry in Namibia

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Abstract

This paper presents findings from a broader PhD study that was undertaken at Rhodes University. The purpose of this qualitative case study was to explore and analyse the Geometry teaching practices of five purposefully selected secondary school teachers in Namibia who are regarded as effective mathematics teachers by the broader professional community including teachers, education ministry officials and University of Namibia lecturers. It also aimed to understand teachers' perceptions of factors that contributed towards their effective teaching of geometry. The selected case study schools where the teachers taught were representative of high performing Namibian schools in terms of learners' mathematics performance in the annual national examinations. This investigation was done through a process of classroom observations where the teachers' instructional practices were observed and analysed using an adapted model of teaching for mathematical proficiency as developed by Kilpatrick, Swafford and Findell (2001) and an enactivist theoretical perspective. The study also used open-ended and semi-structured interviews with the five participating teachers. These interviews took the form of post lesson reflective and stimulated recall analysis sessions with the participating teachers. In this paper, we only focus on the qualitative analysis of videotaped geometry lessons taught by three teachers. We analyse vignettes of selected lessons for effective teaching using the five strands of the Kilpatrick's model for proficient teaching. The analysis indicated that conceptual understanding (CU), procedural fluency (PF) and productive disposition (PD) were reflected regularly by all three teachers. However, the development of strategic competence (SC) or adaptive reasoning (AR) appeared relatively rarely. We observed many occasions where Namibian students were engaged in conceptually rich mathematical activities or invited to solve authentic problems. The tentative conclusion of the study is that the instructional practices enacted by the participating teachers, who were perceived to be effective, aligned well with practices informed by the five strands of the Kilpatrick's model.

Keywords

Teaching proficiency, geometry, mathematical proficiency (MP) strands, effective teachers, mathematical knowledge for teaching for mathematical proficiency, effective teaching practices, Enactivism.

1. Introduction

The notion of effective mathematics teaching including the learning environment inside the mathematics classroom central to this study is critical to quality mathematics education and the Namibian Vision 2030 (Namibia. Office of the president, 2004). A crucial component of effective mathematics teaching in the mathematics classroom is the way in which the teachers actively engage and interact with the students (Clarke and Clarke, 2004). Despite the bad reputation that mathematics teachers have in Namibia (Stephanus and Schäfer, 2011), there are beacons of excellence or scattered pockets of effective mathematics teaching. This study aimed to take a closer look at these beacons of excellence. The study is aimed at exploring and analysing specifically how effective mathematics teachers successfully orchestrate classroom instructional practices develop and use mathematical proficiency (MP) to teach geometry. Even though being effective may not necessarily mean that these teachers are expert or proficient in geometry, their selection as effective teachers was based on their consistent learners' high performance in national grade 10 (JSC) and 12 (NSSC) examinations over time. So, central to this investigation is the question: **What are the instructional practices of effective Namibian mathematics teachers, and what are their views on their teaching of mathematics?** We explored this main research question through the analysis of multiple cases in which selected mathematics teachers approached and handled geometric concepts, including their actions, utterances and interactions with students in their classrooms. The focus was on geometry. This is because geometry is a key area in the secondary school curriculum in which practical, real-life examples and contexts are very important. I thus assume both teachers and learners have everyday knowledge about geometry apart from mathematical knowledge or other mathematical domains.

2. Purpose and context of the study

This paper presents an initial video-based case study analysis of the teaching of geometry taught by one Namibian mathematics teacher construed locally as effective. For the purpose of this study, effective teachers are those whose learners have consistently performed well in the national mathematics examinations. Further, they are teachers who have a high standing and good reputation in the mathematics education community, including the Ministry of Education. Askew, Brown, Rhodes, William and Johnson (1997) defined effective numeracy teachers as "highly performing mathematics teachers who have knowledge and awareness of interrelations between areas of the mathematics curriculum that they teach, and their classes of pupils had, during the year, achieved a high average

gain in numeracy in comparison with other classes from the same year group” (p. 2). The latter is consistent with the selection of effective teachers in this study. Of course, these teachers were representative of effective mathematics teachers in Namibia with regard to their standing both regionally and nationally.

In order to unpack and analyse teachers’ classroom instructional practices, I chose the Kilpatrick et al.’s (2001) model of mathematical teaching proficiency (MTP) as a conceptual framework and analytical tool. The five strands of the Kilpatrick’s model represented the learning outcomes that were observably inferable from the videotapes focused on teachers’ actions, utterances and interactions with students in classroom instruction. The broader study also drew on elements of an enactivist worldview as a theoretical vantage point. As a useful extension of constructivism, enactivism (Maturana and Varela, 1987) was used to complement the Kilpatrick teaching model in order to provide a rich and powerful analytical tool of analysis of teaching practices of effective teachers. For the purpose of this paper, however, the enactivist dimensions will not be discussed as I wish to focus on Kilpatrick’s et al. (2001) strands of MTP as a framework for analysing teachers’ practice. The conceptual and analytical framework underpinning this study is described below.

3. Conceptual and theoretical framework

The presentation of this paper is framed against an adapted model of Kilpatrick et al.’s (2001) five strands of teaching for mathematical proficiency (MP), which framed the broader study as an analytical tool. The Kilpatrick framework helped me to conceptualise the various dimensions of effective teacher practice of teaching mathematics. Specifically, we adapted Kilpatrick et al.’s (2001) five strands of teaching for MP to analyse teacher practice and facilitate our understanding of teaching characteristics and uniqueness of the Namibian teachers that allow confidence of their reputation. The Kilpatrick (2001) framework builds on Shulman’s (1987) dimensions of general pedagogical models of teaching competence. I found this model useful to analyse effective teachers’ teaching proficiency because it was based on the notion of mathematical proficiency - a theoretical concept that is easily operationalised. The Kilpatrick et al.’s (2001: 380) conceptual and analytical framework entails five interwoven and interdependent strands of MTP that guided both the data collection and data analysis. These are:

- *Conceptual understanding* (CU) of core knowledge that encourages comprehension of concepts, operations and relations as required in the practice of teaching;
- *Procedural fluency* (PF) in carrying out basic instructional routines;
- *Strategic competence* (SC) in planning effective instruction and solving problems that arise during instruction;

- *Adaptive reasoning* (AR) in justifying and explaining one’s instructional practices and in reflecting on those practices so as to improve them, and
- *A productive disposition* (PD) towards Mathematics, the teaching, the learning and the improvement of practice.

The five strands of the Kilpatrick’s framework of teaching for MP guides my thinking and research into what selected effective teachers do within their classroom instructional practices in the context of their effective mathematical actions with learners. Specifically, the study focused on how selected mathematics teachers approached and handled certain concepts including their utterances that shape the mathematical ideas at play in classroom interactions.

4. Research methodology and data source

This study is oriented within the qualitative framework, and is anchored within an interpretive paradigm. An interpretive case study research design was employed, involving five selected Namibian mathematics teachers, in order to investigate and deconstruct teachers’ geometric instructional practices “within its real-life context” of their mathematics classrooms and to gain “intensive, holistic description and analysis” (Yin 2003: 13). The Ministry of Education’s (MoE) archived statistical records of learners’ performance in the Junior Secondary Certificate (JSC) and Namibia Senior Secondary Certificate (NSSC) mathematics examinations in the last three years were used to select five teachers in order to analyse their classroom instructional practices.

Sampling was carried out in two stages. In the first stage of the broader study, a purposive sampling (Creswell, 2007) was used to select 10 Namibian secondary school mathematics teachers, who had consistently achieved the top results in the Grade 10 and 12 national examinations for the last three years (2008-2010). The second stage which is the focus of this paper, involved contacting these 10 teachers and inviting five volunteers to participate in my study. In order to secure the participation of the five teachers, I also selected them on the basis of (1) their voluntary participation and willingness to share teaching practice and experiences and (2) their qualifications.

A variety of data gathering methods were used in order to generate an account of the teachers’ practice. The primary data set comprised of classroom lesson observations and video recordings of teachers’ geometry instructional practices. A video camera was used to capture all classroom utterances and actions made by the teachers as well as their interactions with the students. Thus the data reported in this paper included the field notes made in classrooms and transcriptions of lesson videos from one mathematics classroom.

5. Participants

My sample consisted of two males and three females. Even though gender was not a central factor in this research, it is important to point out that the dominance of female mathematics teachers in my sample of participating teachers was neither purposive nor deliberate, and may not be representative of mathematics teachers in Namibia. In this study, the five mathematics teachers and their respective schools have been categorised and coded using pseudonyms. For example, Teacher 1 is referred to as Demis of school A, teacher 2 as Jisa of school B, teacher 3 as Ndara of school C, teacher 4 as Emmis of school D and teacher 5 as Sann of school E. My five case study schools are spread across five regions in Namibia, namely, two (B and C) in the northern regions, two (A and D) in central regions and one (E) in the coastal regions.

Table 1 below summarises information about the participating teachers.

School	School type	Teacher and Name	Sex	Age	Levels of study	Teaching experience in years
A	Public	T1: Demis	Female	> 40	Gr.12, BSc, HED	20
B	Private	T2: Jisa	Female	30-40	Gr.12,B.Sc, MSc, Bed	10
C	Public	T3: Ndara	Male	30-40	Gr. 12, B. Sc, MSc	6
D	Private/Day	T4: Emmis	Male	> 40	Gr. 12, BEd, Med	15
E	Private/Day	T5: Sann	Female	30-40	Gr. 12, HED	8

Table 1: Summary of case study participants and their schools

This paper reports only on geometry lessons taught by one female teacher of particular interest, namely Demis. The table shows that Demis had teaching experience of 20 years in different schools and contexts.

6. Procedures for data analysis

The data analysis was inseparable from the data collection. Data was analysed using descriptive narratives (Cohen, Manion and Morrison, 2007). Data from lesson videos were viewed several times, transcribed verbatim, and thereafter were coded based on the aforementioned concepts of an adapted Kilpatrick's (2001) model of teaching for MP. Thereafter, lesson videos of selected sequences were scrutinised several times and then

colour coded to look for *similarities* and *differences*. This was done in order to get a feel for how different lessons played out and to render a description of teachers' classroom instructional practice in relation to their teaching proficiency as well as their broader identity as effective teachers of mathematics. Accordingly, selected lesson vignettes that show evidence of the five strands of MP were used to elucidate the evidence warranting identification of how Demis teachers addressed development of particular strands of MP in the students. The results section represents the negotiated consensus of the author and the supervisor with regard to how the classroom observation data were coded and categorised.

6.1 The Kilpatrick's classroom observation coding instrument

Table 2 below shows the Kilpatrick's classroom observation coding instrument used to analyse the lesson video transcriptions. We generated a set of observable indicators (codes) that represent a realistic reconstruction of the five strands of MTP. Each strand is recognised by phrases indicating observable indicators that describe identification of classroom interactions and show how each strand was exemplified.

Table 2: Summary of analytical framework for analysing lessons [Note: All fifteen lessons observed were subjected to this analytical tool (Adapted from Kilpatrick et al., 2001)].

The five mathematical teaching proficiency strands of adapted Kilpatrick model	Code	As defined by Kilpatrick et al. (2001: 380)	Observable indicators of the FIVE strands of mathematical teaching proficiency The Teacher:
1. Conceptual Understanding	CU	<p>Conceptual understanding can be thought of as part of the “knowing why” of mathematical knowledge. Understanding of core knowledge or teaching that encourages comprehension of mathematical concepts, operations and relations as required in the practice of teaching. Competency in this strand is defined in terms of “being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes”.</p>	<p>CU1: uses mathematically appropriate and comprehensive definitions and language CU2: provides accurate explanations of concepts that are useful to learners (e.g. teacher explains ideas or procedure to deepen the learners’ conceptual understanding or elaborate on geometric concepts under considerations or related concepts i.e. understanding of locus, angles, theorem, quadrilateral properties through problem solving) CU3: emphasises the links or connections between different geometric concepts, ideas, such as the interrelationships of properties of shapes CU4: makes reference to interesting contexts, giving/using examples from learners’ contexts that they can relate to. CU5: makes links to learner’s prior knowledge. CU6: makes conceptual links to other areas of Mathematics such as algebra CU7: engages learners in tasks that are conceptually rich Non examples: Giving explanation of a concept in a form of telling and does not make conceptual links (showing a lower or insufficient conceptual understanding with some errors), and asking questions that require learners to guess what teachers is asking.</p>

<p>2. Procedural Fluency</p>	<p>PF</p>	<p>Procedural fluency can be thought of as part of the “knowing when and how” of mathematical knowledge, i.e. quick recall and accurate execution of procedures and “fluency in carrying out basic instructional routines”. Teaching that encourage knowledge or skills of rules, algorithms, or procedures used to solve mathematical problems or tasks flexibly, accurately, efficiently and appropriately in different ways and contexts (knowledge of step-by-step instructions that prescribe how to complete a task). Knowledge of when and how to use procedures appropriately and skills in performing them efficiently and accurately.</p>	<p>PF1: asks learners questions that solicit the procedures for solving a problem or the next step in the process of solving mathematical tasks or problems under discussion. PF2: asks learners to explain and justify their answers or methods for solving problems. PF3: explains procedures and provides algorithms to solve mathematical problems at stake (i.e. how a procedure should be used) PF4: elaborates on solving procedures suggested by learners. PF5: encourages learners to use mathematical formulae, procedures and techniques accurately PF6: encourages learners to use mathematical formulae, procedures and techniques correctly PF7: encourages learners to use mathematical formulae, procedures and techniques appropriately PF8: encourages multiple procedures in solving problems to develop learners’ procedural fluency and understanding of geometric concepts Non examples: Performing strict procedures or algorithmic type of problem that have no connections to the underlying geometric concepts or ideas and lacking justification or explanations (call on learners to recall procedures, fact or definition).</p>
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3. Strategic Competence

SC

Strategic competence can be thought of as part of the “knowing ‘what’ to teach, ‘when’ and ‘how’ to teach it”. Teacher’s ability in planning effective instructions and instructional activities as well as formulating, representing and solving problems that arise during instructions. Competency in this strand is linked to what commonly called problem solving and problem formulation. A theme characteristic in the above conception is the heuristics embedded in the problem solving process.

SC1: plans the lesson carefully
SC2: provides tasks/activities that allow multiple solving strategies and evaluation of different solution method strategies.
SC3: emphasise and encourages learners’ engagement with the solution of non-routine tasks i.e. formulating own theorems in circle geometry
SC4: encourages learners to discuss and solve problems collaboratively
SC5: asks probing questions for learners to reflect critically and provide critical reasoning and argumentation (to move the lesson on).
SC6: represents ideas carefully using multiple representations/notations such as mapping graphical, symbolic representations, algebraic notations and pictures.
SC7: engages with expected and unexpected learners’ mathematical ideas and suggestions
Non examples: Using (or forcing students to use) a particular problem solving strategy to solve routine problem rather than developing strategies with students by allowing them to determine a way to solve non-routine problems that make sense to them

<p>4. Adaptive Reasoning</p>	<p>AR</p>	<p>Reasoning in justifying, explaining one's instructional practices and reflecting on those practices so as to improve them. Teaching that encourages or emphasises capacity for logical thought, for reflection on, explanation of and justification of mathematical arguments. That is, teacher's capacity to think logically about the relationship among concepts and situations, and ability to consider and select from alternatives.</p>	<p>AR1: provides situations and activities that require logical reasoning. AR2: asks questions that solicit learners to explain or justify their solution strategies. Examples: why are two angles equal, how you did that, why, why are you saying so, what would be the reason, how did you come to that answer to encourage the learners' development and articulation of justification and argumentation. AR3: engages with learners that encourages reflection. AR4: encourages learners to think deductively. AR5: provides learners with geometric activities that require and emphasise deductive reasoning AR6: invites constructive criticism and feedback from learners. Non examples: Explanations of concepts or procedures are not immediately followed with informal proof, justification and/or deductive reasoning. Learners are not encouraged to consistently explain or justify their answers or claims.</p>
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5. Productive Disposition

PD

Disposition towards Mathematics, teaching, learning and the improvement of practice. Teaching that encourage or instil habitual inclination to see senses in Mathematics, to perceive it as both useful and worthwhile subject to be learned, coupled with a belief in the value of diligent work and in one's own efficacy, to believe that steady effort in learning Mathematics pays off, and to see oneself as an effective learner and doer of Mathematics.

PD1: provides learners with homework tasks to encourage learners to do Mathematics outside of the classroom.
PD2: encourages and affirms learners
PD3: adopts an attitude with learners that making mistakes in the math classroom is OK
PD4: teaches with passion and enthusiasm
PD5: expects learners to be enthusiastic
PD6: expects learners to work hard
PD7: motivates learners by making the lessons interesting with hands-on activities.
PD8: has high expectations of learners
PD9: enforces the notion that everybody has a contribution to make in the math classroom.
Non examples: Have a negative disposition a lot of the time, e.g., displays anger, aloofness, and sarcasm. Does not praise effort and performance or encourage learners to persist or persevere, and constantly seeking the correct answers from certain learners.

7. Results and discussion

Observably the mathematics instructions observed in the classroom of the teacher of interest represent the daily teaching practices of the effective Namibian mathematics teachers, and captured well the mathematical discourse patterns that dominated their instructions. The data analyses and respective lesson videos indicated that Demis focused a considerable attention on the development of her students' conceptual understanding (CU), procedural fluency (PF) and productive disposition (PD). Though evidence emphasising or addressing strategic competence (SC) and adaptive reasoning (AR) appeared relatively rare, it was interesting to note development of these strands in my study. In what follows, I identify "uniqueness" in ways in which Demis developed the five strands of MP in teaching geometry and offer some recommendation based on teaching evidence in the videotaped lessons. The selected vignettes illustrate the ways in which Demis interacted with the students.

7.1 The development of students' conceptual understanding (CU)

Demis's first videotaped lesson on circle geometry revolved around measurement of plane shapes. Demis, in her introduction, addressed the development of students' CU of perimeter and area in a variety of ways entailing, for instance, questioning, exposition and whole class reflections on two dimensional combined shapes. To begin with the first activity and to ensure that learners were using correct mathematical terminologies, Demis opened the lesson by asking if students had an idea of what the "perimeter" and "area" are. The following transpired.

- 1 Demis: Tell me quickly, what is the perimeter? (CU5)
- 2 Student 1: The perimeter is a length around the figure.
- 3 Demis: Excellent (PD2), the perimeter is a distance around the figure (CU1). And what is the area?
- 4 Student 2: Area is the space inside.
- 5 Demis: That is good (PD2), area is the inside space of the figure or boundary. (CU1; CU2)

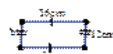
This exchange showed evidence of CU, with some elements of PD at play. This was conceptual in nature because Demis engaged the learners with a clear introduction of two key concepts, namely perimeter and area. Demis asked questions that solicited previously learned definitions of perimeter and area in order to obtain clarity about what students knew (lines 1 and 3). Also, in lines 3 and 5, Demis provided accurate explanations of ideas and terminologies that were useful to learners (CU2). While Demis challenged her learners to articulate their mathematical ideas (line 1 and 3), she also used mathematically appropriate and comprehensible definitions and language (CU1) to explain the relationship between the perimeter and the area. In this way, Demis demonstrated fluency in mathematical language as she provided precise and accurate explanations of the difference between perimeter and area in that a perimeter is the distance around the figure (line 3) while the area is the inside space of the figure (line 5).

With regard to elements of PD within this vignette, Demis affirmed the learners' responses by saying "excellent" (line 3) and "that is good" (line 5), which is PD2; to create a positive productive disposition towards mathematics or interest in the mathematical ideas they were engaged with. This finding resonates strongly with Kilpatrick et al. (2001) who claim that students are more likely to hold productive dispositions (autonomy, belief that mathematical competence is malleable rather than fixed) in a mathematics classroom in which the teacher transfers responsibility to students, solicits multiple solution strategies, provides process scaffolding and presses for conceptual understanding (CU).

7.2 The development of students' procedural fluency (PF)

Indeed, concepts alone do not make mathematics, and a considerable amount of lesson time was spent on addressing students' procedural skills (PF). During the whole-class reflection, Demis focused learners' attention on the concept of perimeter that demanded procedural knowledge, to move the lesson on and get learners ready for the next phase of the learning process. For example, she asked learners to determine the procedure for finding the perimeters of the rectangle that they cut out from the graph sheet. The first stage of the procedure involved counting the square blocks on the square grid paper where the rectangle was drawn. This conversation went as follows.

12 Demis: [*While moving around the class*] I can see some people have numbered the blocks which is a good thing. [*She then drew a rectangle on the board*] okay, I just want to know, what is your distance here, how many square blocks or centimetres are here [*pointing to the length side*]? (PF1)



13 Students: 16 cm (*length*).

14 Demis: 16 cm so basically the perimeter is 16cm plus...what are the blocks here [*width*]? (PF3)

15 Students: 12cm

16 Demis: [*Writing on the board*] so we can say $P=2l+2b$ (PF3). Now your perimeter will be two times 16cm plus two times 12cm, (CU2) that is equal to $2(16cm)+2(12cm)$, so that will be $32cm+24cm$ (PF4).

People, can you think about algebra, is this like or unlike terms (PF6; CU6)?

In the above interaction, the strand of PF dominated this vignette, but CU was also evident. In line 12, Demis asked questions that solicited the procedure for determining the perimeter of a rectangle, which is PF1. She also explained and elaborated on the procedure suggested by the learners (PF4) and then focused their attention on the number of square blocks to determine the dimensions of a rectangle (line 16). As she did this, Demis offered accurate mathematical explanations to give meaning to the ideas and procedures under discussion. For example, she explained how the formula or procedure should be used and why the solution method makes sense (line 16). Here, Demis encouraged her learners to use mathematical procedures and formulae accurately and appropriately (PF6) as she explained and showed them how the procedures should be used by making conceptual links or reference to algebra, which is CU6 (line 16). This further indicated Demis's

attention to the development of CU as she was able to provide accurate mathematical explanations, which is CU2, to give meaning to formula, steps and procedures (line 16). Such explicit explanations enabled students to exhibit skills of procedural fluency when they determined both the dimensions and perimeter of a rectangle as they were working on the given task.

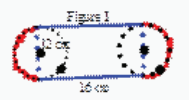
7.3 The development of students' strategic competence (SC)

During her second lesson, Demis then focused students' attention on mathematical procedures, thereby forging a conceptual link to the conventional formula for determining the perimeter and area of constructed shapes. The following episode shows how Demis engaged her class in a procedure-focused discussion in which all five strands of MP were evident.

20 Demis: It is always good people especially to structure your working plan. When you are doing geometry, it is very, very important that you present your work nicely. The perimeter of this combined figure 1 will be $P = l + \frac{1}{2} \text{ circle} + l + \frac{1}{2} \text{ circle}$ (SC6). So one length plus another length will be equal to what? (PF1)

21 Students: two lengths plus a circle.

22 Demis: $P = 2l + 1 \text{ circle}$ (PF3).



The perimeter of the circle is $2\pi r$. Okay, people remember like what I always say let us put the brackets around the variables to make our working easy. $2\pi r = 2(\pi)(r)$ Can you still remember this?

$\Rightarrow P = 2l + 1 \text{ circle}$ (PF3) $\Rightarrow P = 2(l) + 2(\pi)(r)$ (PF5) $\Rightarrow P = 2() + 2()$ (PF6; PF7).

Just to get your work nicely done, you do it like this.

This third vignette, typical of longer sequence of interactions, suggests that the focus had shifted to address students' SC, allowing multiple solving strategies and evaluation of different solution strategies. In line 20, for instance, Demis engaged learners in a discussion to find ways to devise a working plan, represent ideas carefully using multiple representations and notations (i.e. symbolic representations, algebraic notations, formulae) (SC6) and to solve the problem. In her attempt to explain the procedures (PF3) and find the formula that allowed them to find the perimeter of combined figures, Demis showed a flexible approach to problem solving in explicit ways (line 22). For instance, she represented the perimeter formula carefully using multiple representations such as symbolic representations and algebraic notations, which show SC6 (line 22). Within this vignette, Demis showed high facility with SC in formulating, representing and solving mathematical problems. This is because her main focus was to explain and structure the solving working plan, formulate the perimeter formula expression based on the rectangle and circle, and substitute into the formula to work out the perimeter of the geometric shapes.

7.4 The development of students' adaptive reasoning (AR)

During her third lesson on trigonometry, Demis provided learners with several tasks that required and emphasised deductive reasoning (AR). The task in the extract below involved a right-angled triangle with two side lengths given and required learners to find an angle. The conversation that elicited or showed adaptive mathematical reasoning (AR) went as follows.

31 Demis: [Task 1] you get 53.1° . If someone writes this answer $\tan A = \frac{4}{3}$ (adj/hyp) like this $\tan A = \frac{4}{3} = 53.1^\circ$, is this answer mathematically correct? **(AR3)**
32 Student: Not correct.
33 Demis: Why not? This answer 53.1° is correct. But why is writing the answer $\tan A = \frac{4}{3} = 53.1^\circ$ not correct? **(AR2; AR3)**
34 Student: Because of the equal sign [=].
35 Demis: Very good. Listen people. If you just write equal sign it means that you say $\tan A = 53.1^\circ$. And $\tan A$ is indicating that ratio which is equal to $(\frac{4}{3})$. So, if you just still keep on writing equal sign (=), you still say it is $\tan A = 53.1^\circ$, which is incorrect because $\tan A$ is not equal to 53.1° . It is angle A which is equal to 53.1° [$A = 53.1^\circ$]. Okay, do not just write the equal sign; rewrite the angle to show that it is equal to that answer. **(CU2)**

The interaction showed evidence of AR. That is, Demis was able to provide students with worthwhile mathematical tasks that elicited mathematical reasoning. She also engaged learners through high level questioning that encouraged reflections and required learners to explain and justify their solution strategies which are AR3 (line 31). In line 33, interaction further displayed a high level of CU and PF. For example, the teacher confirmed learners' answers by providing accurate explanations of concepts, which is CU2 (line 35). This discussion helped to clarify the distinction between opposite and adjacent sides. It also encouraged learners to use their mathematical reasoning to justify their answers, and use procedures efficiently and appropriately.

7.5 The development of students' productive disposition (PD)

Another important strand of MP that was evident in this lesson was that of PD. That is, towards the end of the lesson, Demis provided positive feedback for learners to see the worth of the lesson (line 31). She also assigned a homework task to encourage learners to do mathematics outside of the classroom, which is PD1 (line 31). The homework task was purposely selected to allow learners to understand the concepts and the way they appear in national examinations.

31 Demis: okay, people we are going to continue with this task tomorrow. But before we go, check here, we said the perimeter is the distance around the figure. So, the perimeter of figure 1 will be: $P = \text{halfcircle} + \text{length} + \text{halfcircle} + \text{length}$. The perimeter of figure 2 will be: $P = \text{length} + \text{halfcircle} + \text{length} + \text{halfcircle}$. That is exactly the same. Please check your homework **(PD1)** on page 220. Okay, good bye class.

8. Concluding comments

The purpose of this study was to investigate and analyse the Geometry teaching practices of purposefully selected secondary school teachers in Namibia, and provide evidence of the ways in which their classroom practice and teaching proficiency supported the development students' MP, as effective teachers of mathematics. The analysis of Demis' lesson videos indicated that she was strongly oriented towards conceptual and procedural teaching in mathematics. Demis flexibly moved learners from concrete to more abstract representations in teaching geometry. She also demonstrated mathematical procedures with examples for learners to grasp the algorithms and techniques. There were opportunities for development of AR and SC which Demis engaged with through carefully selected mathematics tasks or problems that could produce multiple ways of seeing relationships and connections among geometric concepts and ideas. These mathematical tasks acted as a lead in for using different strategies for solving high level demanding problems. The strand of PD emerged as a "character trait in the service of the teacher's mathematics teaching" (p. 380), and was addressed in virtually every lesson. PD was the strand that holds other four strands of MP together as emerged in the lesson videos. In many cases, much evidence was seen of Demis presenting or emphasising students' engagement with the solution of non-routine or authentic mathematical tasks. Hence, the analysis of lesson videos allowed me to conclude tentatively that despite the fact that Demis clearly taught at conceptual and procedural levels, evidence strongly showed that it was possible for her to teach in a classroom environment where all five strands of MP were manifested and reinforced each other.

9. Recommendation

On the strength of the findings from this study, I encourage both prospective and practising teachers to structure classroom learning activities so that all five strands are emphasised and synchronised. A starting point might be to use the Kilpatrick's classroom observation instrument/checklist descriptors developed for this study (Table 2) to evaluate their own teaching practices in the mathematics classroom. Peer assessment of one's own instructional practice using the lesson observation tool is an effective form of research that allows teachers to take a central role as an investigator of their own classroom practices and become autonomous thinkers/researchers of teaching and learning in the classroom. As research demonstrated strongly the truth and usefulness of Kilpatrick's position in this regard.

10. Acknowledgement

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